CCAUV/19-44

WORKSHOP OF THE CONSULTATIVE COMMITTEE FOR ACOUSTICS, ULTRASOUND AND VIBRATION

ACOUSTIC TRANSFER ADMITTANCE OF CYLINDRICAL CAVITIES IN INFRASONIC FREQUENCY RANGE

P. VINCENT, D. RODRIGUES, F. LARSONNIER, C. GUIANVARC'H, S. DURAND

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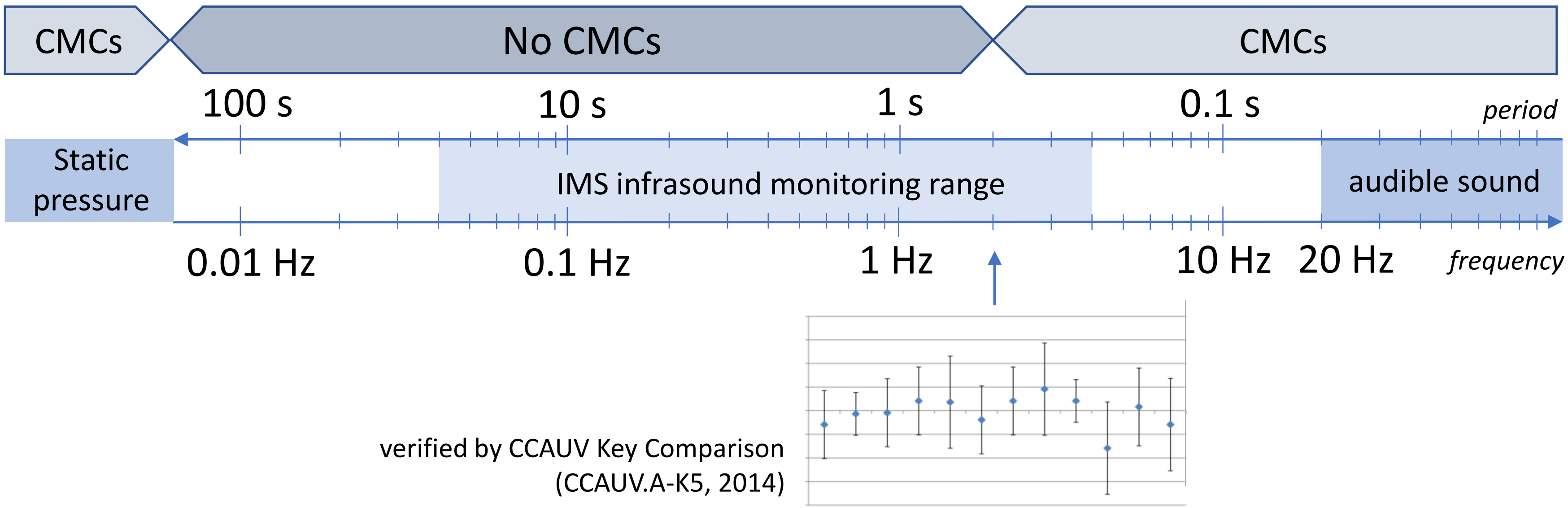
LABORATOIRE NATIONAL DE MÉTROLOGIE ET D'ESSAIS

INTRODUCTION

State of the art

Traceability

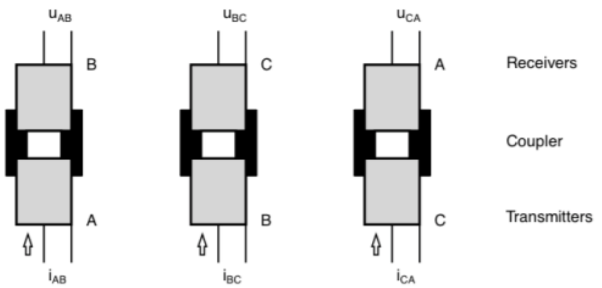
- The “reference” for conventional acoustic measurements is established by the primary calibration of laboratory standard microphones using the reciprocity method; IEC 61094-2:2009.
- Current CMCs (BIPM KCDB) in primary standards for sound pressure have a lower limiting frequency of 2 Hz.



INTRODUCTION

Pressure reciprocity method: The infrasound challenge

Heat conduction modelling



- IEC 61094-2 standard provides 2 formulations:
  a) Broad band solution
  b) Low frequency solution

- Significant behavioral differences between the standardised models at very low frequencies have been highlighted (E. Sandermann Olsen, R. Jackett).

$$\underline{M}_{p,1}\,\underline{M}_{p,2} = \frac{1}{\underline{Z}_{a,12}}\,\frac{\underline{U}_2}{\underline{i}_1}$$

- These discrepancies yield inconsistent calibration results.

Need for an alternative and accurate low frequency formulation

SUMMARY

- **Introduction**
- **Validation of acoustic transfer admittance formulations**
  - Methodology
  - Measurement Setup
- **Results**
  - Measurement processing
- **Acoustic transfer admittance: Model presentation**
  - IEC 61094-2:2009: The `broadband solution'
  - IEC 61094-2:2009: The `low-frequency solution'
  - Alternative solutions
- **Conclusion & recommendations**

VALIDATION OF ACOUSTIC TRANSFER ADMITTANCE FORMULATIONS

Methodology

Based on the reciprocity method

- Use of 2 cavities of different lengths
- Product of sensitivities:

$$M_t M_r |_s = Z_{e,s} Y_{a,s}$$
$$M_t M_r |_\ell = Z_{e,\ell} Y_{a,\ell}$$

- By considering the microphones as stable during the experiment, the products of the sensitivities should be invariant as functions of the cavity, insofar as the models of the acoustic transfer admittances are perfectly valid.
- Error estimator:

$$\delta_m = \frac{M_t M_r |_s}{M_t M_r |_\ell} = \frac{Z_{e,s} Y_{a,s}}{Z_{e,\ell} Y_{a,\ell}} \xrightarrow{\text{Insert voltage not required}} \delta_m = \frac{u_{r,s} / u_{t,s}\, Y_{a,s}}{u_{r,\ell} / u_{t,\ell}\, Y_{a,\ell}}$$

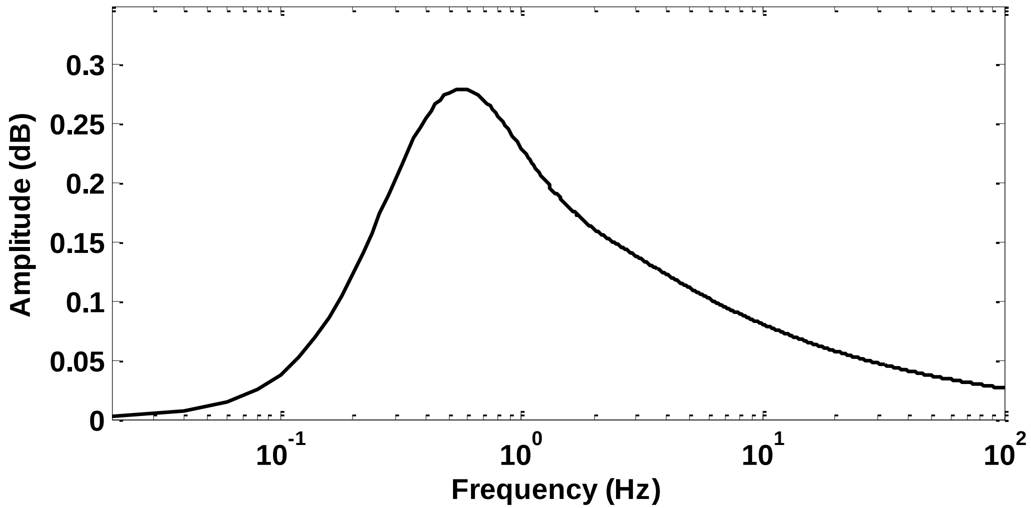
- The error estimator should tend towards unity (or 0 dB) for a perfect model of the acoustic transfer admittances.

VALIDATION OF ACOUSTIC TRANSFER ADMITTANCE FORMULATIONS

Measurement setup

Cavities

- The cavity lengths (6 mm & 10 mm) were chosen to be sufficiently different to allow measurement of the deviation between the thermal corrections.



Microphones

- 2 Brüel & Kjær Type 4160

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VALIDATION OF ACOUSTIC TRANSFER ADMITTANCE FORMULATIONS

Features of the measuring setup

- **Impacting factors in the validation process**

$$Y_T = \underbrace{Y_c}_{\text{Coupler}} + \underbrace{Y_r + Y_t}_{\text{Receiver \& transmitter microphones}} + \underbrace{\vartheta(f)}_{\text{Leakage}}$$

This study focuses only on the validation of the formulation for the admittance of the coupler.

Difference of heat conduction effects in Y_C reach 0.3 dB at the maximum for the chosen coupler dimensions.

Uncertainties due to unknowns in Y_r, Y_t and leakages shall be reduced to an order of magnitude << 0.3 dB

VALIDATION OF ACOUSTIC TRANSFER ADMITTANCE FORMULATIONS

Features of the measuring setup

- **Impacting factors in the validation process**

$$Y_T = Y_c + Y_r + Y_t + \cancel{\vartheta(f)}$$

a) The cavities were designed with gaskets to ensure optimal sealing conditions.

b) The back cavity vents of both microphones were sealed.

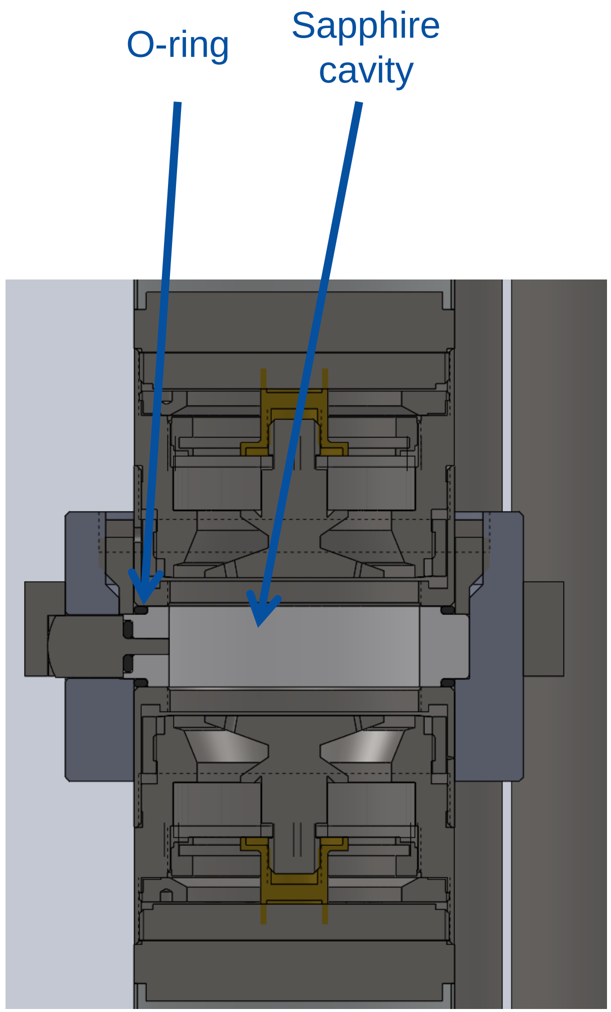
Increase Signal-to-noise ratio

Microphone admittances are given in their simplest form:

$$Y_{r,t} = \frac{j\omega V_{eq,(r,t)}}{\gamma P}$$

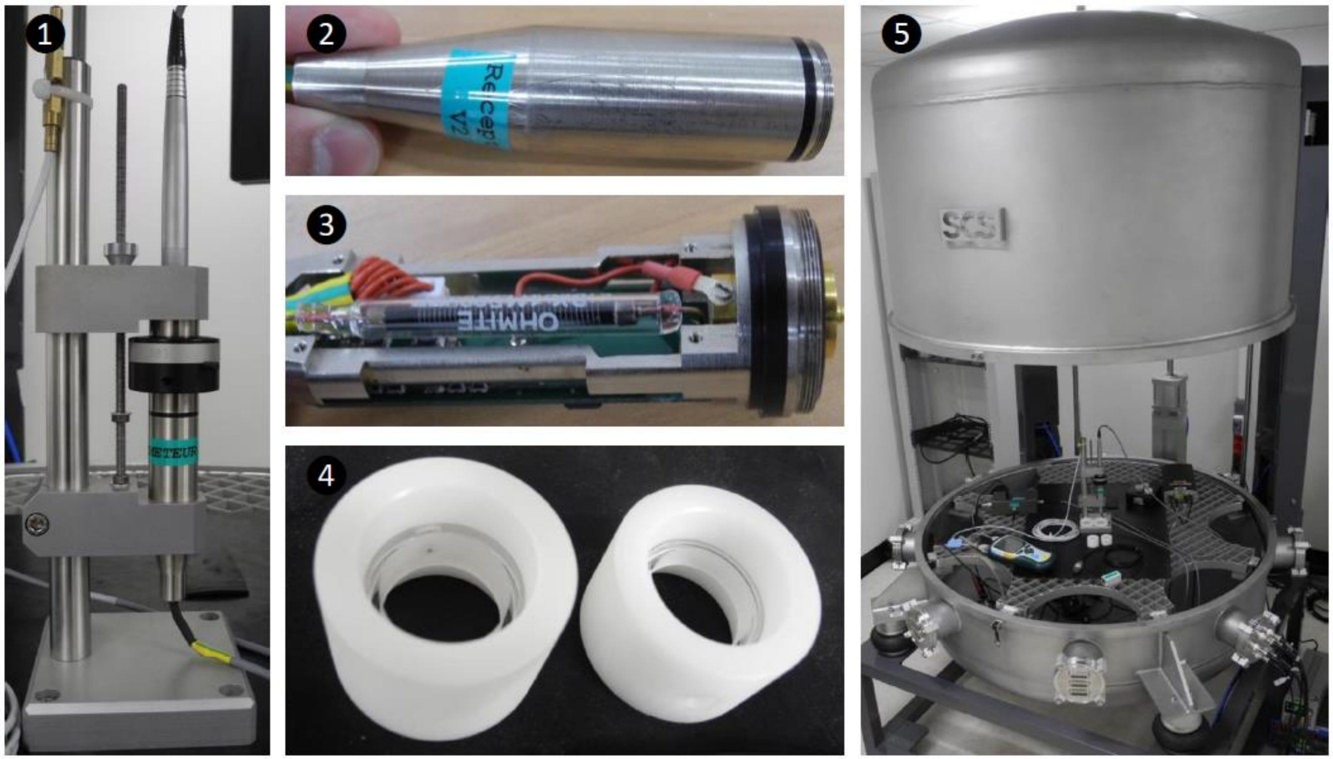
$$V_{eq,(r,t)} \approx V_{cavity}$$

Heat conduction effects are neglected in the back cavity.



VALIDATION OF ACOUSTIC TRANSFER ADMITTANCE FORMULATIONS

Measurement setup



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RESULTS

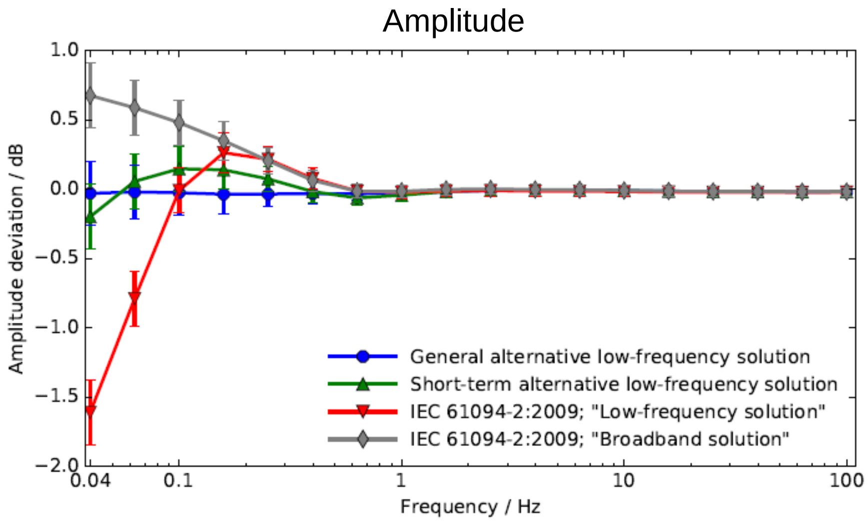
Error estimator

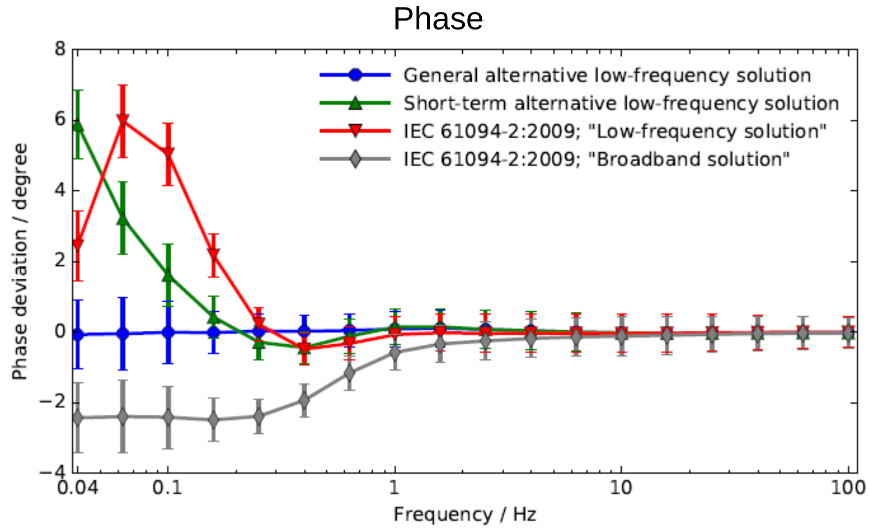
- Analysis of the error estimator δ_m :

a) $\delta_m \nrightarrow 0$ (in dB): the product of sensitivities depends on the cavity dimensions. The formulation of the acoustic transfer admittance is invalid.

b) $\delta_m \rightarrow 0$ (in dB): the product of sensitivities does not depend on the cavity dimensions. The formulation of the acoustic transfer admittance is valid.

c) Other effects mask or compensate for each other by coincidence, for the chosen coupler sizes. It should be unlikely.





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RESULTS

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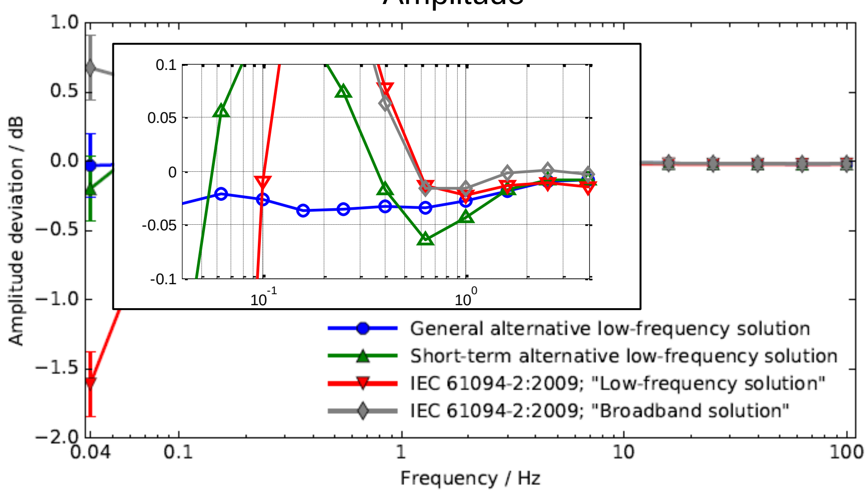
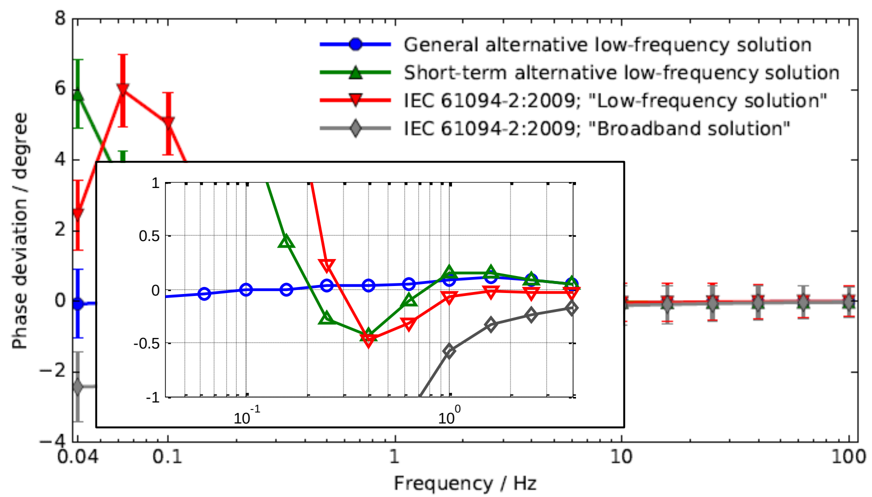
Amplitude



Phase



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ACOUSTIC TRANSFER ADMITTANCE: MODEL PRESENTATION

IEC 61094-2:2009: The “broadband solution” & “low-frequency solution”

Modelling & low frequency limitation

- Explanations about low frequency limitations are provided in the *Metrologia* paper:

  P. VINCENT et al., Acoustic transfer admittance of cylindrical cavities in infrasonic frequency range. Metrologia, 56(1):015003, 2019.

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ACOUSTIC TRANSFER ADMITTANCE: MODEL PRESENTATION

Alternative low-frequency solution for cylindrical cavities

- **Dissipative fluid = shear/volume viscosity and heat conduction**

Set of 4 equations for 4 quantities

$\boldsymbol{p}$: pressure variation

$\boldsymbol{v}$: particle velocity

$\boldsymbol{\tau}$: temperature variation

$\boldsymbol{\rho}$: density variation

1. Navier-Stokes equation

$$\frac{1}{c_0}\frac{\partial \vec{v}}{\partial t}+\frac{1}{\rho_0 c_0}\,\vec{\mathrm{grad}}\;p = \ell_v \vec{\mathrm{grad}}\;\mathrm{div}\;\vec{v}-\ell'_v \vec{\mathrm{rot}}\;\vec{\mathrm{rot}}\;\vec{v}+\frac{\vec{\mathrm{F}}}{c_0}$$

ℓ_v and ℓ'_v : Characteristic lengths of shear and volume viscosity

2. Conservation of mass equation

$$\rho_0\;\mathrm{div}\;\vec{v}+\frac{\partial \rho}{\partial t}=\rho_0 \mathrm{q}$$

3. Thermodynamic law

$$\rho=\rho_0\chi_T\left(p-\beta\tau\right)$$

4. Fourier equation

$$\left[\frac{1}{c_0}\frac{\partial}{\partial t}-\ell_h\Delta\right]\tau=\frac{\gamma-1}{\beta\gamma}\frac{1}{c_0}\frac{\partial p}{\partial t}+\frac{\mathrm{h}}{c_0 \mathrm{C_p}}$$

ℓ_h : Characteristic length of thermal diffusion

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ACOUSTIC TRANSFER ADMITTANCE: MODEL PRESENTATION

Alternative low-frequency solution for cylindrical cavities

Modelling: The basic equations

1. Navier-Stokes equation

- No force source inside the cavity.
- Low frequency assumption: $\lambda >> \sqrt[3]{V}$

  Pressure field uniform inside the cavity:

  grad p=0 and v≈0 at any point of the cavity.

$$\xcancel{\frac{1}{c_0}\frac{\partial \vec{v}}{\partial t}+\frac{1}{\rho_0 c_0}\,\mathrm{gr\vec{a}d}\,p=\ell_v\,\mathrm{gr\vec{a}d}\,\mathrm{div}\,\vec{v}-\ell'_v\,\mathrm{r\vec{o}t}\,\mathrm{r\vec{o}t}\,\vec{v}+\frac{\vec{\mathrm{F}}}{c_0}}$$

The Navier-Stokes equation is not required in this formulation.

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ACOUSTIC TRANSFER ADMITTANCE: MODEL PRESENTATION

Alternative low-frequency solution for cylindrical cavities

Modelling: The basic equations

2. Conservation of mass equation + thermodynamic law

- No mass source inside the cavity

$$\rho_0 \operatorname{div} \vec{v} + \frac{\partial \rho}{\partial t} = \cancel{\rho_0 q}$$

- The conservation of mass equation can be integrated over the entire cavity volume

$$\iiint_V \frac{\partial \rho}{\partial t} \mathrm{d}V + \rho_0 \iint_A v \mathrm{d}A = 0$$

v≈0 in the field, expected at the end boundaries (microphones)

$$+ \quad \rho = \rho_0 \chi_T \left(p - \beta \tau \right)$$

$$\left(1 + \frac{A}{j\omega Z \chi_T V} \right) p - \frac{\beta}{V} \iiint_V \tau \mathrm{d}V = \frac{\delta V}{\chi_T}$$

Volume variation due to the displacement field of the transmitting diaphragm

Impedance of the receiver microphone

ACOUSTIC TRANSFER ADMITTANCE: MODEL PRESENTATION

Alternative low-frequency solution for cylindrical cavities

Modelling: The basic equations

3. Fourier equation for heat conduction

- No heat source inside the cavity

$$\left[\frac{\partial}{\partial t} - \alpha_t \Delta\right]\tau = \frac{\gamma - 1}{\beta\gamma}\frac{\partial p}{\partial t} + \cancel{\frac{\mathrm{h}}{\mathrm{C_p}}}$$

+ Pressure field uniform in the cavity + τ=0 on the boundaries + axisymmetric problem

- Solution provided by Gerber (1964)

$$\iiint_V \tau \mathrm{d}V = p\frac{\gamma - 1}{\beta\gamma}E_p$$

$$E_p = \sum_{m=0}^{+\infty}\sum_{n=1}^{+\infty}\left[\frac{8/\pi^2}{(m+1/2)^2\lambda_n^2}F_{m,n}\right]$$

Short-term Laplace asymptotic development

$$E_P = 1 - X_P + \frac{\pi R^2 + 8R}{\pi(2R+1)^2}X_P^2 + \frac{3}{4}\sqrt{\pi}\frac{R^3 - 6R^2}{3\sqrt{\pi}(2R+1)^3}X_P^3$$

ACOUSTIC TRANSFER ADMITTANCE: MODEL PRESENTATION

Alternative low-frequency solution for cylindrical cavities

Modelling: The basic equations

4. Solutions

$$\left(1+\frac{A}{j\omega Z\chi_T V}\right)p-\frac{\beta}{V}\iiint_V \tau \mathrm{d}V=\frac{\delta V}{\chi_T}$$

$$\iiint_V \tau \mathrm{d}V = p\frac{\gamma-1}{\beta\gamma}E_p$$

$$p=-\frac{\delta V}{\chi_T V}\frac{1}{1+\frac{1}{j\omega Z\chi_T V}Y_r-\frac{\gamma-1}{\gamma}E_P}$$

- Acoustic transfer admittance definition:

Ratio of the shortcircuit volume velocity produced by the transmitter microphone to the sound pressure acting on the diaphragm of the receiver microphone

$$Y_T=\frac{Sv_t+Y_t p(0)}{p(\ell)}=\frac{-j\omega\delta V}{p}+Y_t$$

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ACOUSTIC TRANSFER ADMITTANCE: MODEL PRESENTATION

Alternative low-frequency solution for cylindrical cavities

Modelling: The basic equations

4. Solutions

$$Y_T = \frac{j\omega V}{\gamma P_0}\left[\gamma - (\gamma - 1) E_P\right] + Y_r + Y_t$$

Note:
This solution is equivalent to that provided by the first Gerber interpretation.
However, it incorporates the admittance of the receiver microphone by considering the formulation of the conservation of mass equation.

- The general alternative low-frequency solution

where Ep is given by the general formulation

$$E_p = \sum_{m=0}^{+\infty}\sum_{n=1}^{+\infty}\left[\frac{8/\pi^2}{(m+1/2)^2\,\lambda_n^2}F_{m,n}\right]$$

- The short-term alternative low-frequency solution

where Ep is given by the Laplace asymptotic development

$$E_P = 1 - X_P + \frac{\pi R^2 + 8R}{\pi(2R+1)^2}X_P^2 + \frac{3}{4}\sqrt{\pi}\,\frac{R^3 - 6R^2}{3\sqrt{\pi}(2R+1)^3}X_P^3$$

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CONCLUSION & RECOMMENDATIONS

Conclusion

- The "general alternative low-frequency solution" provides the best results among the studied formulations.
- These results highlight the limitations of the current standardised formulations of acoustic transfer admittance for the purpose of microphone infrasound calibration.

Recommendations for future revision of the IEC standard 61094-2

- Use of the "general alternative low-frequency solution" instead the current standardised "low-frequency solution".

$$Y_T = \frac{j\omega V}{\gamma P_0}\left[\gamma - \left(\gamma - 1\right) E_P\right] + Y_r + Y_t$$

$$E_p = \sum_{m=0}^{+\infty} \sum_{n=1}^{+\infty} \left[\frac{8/\pi^2}{\left(m + 1/2\right)^2 \lambda_n^2} F_{m,n} \right]$$

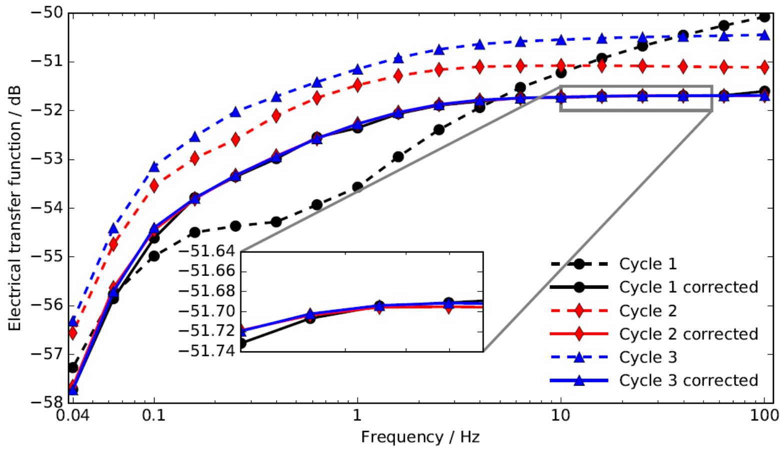
Thank you for your attention

RESULTS

Measurement processing

- As the cavities and microphones are necessarily sealed, the local environmental variations inside the reciprocity system have an important effect on its stability (in amplitude).
- To overcome this problem, a specific correction process was implemented (only on amplitude results), based on the hypothesis that the environmental coefficient of the microphones sensitivities (temperature and static pressure) tends towards a fixed value at low frequencies.

Frequencies / Hz:	$\mathbf{f_{i-1}}$	100 Hz	$\mathbf{f_i}$	100 Hz	$\mathbf{f_{i+1}}$	100 Hz	...
Raw amplitude / dB:	$\mathbf{A_{i-1}}$	B_j	$\mathbf{A_i}$	B_{j+1}	$\mathbf{A_{i+1}}$	B_{j+2}	...
Correction / dB:	...		$\boxed{C_i = C_0 - \frac{B_j + B_{j+1}}{2}}$		...		
Corrected amplitude / dB:	...		$\mathbf{A'_i = A_i} + C_i$		...		



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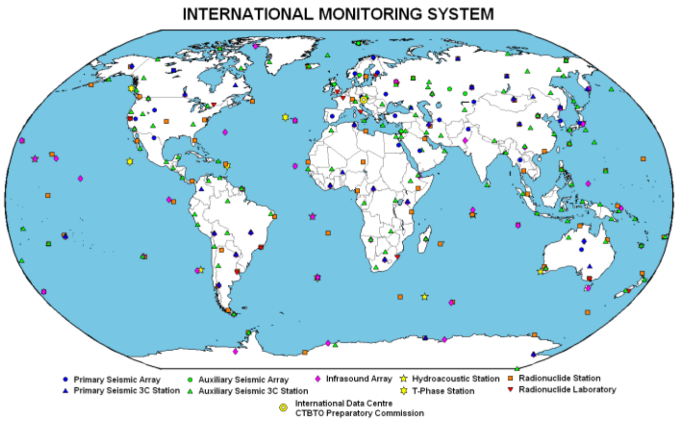
INTRODUCTION

Context

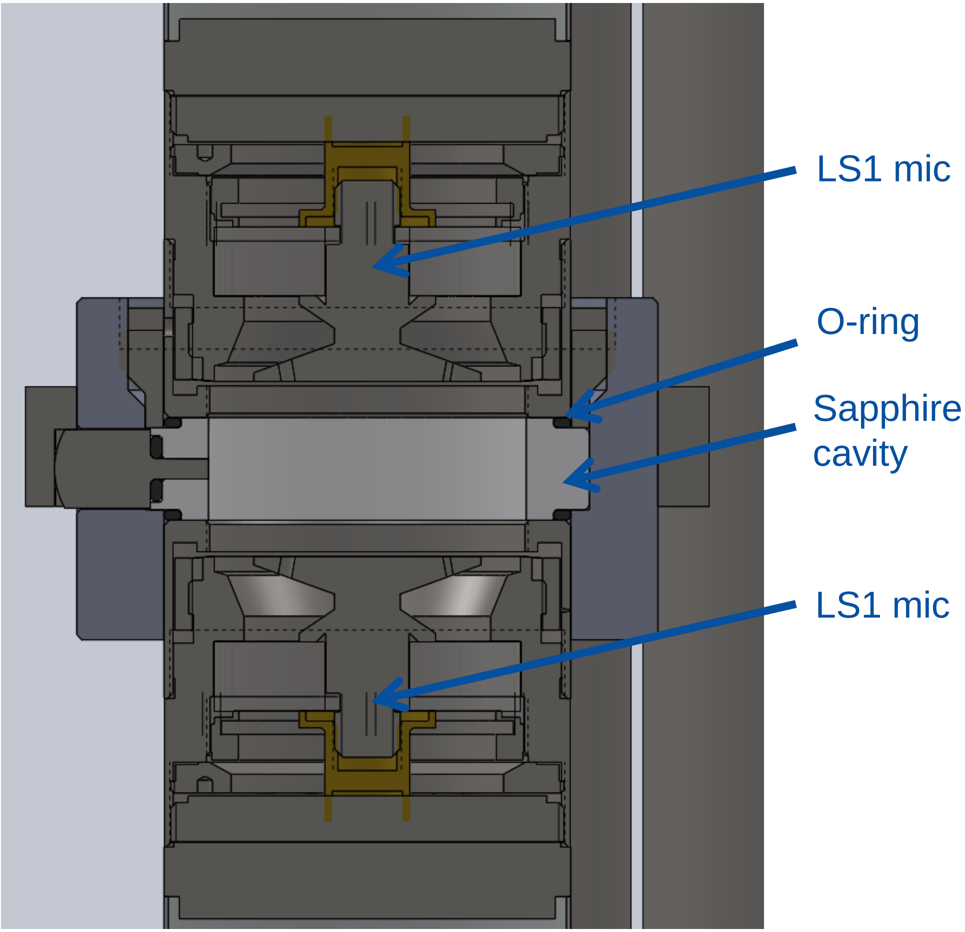
- **Demand for measurements and calibration at infrasonic frequencies has recently emerged**
  - In response to issues such as volcano, tsunami, avalanche, wind turbine, and transportation monitoring.
  - In response to the requirements of the CTBTO, which provides global international coverage to help enforce nuclear testing bans. The International Monitoring System of the CTBTO requires calibration (amplitude and phase) of its infrasound sensor network in the frequency range of 0.02 Hz - 4 Hz.



ACOUSTIC MODELLING

New acoustic modelling: Validation

- **New sealed cavities**

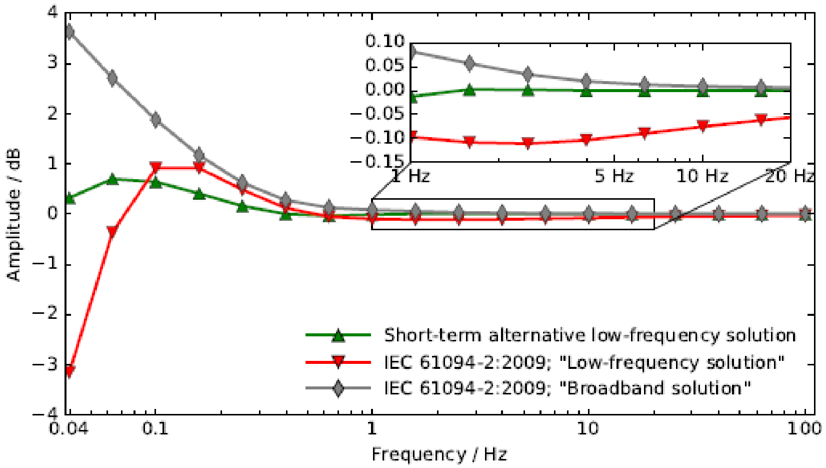


CONCLUSION & RECOMMENDATIONS

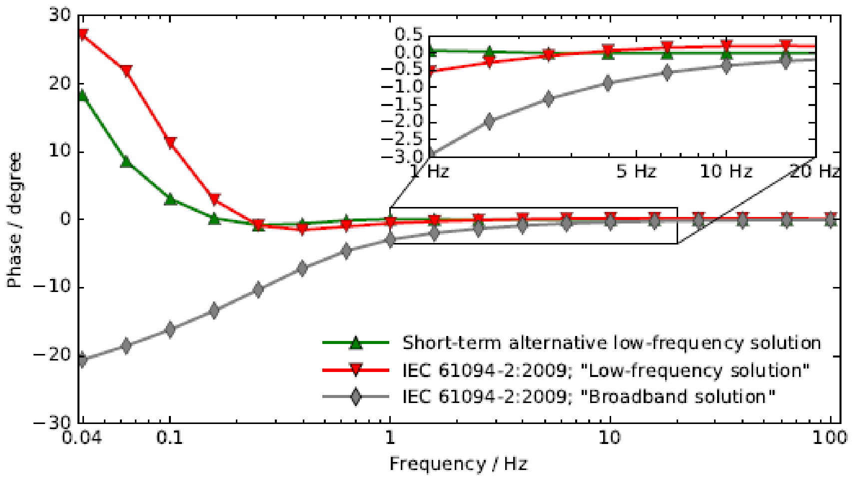
- The general alternative low-frequency solution provide the best results among the studied formulations.
- The short-term alternative low-frequency solution provide better results than the standardised solutions.
- These results highlight the limitations of the current standardised formulations of acoustic transfer admittance for the purpose of microphone infrasound calibration.
- Possible error on microphone calibration:

:

Amplitude



Phase



Other models relative to the general alternative LF solution

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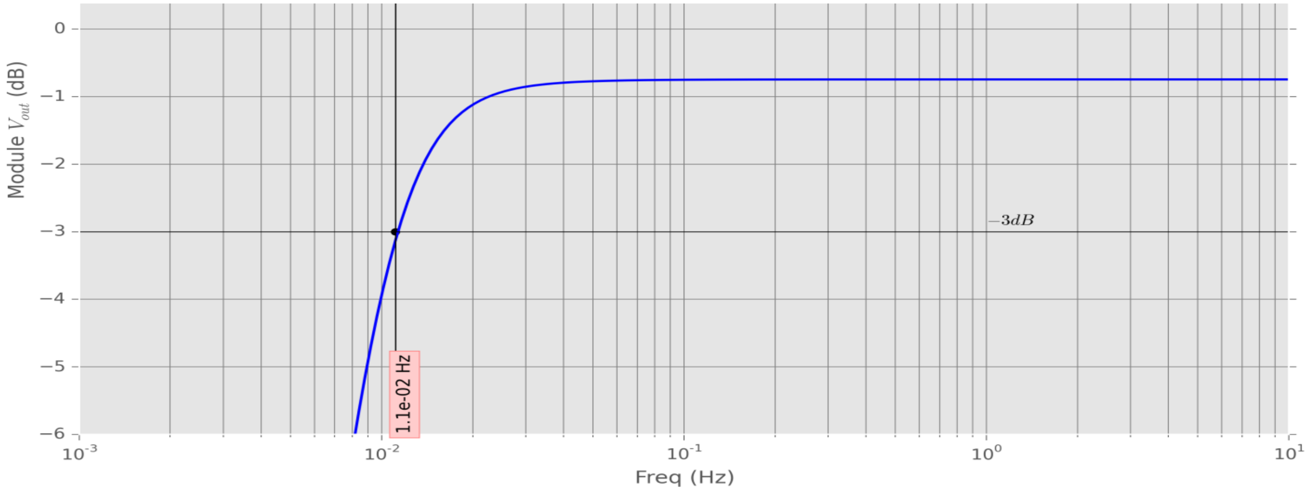
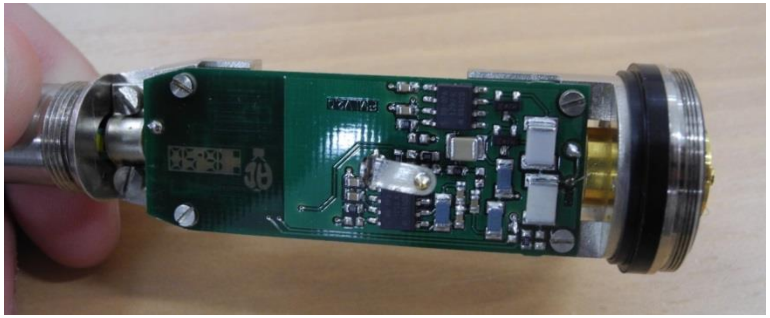
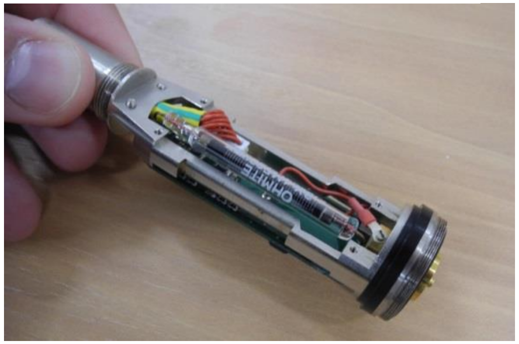
ACOUSTIC MODELLING

New acoustic modelling: Validation

- **New preamplifiers**







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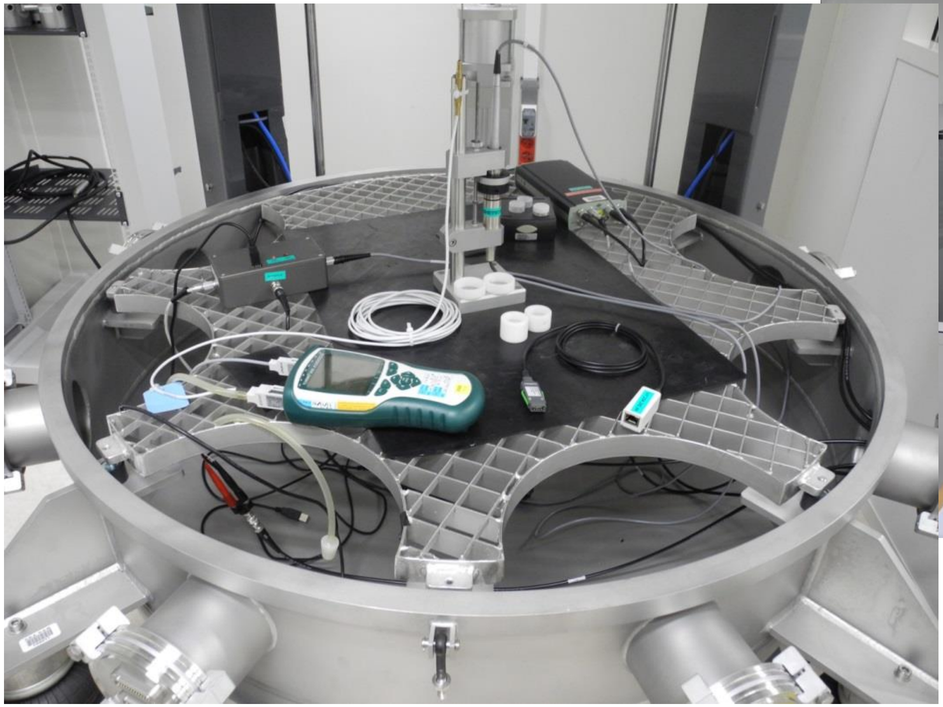
ACOUSTIC MODELLING

New acoustic modelling: Validation

- **Laboratory (CEA)**



ACOUSTIC MODELLING

E_P general vs E_V general