Note to reader.

In preparation for the possible redefinition of the kilogram, ampere, kelvin and mole by the 26th CGPM in 2018 in terms of the Planck constant, elementary charge, Boltzmann constant, and Avogadro constant the CCEM Working Group on Proposed Changes to the SI has prepared this draft of a *Mise en pratique* for the ampere and other SI electric units in order to indicate how the electrical units may be realized in practice based on the new definitions.

All characters in the color blue are place holders of the final characters to be inserted at the time of implementation of this *Mise en pratique*.

At the time of the preparation of this *Mise en pratique*, single electron transport (SET) implementations still have technical limitations and often larger relative uncertainties than some other competitive techniques. However, SET implementations are included in this *Mise en pratique* because they offer unique and elegant approaches to realizing SI units, and their uncertainties have been improving in recent years, and they promise to improve further in the future.

Virtually all numbers are truncated in modern calculations. This is true with limitations on the number of significant digits at each stage of the calculation, as well as in transfers from decimal to binary and back to decimal. This merely emphasizes that truncation of the value of any number is a universally accepted process and that the impact is really focused on the uncertainty imposed by that process. So long as the error due to truncation is kept substantially smaller than the uncertainty of the overall result, then this process will have no measurable effect.

However, it is probably still helpful to the metrological community to recommend that they use a truncated value with a sufficiently large number of significant digits so that its use will have no measurable uncertainty impact for practical applications. Sixteen significant digits seems a reasonable level at the moment and this *Mise en pratique* may be revised later as technical advances dictate.

Comments and suggestions are welcome and may be sent to Barry Wood, CCEM WGSI chair: barry.wood@nrc-cnrc.gc.ca.

Mise en pratique for the ampere and other electric units in the International System of Units (SI)

CCEM Working Group on the SI
Draft #4

1. Introduction.

Resolution AA of the 26th General Conference on Weights and Measures (CGPM), which convened in November 2018, abrogated the then existing definitions of the four base units kilogram, ampere, kelvin, and mole of the International System of Units (SI) and replaced them with new definitions that fix the values of the Planck constant h, the elementary charge e, the Boltzmann constant k, and the Avogadro constant N_A, respectively. As a consequence, all base and derived units can be established from a set of seven reference constants. The distinction between base and derived units is no longer fundamental, but is maintained mainly for historical continuity and pedagogical purposes. The exact values of the seven constants, including the speed of light in vacuum, c, the hyperfine splitting of Cs, $\Delta\nu_{Cs,}$ and the luminous efficacy, K_{cd}, are [1]:

$$h = 6.626\ 070\ 04 \times 10^{-34}\ \text{J s} \quad (1)$$
$$e = 1.602\ 176\ 621 \times 10^{-19}\ \text{C} \quad (2)$$
$$k = 1.380\ 648\ 5 \times 10^{-23}\ \text{J K}^{-1} \quad (3)$$
$$N_A = 6.022\ 140\ 857 \times 10^{23}\ \text{mol}^{-1} \quad (4)$$
$$c = 299\ 792\ 458\ \text{m s}^{-1} \quad (5)$$
$$\Delta\nu_{Cs} = 9\ 192\ 631\ 770\ \text{Hz} \quad (6)$$
$$K_{cd} = 683\ \text{lm W}^{-1} \quad (7)$$

The purpose of this *Mise en pratique*, prepared by the Consultative Committee for Electricity and Magnetism (CCEM) of the International Committee for Weights and Measures (CIPM) and formally adopted by the CIPM, is to indicate how the SI base unit, the ampere, symbol A, and the derived SI electric units with names and symbols, the volt V, ohm **Ω**, siemens S, coulomb C, farad F, henry H, watt W, tesla T, and weber Wb, may be realized in practice based on the new definitions.

In general, the term "to realize a unit" is interpreted to mean the establishment of the value and associated uncertainty of a quantity of the same kind as the unit that is consistent with the definition of the unit. It is important to recognize that any method consistent with the laws of physics and based on the set of seven reference constants (which is equivalent to being consistent with the base unit definitions) can be used to realize any SI unit, base or derived. Thus, the list of methods given is not meant to be an exhaustive list of all possibilities, but rather a list of those methods that are easiest to implement and/or that provide the smallest uncertainties.

2. Definition of the ampere.

The definition of the ampere, SI base unit of electric current, as adopted by the 26th CGPM, is as follows [1]:

- **The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge *e* to be 1.602 176 621 $\times 10^{-19}$ when**

expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{Cs}$.

3. Practical realization of the ampere, A, SI base unit of electric current.

As already noted in Sec. 1, to realize a unit generally means to establish the value and associated uncertainty of a quantity of the same kind as the unit that is consistent with the definition of the unit. In practice, the ampere A can be realized:

(a) by using Ohm's law, the unit relation A = V/**Ω**, and using practical realizations of the SI derived units the volt V and the ohm **Ω**, based on the Josephson and quantum Hall effects, respectively, as discussed in Secs. 4 and 5 below; or

(b) by using a single electron transport (SET) or similar device, the unit relation A = C/s, the value of *e* given in Eq. (2) and a practical realization of the SI base unit the second s; or

(c) by using the relation $I = C{\cdot}\mathrm{d}U/\mathrm{d}t$, the unit relation A = F·V/s, and practical realizations of the SI derived units the volt V and the farad F and of the SI base unit second s.

4. Practical realization of the volt, V, SI derived unit of electric potential difference (voltage) and electromotive force.

The volt V can be realized using the Josephson effect and the following value of the Josephson constant K_J:

$$K_J = 2e/h \approx 483\,597.852\,500\,000\,0 \text{ GHz V}^{-1}. \quad * \tag{8}$$

* The truncated numerical value has been calculated to 16 significant digits.

This value follows from the assumption of the accuracy of the equation $K_J = 2e/h$, which is strongly supported by a large body of experimental and theoretical works, and the values of *h* and *e* given in Eq. (1) and (2). Although the quotient 2*e*/*h* can obviously be calculated with any number of digits, this truncated recommended value is in error by less than 1 part in 10^{15}, which is intended to be negligible in the vast majority of applications. In those rare cases where this error may not be negligible, additional digits should be employed. The advantage of recommending a particular numerical value of K_J for practical use is that it ensures that virtually all realizations of the volt based on the Josephson effect employ exactly the same value.

Note that the value of K_J in Eq. (8) is smaller than the value $K_{J\text{-}90} = 483\,597.9$ GHz V^{-1}, which was adopted by the CIPM starting 1 January 1990 for the international realization of the volt using the Josephson effect, by the fractional amount 98.222×10^{-9}. This implies that the unit of voltage realized using $K_{J\text{-}90}$ was larger than the present SI unit as realized using the value in Eq. (8) by the same fractional amount. Thus, the numerical value of a voltage measured in terms of $K_{J\text{-}90}$ would have been smaller by the same fractional amount as the numerical value of the identical voltage measured today in terms of the present SI volt realized using the value of K_J given in Eq. (8).

5. Practical realization of the ohm, Ω, SI derived unit of electric resistance and impedance.

The ohm Ω can be realized as follows:

(a) by using the quantum Hall effect in a manner consistent with the CCEM Guidelines [2] and the following value of the von Klitzing constant R_K:

$$R_K = h/e^2 \approx 25\,812.807\,455\,500\,00\ \Omega. \quad * \tag{9}$$

* The truncated numerical value has been calculated to 16 significant digits.

This value follows from the assumption of the accuracy of the equation $R_K = h/e^2$, which is strongly supported by a large body of experimental and theoretical works, and the values of h and e given in Eqs. (1) and (2). Although the quotient h/e^2 can obviously be calculated with any number of digits, this truncated recommended value is in error by less than 1 part in 10^{15}, which is intended to be negligible in the vast majority of applications. In those rare cases where this error may not be negligible, additional digits should be employed. The advantage of recommending a particular numerical value of R_K for practical use is that it ensures that virtually all realizations of the ohm based on the quantum Hall effect employ exactly the same value; or

(b) by comparing an unknown resistance to the impedance of a known capacitance using, for example, a quadrature bridge, where, for example, the capacitance has been determined by means of a calculable capacitor and the value of the electric constant given by Eq. (10).

Note that the value of R_K in Eq. (9) is larger than the value $R_{K\text{-}90} = 25\,812.807\ \Omega$, which was adopted by the CIPM starting 1 January 1990 for the international realization of the ohm using the quantum Hall effect, by the fractional amount 17.646×10^{-9}. This implies that the unit of resistance realized using $R_{K\text{-}90}$ was larger than the present SI unit as realized using the value in Eq. (9) by the same fractional amount. Thus, the numerical value of a resistance measured in terms of $R_{K\text{-}90}$ would have been smaller by the same fractional amount as the numerical value of the identical resistance measured today in terms of the present SI ohm realized using the value of R_K given in Eq. (9).

6. Practical realization of the siemens, S, SI derived unit of electric conductance.

The siemens S can be realized from a realization of the ohm (see Sec. 5) since S is related to Ω by the unit relation $\mathrm{S} = \Omega^{-1}$.

7. Practical realization of the coulomb, C, SI derived unit of electric charge.

The coulomb C can be realized as follows:

(a) by measuring the duration in terms of the SI unit of time, the second s, of the flow of an electric current known in terms of the ampere realized as indicated in Sec. 3; or

(b) by determining the amount of charge placed on a capacitance known in terms of the farad F realized by method 8(a) or 8(b), using the unit relation C = F·V and by measuring the voltage across the capacitance in terms of the volt V as realized by the Josephson effect and the value of the Josephson constant given in Eq. (8) (see Sec. 4); or

(c) by using a SET or similar device to transfer a known amount of charge based on the value of *e* given in Eq. (2) onto a suitable circuit element.

8. Practical realization of the farad, F, SI derived unit of capacitance.

The farad F can be realized as follows:

(a) by comparing the impedance of a known resistance obtained using the quantum Hall effect and the value of the von Klitzing constant given in Eq. (9) (see Sec. 5), including a quantized Hall resistance itself, to the impedance of an unknown capacitance using, for example, a quadrature bridge; or

(b) by using a calculable capacitor and the value of the electric constant given by Eq. (10).

9. Practical realization of the henry, H, SI derived unit of inductance.

The henry H can be realized as follows:

(a) by comparing the impedance of an unknown inductance to the impedance of a known capacitance with the aid of known resistances using, for example, a Maxwell-Wien bridge, where the known capacitance and resistances have been determined, for example, from the quantum Hall effect and the value of R_K given in Eq. (9) (see Secs. 5 and 8); or

(b) by using a calculable inductor of, for example, the Campbell type of mutual inductor and the value of the magnetic constant μ_0 given by Eq. (14).

10. Practical realization of the watt, W, SI derived unit of power.

The watt W can be realized using electrical units by using the fact that electric power is equal to current times voltage, the unit relation based on Ohm's law, $W = V^2/\Omega$, and realizations of the volt and ohm using the Josephson and quantum Hall effects and the values of the Josephson and von Klitzing constants given in Eqs. (8) and (9) (see Secs. 4 and 5).

11. Practical realization of the tesla, T, SI derived unit of magnetic flux density.

The tesla T can be realized as follows:

(a) by using a solenoid, Helmholtz coil or other configuration of conductors of known dimensions carrying an electric current determined in terms of the ampere realized as discussed in Sec. 3, and the value of the magnetic constant μ_0 given in Eq. (14) in the calculation of the magnetic flux density generated by the current carrying conductors; or

(b) by using nuclear magnetic resonance (NMR) with a sample of known gyromagnetic ratio, for example, a spherical sample of pure H_2O at 25 °C and the most recent recommended value of the shielded gyromagnetic ratio of the proton $\gamma_{p'}$ given by CODATA.

12. Practical realization of the weber, Wb, SI derived unit of magnetic flux.

The weber Wb can be realized from the tesla based on the unit relation Wb = T m^2 or from the volt based on the unit relation Wb = V s. Use can also be made of the fact that the magnetic flux quantum Φ_0, which characterizes the magnetic properties of superconductors, is related to h and e as given in Eqs. (1) and (2) by the exact relation $\Phi_0 = h/2e$.

13. Magnetic constant μ_0 and related quantities.

The new definitions of the kilogram, ampere, kelvin and mole do not alter the relationships among the magnetic constant (permeability of vacuum) μ_0, electric constant (permittivity of vacuum) ε_0, characteristic impedance of vacuum Z_0, admittance of vacuum Y_0 and speed of light in vacuum c. Moreover, they do not change the exact value of c, which is explicit in the definition of the SI base unit of length, the metre m. The relationships among these constants are

$$\varepsilon_0 = 1/\mu_0 c^2 \tag{10}$$
$$Z_0 = \mu_0 c = (\mu_0/\varepsilon_0)^{1/2} \tag{11}$$
$$Y_0 = 1/\mu_0 c = (\varepsilon_0/\mu_0)^{1/2} = 1/Z_0 \tag{12}$$

However, the new definitions do affect the value of μ_0, and hence the values of ε_0, Z_0, and Y_0. In particular, μ_0 no longer has the exact value $4\pi \times 10^{-7}$ N A^{-2} and must be determined experimentally. The value of μ_0 can be obtained with a relative standard uncertainty u_r, identical to that of the fine structure constant α from the exact relation

$$\mu_0 = \alpha \, \frac{2h}{ce^2} \tag{13}$$

Since h, c, and e have fixed numerical values, it follows from Eqs. (10)-(13) that

$$u_r(Y_0) = u_r(Z_0) = u_r(\varepsilon_0) = u_r(\mu_0) = u_r(\alpha).$$

The recommended values of h, e, k and N_A resulting from the 2017 CODATA special least-squares adjustment of the values of the fundamental constants [3] were the basis of the exact values used for these four constants in the new definitions of the kilogram, ampere, kelvin and mole adopted by the 26th CGPM. The 2018 adjustment but with h, e, k and N_A taken to have the exact values used in the new definitions, yields the following currently recommended value of the magnetic constant [4]:

$$\mu_0 = 4\pi\,[1 + 0.0(2.3) \times 10^{-10}] \times 10^{-7}\ \text{N A}^{-2} \tag{14}$$
$$\quad = 12.566\,370\,6144(18) \times 10^{-7}\ \text{N A}^{-2}$$

The values and uncertainties of the electric constant, characteristic impedance of vacuum and characteristic admittance of vacuum may always be obtained from the relationships of Eqn. (10)-(13).

It should be recognized that the recommended values for μ_0, ε_0, Z_0 and Y_0 are expected to change slightly from one future CODATA adjustment to the next, as new data that influence the value of α become available. Users of this document should, therefore, always employ the most up-to-date CODATA recommended values for these constants in their calculations. Of course, the values of h, e, k and N_A fixed by the new definitions will be unchanged from one adjustment to the next.

References

[1] In the final version of this document reference [1] will be the proceedings of the 26th CGPM (Comptes Rendus des Séances de la Conférence Générale des Poids et Mesures). All numerical values of the constants given in this document are presently based on documents CCU/16-0.14 and CCU/16-29

[2] F. Delahaye and B. Jeckelmann, Metrologia 40(5), 217-223 (2003).

[3] In the final version of this document, reference [3] will be the 2017 CODATA special adjustment and perhaps the 2018 CIPM recommendation.

[4] In the final version of this document, reference [4] would be the 2018 CODATA adjustment employing the fixed values of h, e, k and N_A used to define the kilogram, ampere, kelvin and mole. It likely will initially be listed as ‘To be published’