Electronic determination of the Boltzmann constant with Johnson Noise Thermometry

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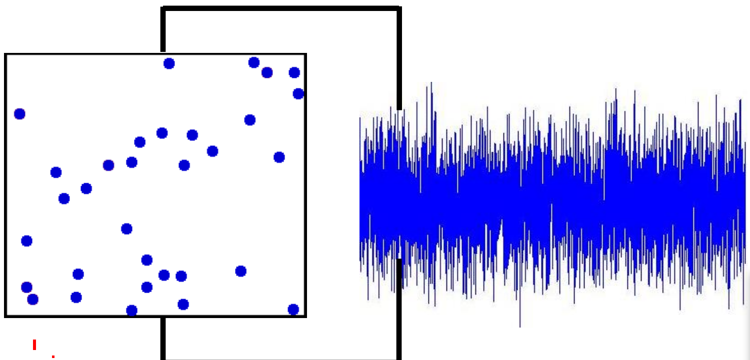
June. 1, 2017, 28th CCT meeting at BIMP

Outline

1. Johnson noise thermometry
2. Absolute JNT
3. Electronic measurement of k_B
4. Conclusion

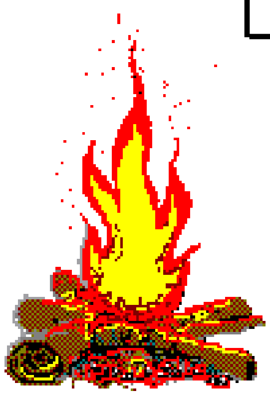
Outline

Johnson noise



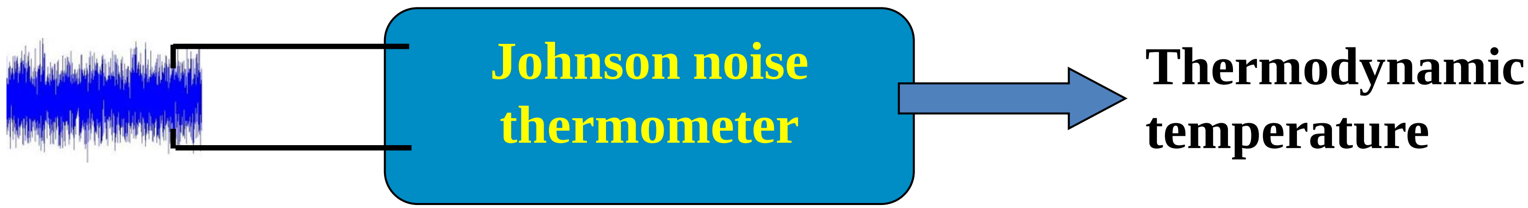
$$S_R = 4hfR\left[\frac{1}{2}+\frac{1}{\exp(hf/kT)-1}\right] \quad (1)$$

$$\langle V^2 \rangle = 4\,kT\,R\,\Delta f \quad (2)$$



- **random thermal motion of electrons in a conductor causes both electrical resistance and a fluctuating voltage**
- **predicted by Einstein in 1906, measured by Johnson in 1927, and theoretically described by Nyquist in 1928**
- **fluctuation-dissipation theorem**

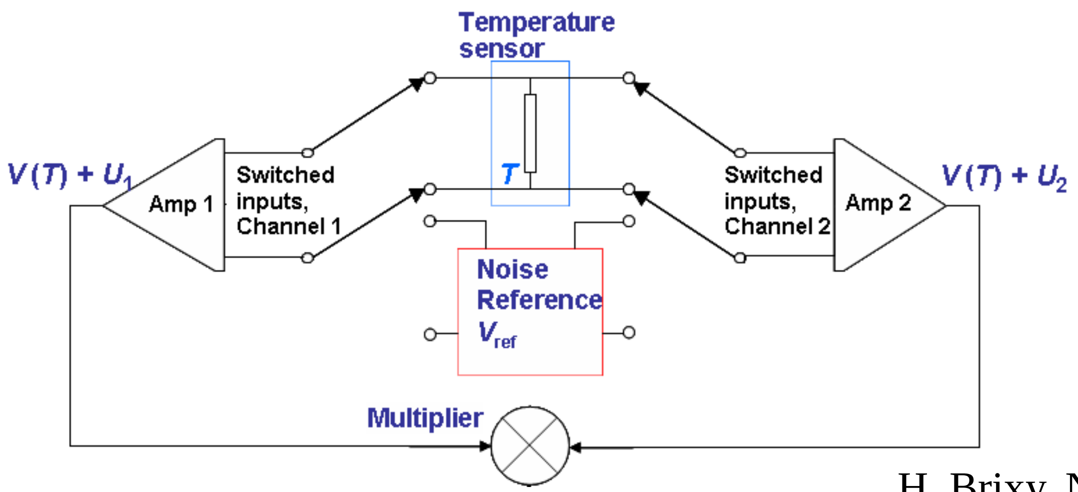
Johnson noise thermometry



- **Pros:**
  - pure electronic measurement of thermodynamic temperature
  - immune from chemical and mechanical changes in the material properties
  - periodic calibration is not necessary

- **Cons:**
  - extremely small voltage, 100 ohm, 273K, ~1.2nV/√Hz (amplify by 10^5)
  - random, very long time integration ($\sigma \sim 1/\sqrt{t}$) (weeks or months)
  - distributed over wide bandwidths ($\sigma \sim 1/\sqrt{\Delta f}$) (a few hundred kHz)

Switching correlator



$$\frac{T}{T_{ref}} = \frac{\langle V^2(T)\rangle / R}{\langle V_{ref}^2 \rangle / R_{ref}}$$

H. Brixy, Nucl. Instrum. Methods, 97, 75-80 (1971)

- **four wire connection defines the source impedance**
- **eliminates uncorrelated noise by cross-correlation**
- **eliminates the effect of amplifier gain drift by switching**
- **impossible to match both the noise power and frequency response**
- **affected by electronic nonlinearity or narrow bandwidth**
- **relative measurement, uncertainty limited to 10^{-5}**

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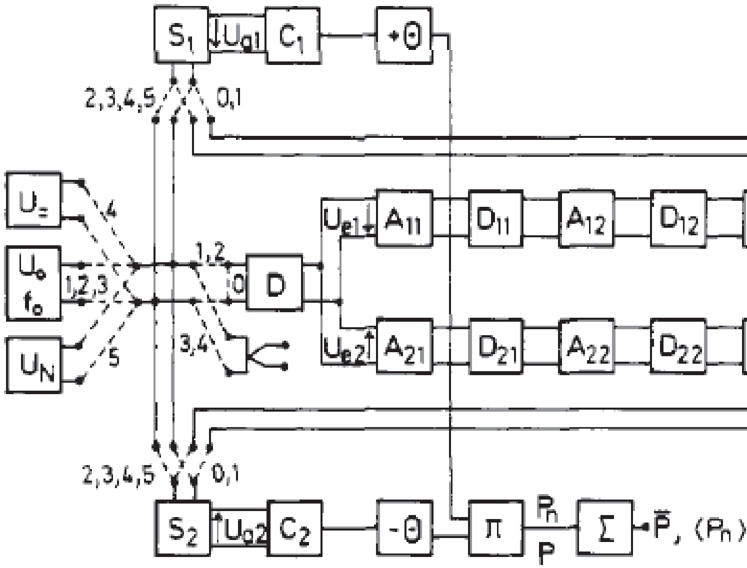
2 **Absolute JNT**

3 Electronic measurement of k_B

4 Conclusion

Early attempt to measure k_B

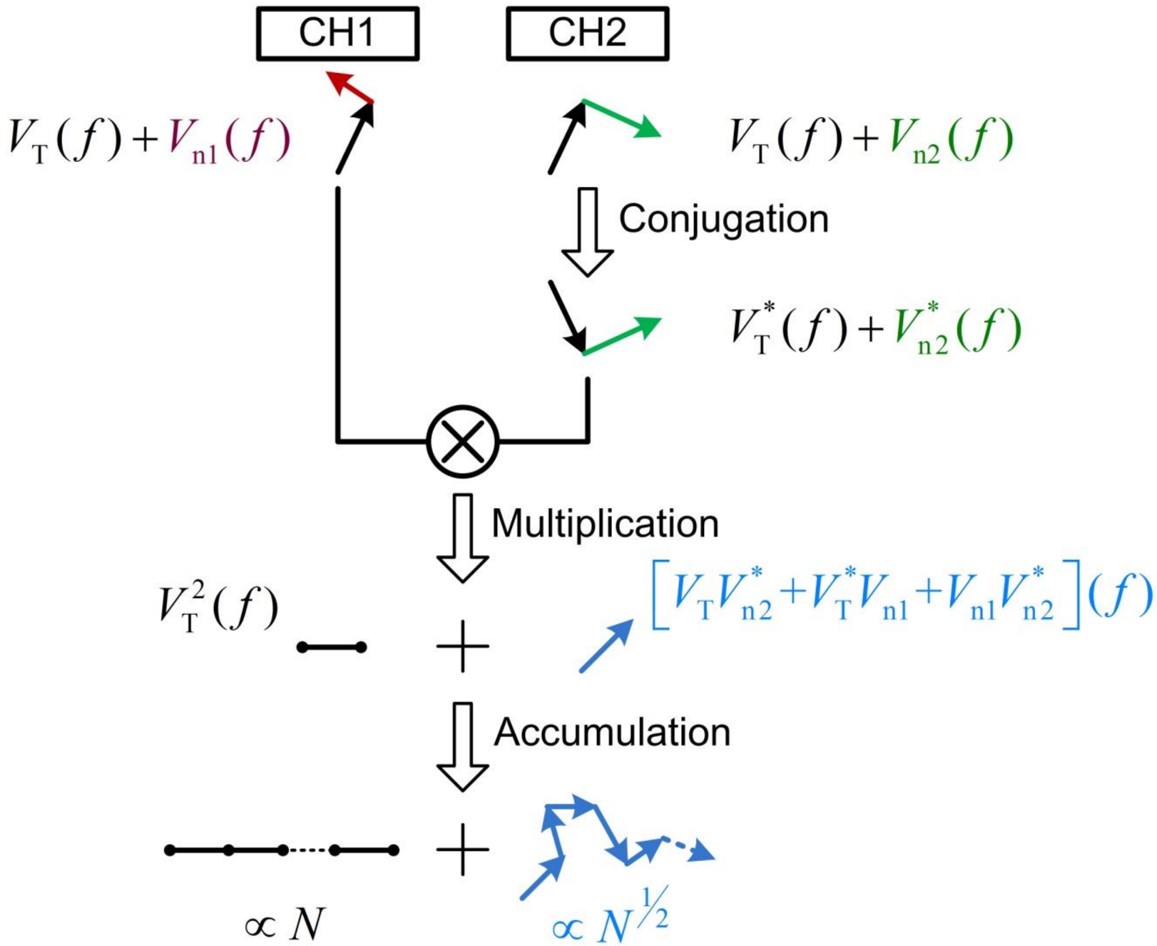


L. Strom, Metrologia 22, 229 (1986)

To achieve 10 ppm in k_B determination:

- **5 additional connections for calibration**
- **keep the temperature of all electronics constant within 0.02 K**
- **measure the divider factors with uncertainty less than 0.5 ppm**
- **accumulate data for more than 1 year!**

Digital signal processing in frequency domain

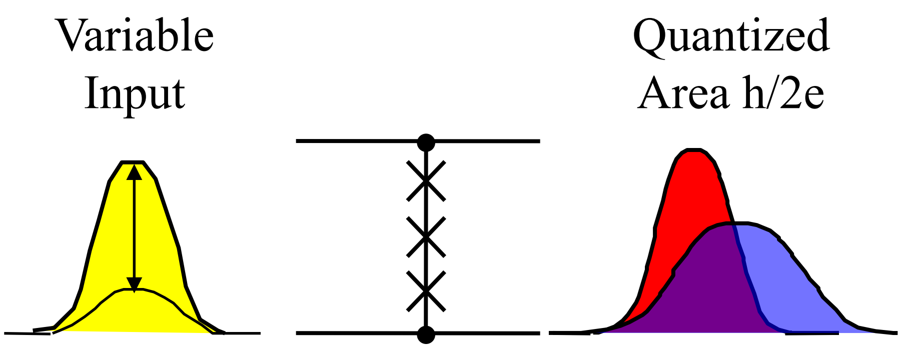


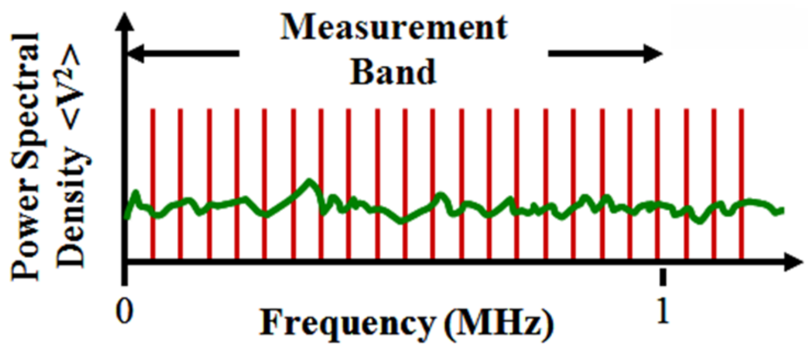
- **Brixy introduced fast and accurate ADC to JNT**
- **Digital signal processing in frequency domain**
- **Bandwidth can be defined accurately**

H. Brixy et. al., Temperature: It’s measurement and control in science and industry, vol 6, 993 (1992)

Quantum voltage noise source

Josephson Pulse Quantizer



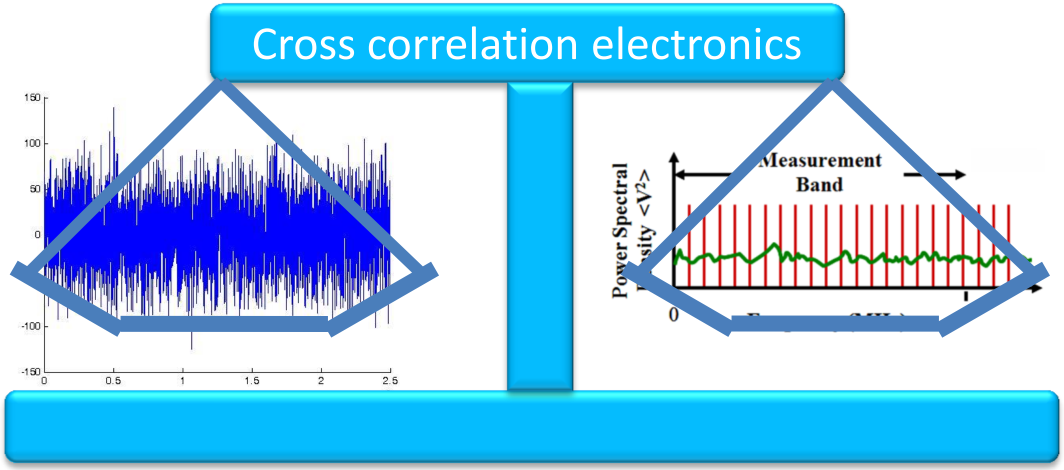


$$\int V(t)dt = \frac{h}{2e}$$

- **Samuel Benz, Clark Hamilton (NIST)**
- **quantum accurate (<<1 ppm up to 4 MHz)**
- **calculable PSD**
- **arbitrary distribution**

S.P. Benz, and C. A. Hamilton Appl. Phys. Lett. 68, 3173 (1996)

Quantum voltage calibrated noise thermometer



Johnson noise

$$S_T \Rightarrow R_K = \frac{h}{e^2}$$

Quantum voltage noise

$$S_V \Rightarrow K_J^2 = \frac{4e^2}{h^2}$$

- **John, Martinis (NIST) suggested**
- **NIST, NIM, NMIJ**

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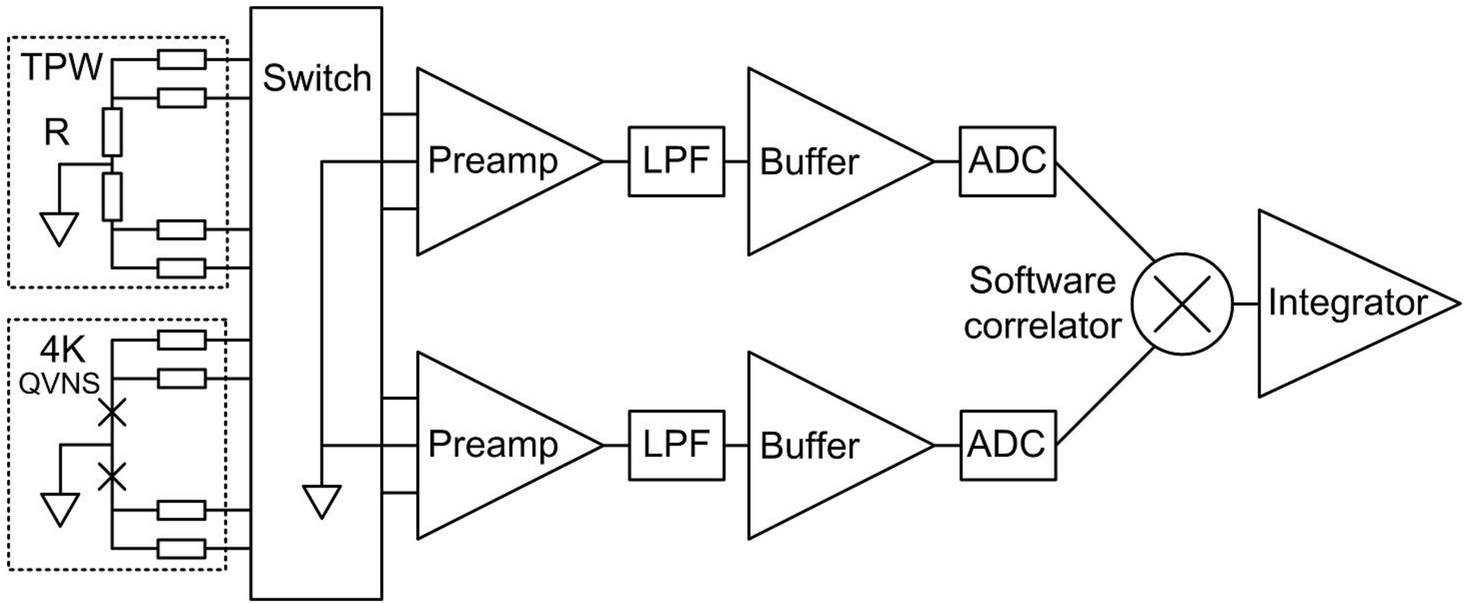
QVNS-JNT system



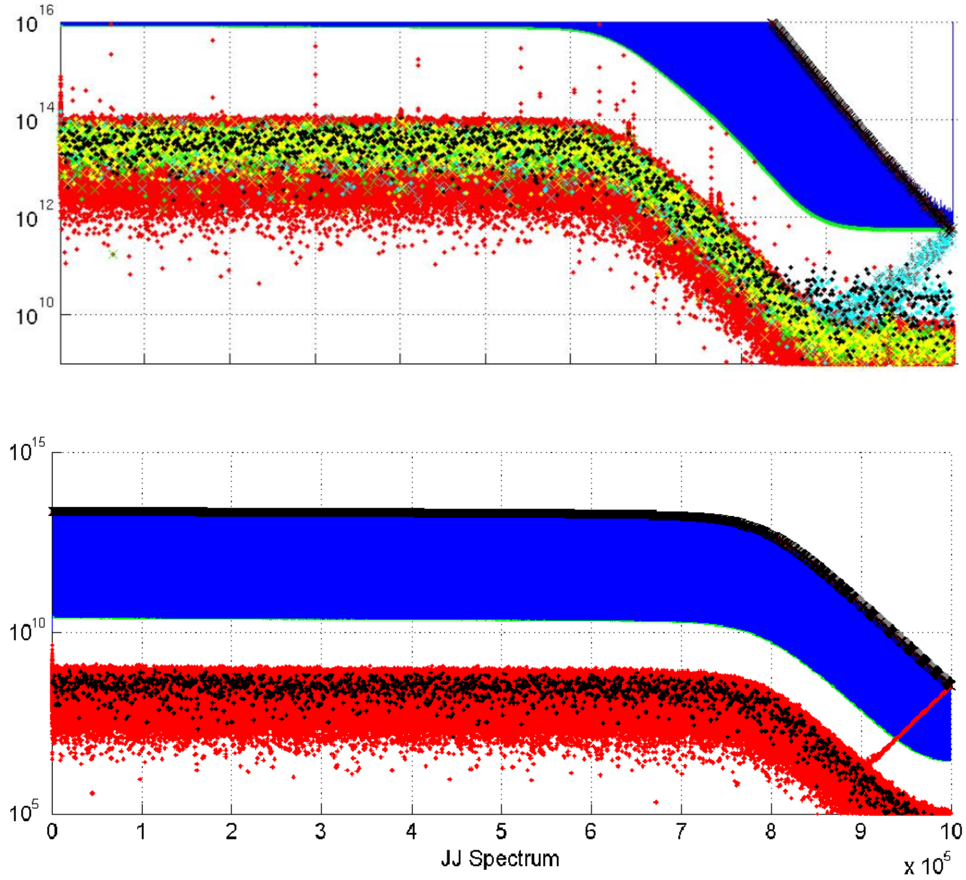
$$S_R = 4kT_W X_R R_K$$

$$S_{\text{Q-calc}} = D^2 N_J^2 f_s M / K_J^2$$

$$k = \frac{D^2 N_J^2 f_s M}{4T_W X_R R_K K_J^2} \frac{\langle S_R \rangle}{\langle S_Q \rangle}$$

Qu et al., *Metrologia* 52 S242 (2015)

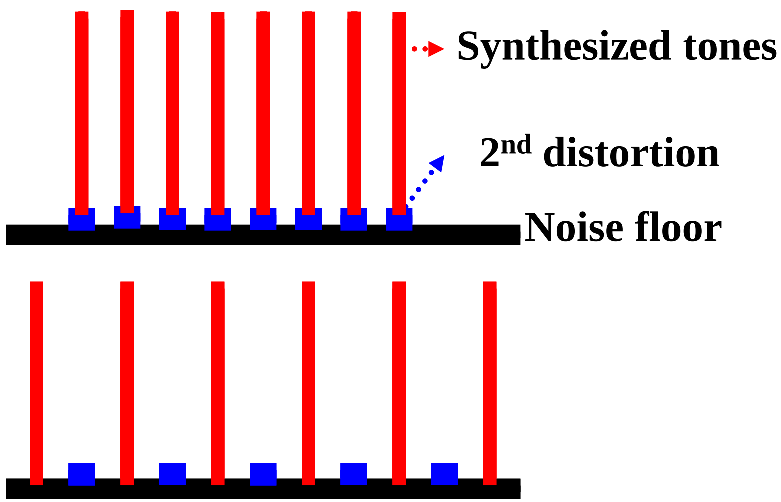
Shielding and grounding

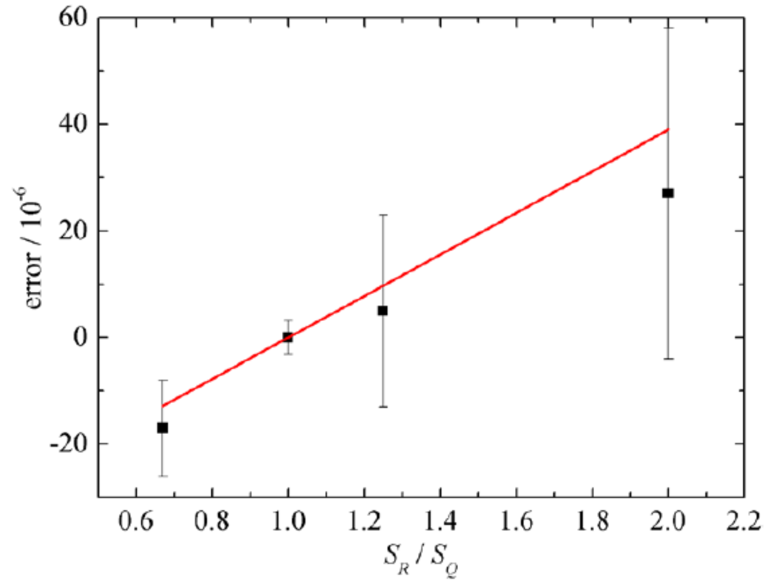


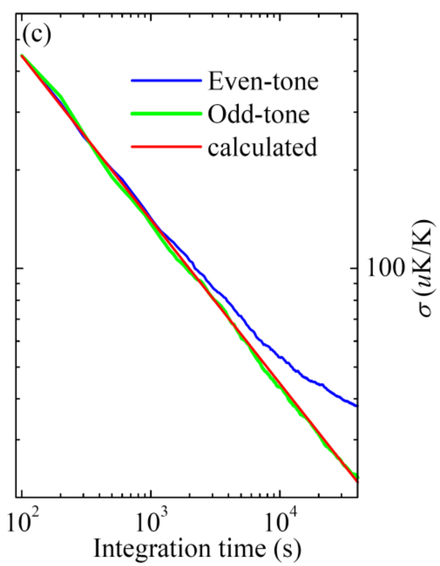
- underground screened room
- shielding with aluminum and high-permeability nickle-alloy boxes
- powered by batteries
- eliminate ground loop

Measured spectra of the synthesized quantum noise waveform with (upper) and without (lower) observable EMI, blue green, and red are auto-correlation in each channel, and correlation spectra, respectively, and black × is the synthesized tones.

Effect of nonlinearity

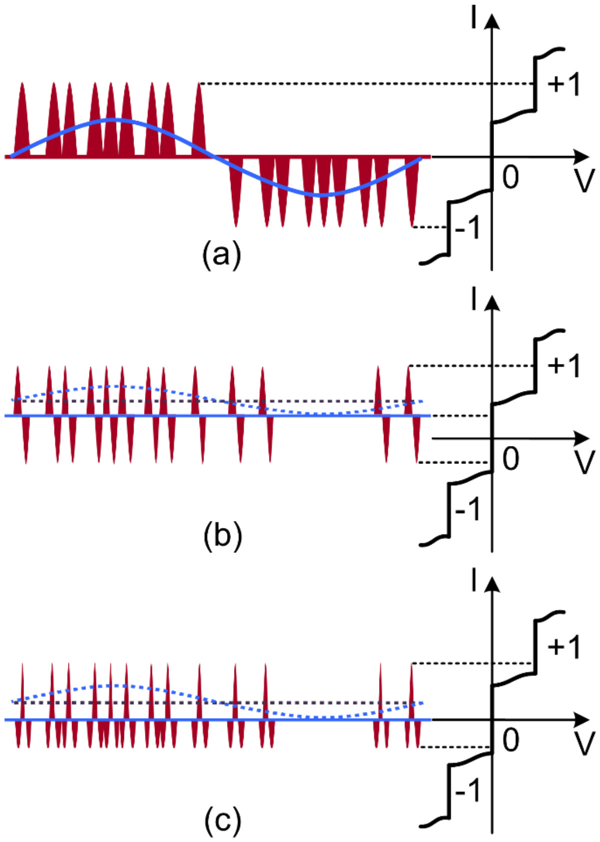


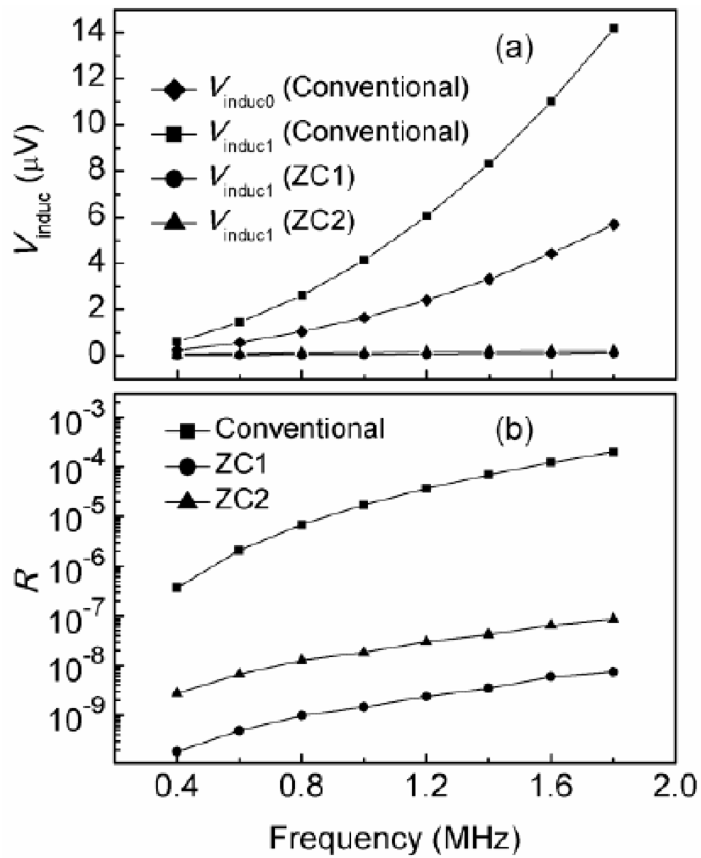




- **nonlinearity introduces significant errors**
- **PSDs are the same, Gaussian distribution, uncorrelated noise power are the same**
- **change the voltage of QVNS without changing any other parameters to measure the nonlinearity effect**
- **~0. 4×10^{-6} error for 1% mismatch**

Zero-compensation method for QVNS synhtesis

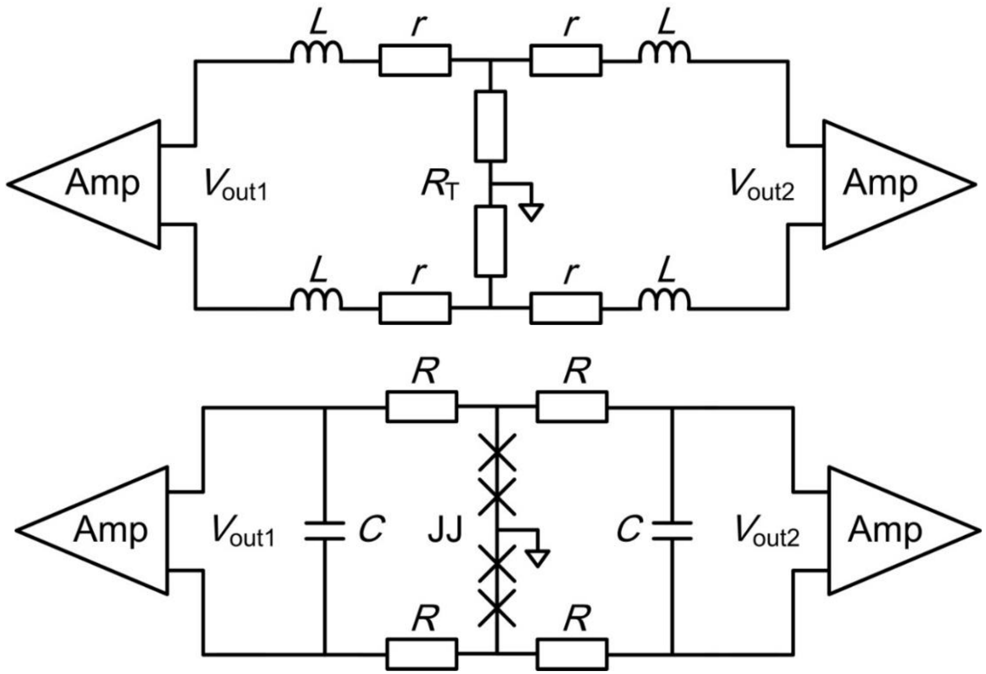




Qu et al., *IEEE Trans. Appl. Supercon.* (2015)

Match the noise sources and transmission lines

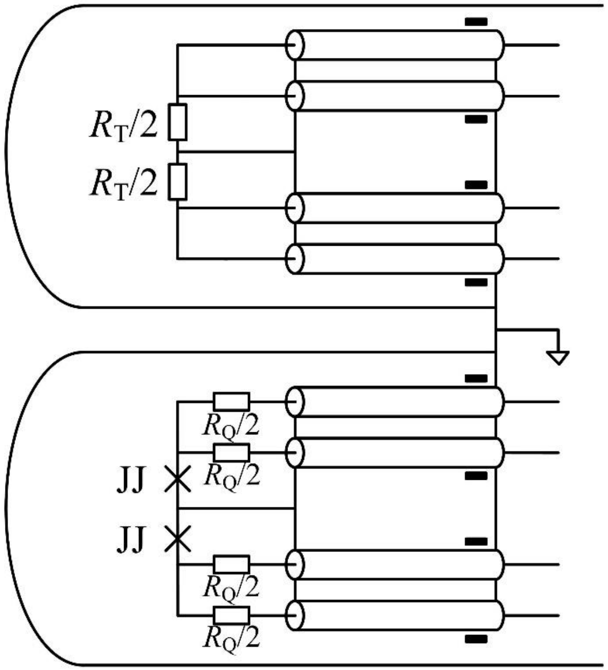
The 2015 determination： $R_T = R_Q = 200\Omega$

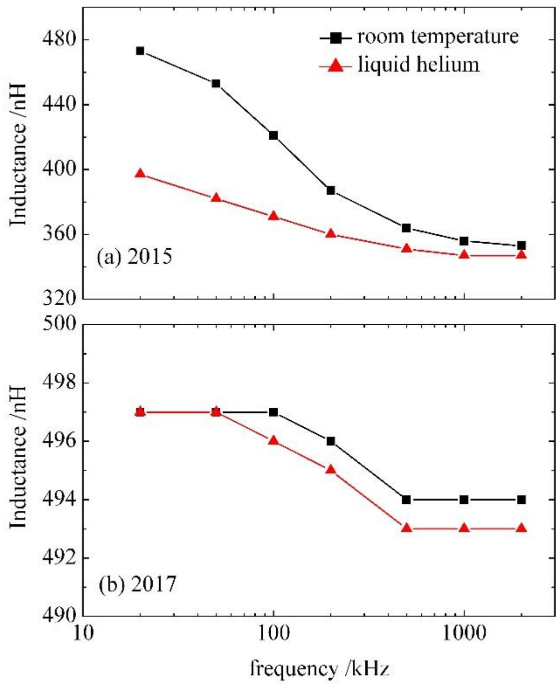


- **insert uncorrelated resistor to match both the noise powers and impedances**
- **insert trimming inductance and capacitance to match the transmission lines**

Match the noise sources and transmission lines

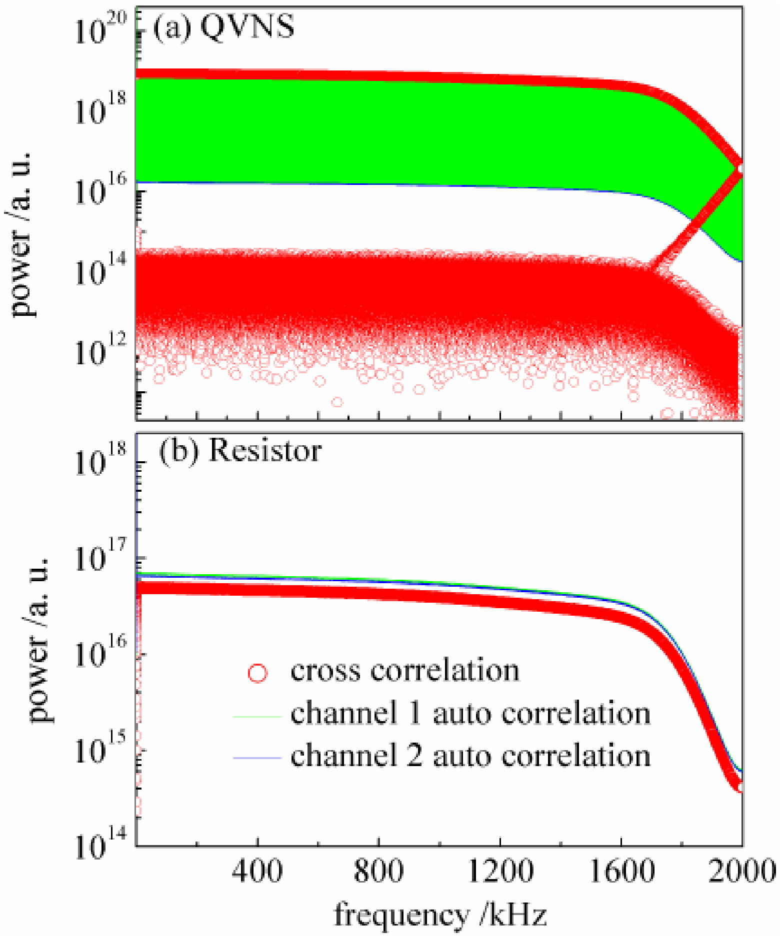
The 2017 determination： $R_T = R_Q/2 = 100\Omega$

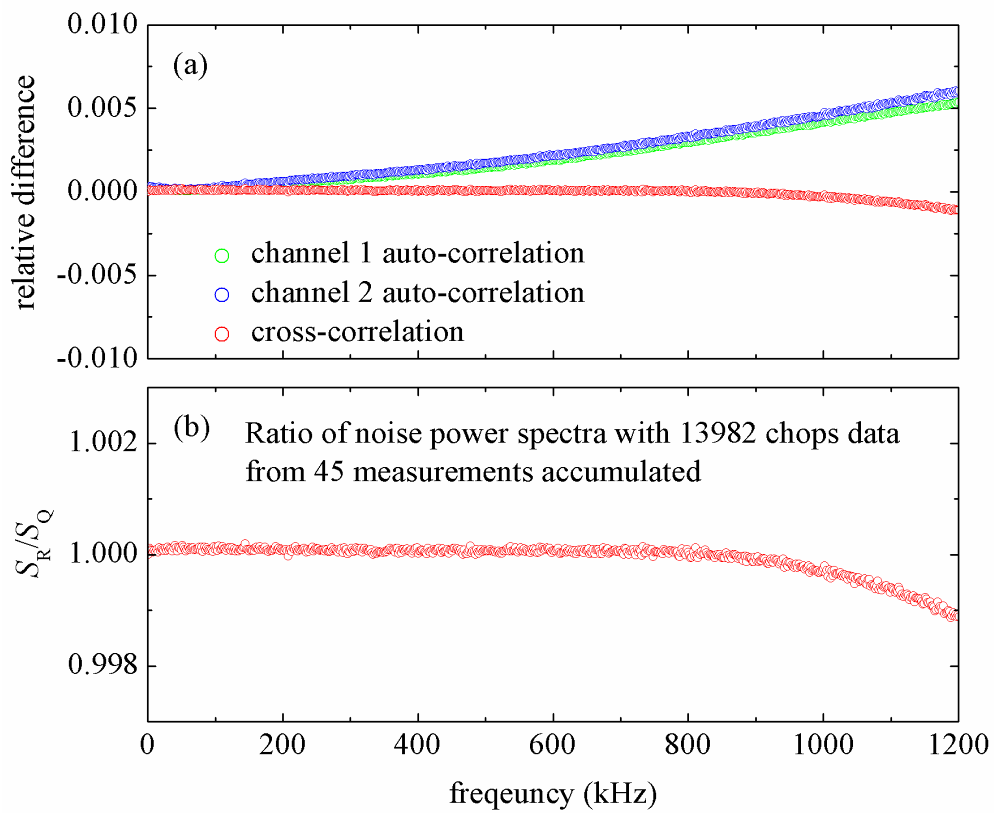




- **Remove all the trimming components, keep the identical connecting leads**
- **Coaxial input networks to get well defined transmission line impedances**
- **Use Be-Cu coaxial cables to get the same inductance at different temperature**

Measurement result

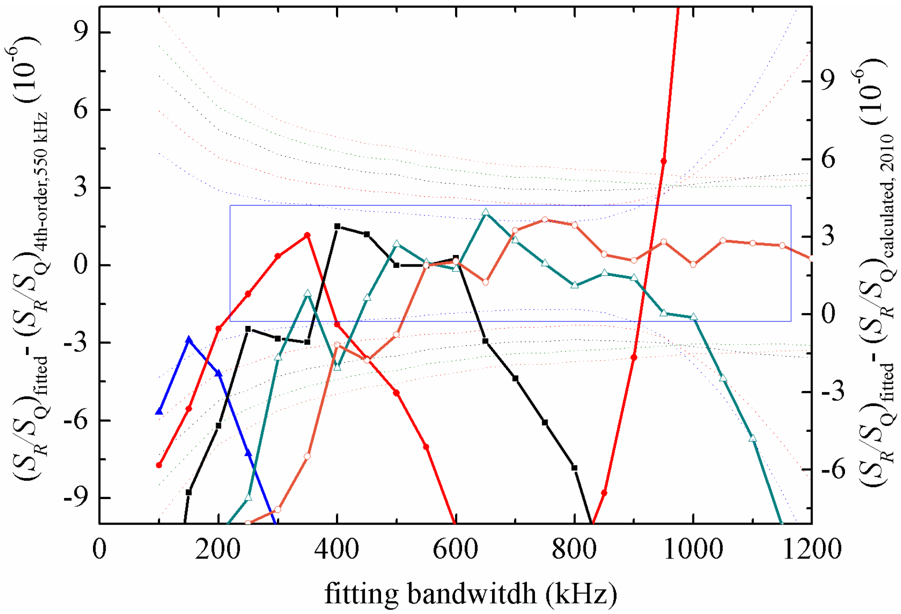




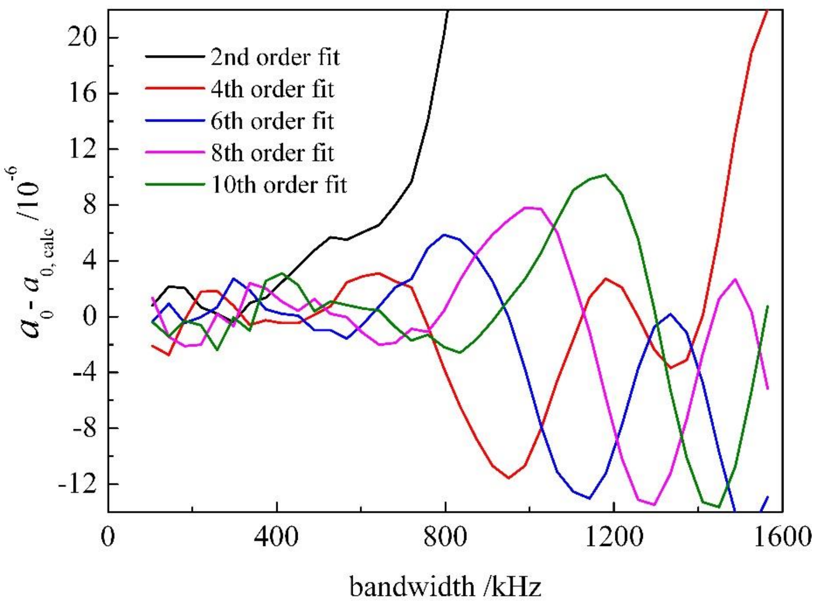
$$k = \left. \frac{\left\langle V_R^2 \right\rangle}{\left\langle V_Q^2 \right\rangle} \right|_{f=0} \frac{\left\langle V_Q^2 \right\rangle_{cal}}{4TR}$$

Polynomial fit

2015



2017



$$R(f) = \frac{S_R}{S_Q}\left(1 + a_2 f^2 + a_4 f^4 + a_6 f^6 + \ldots\right)$$

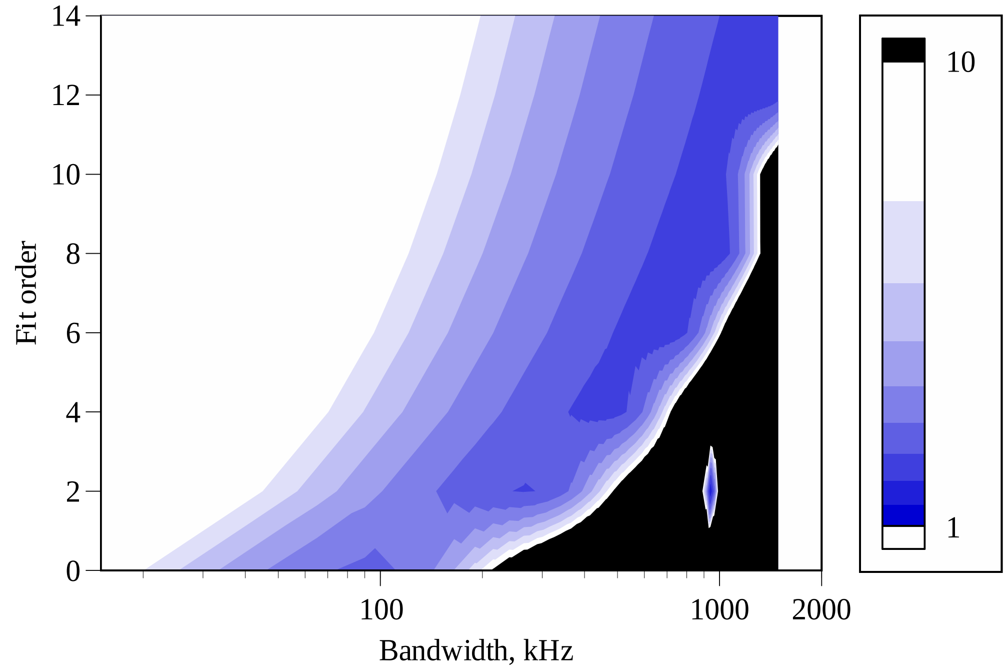
- **short connections-lumped components**
- **ambiguity–which model and bandwidth to select?**

Model selection method

$$\hat{\sigma}^2_{\text{tot}} = \hat{\sigma}^2_{\alpha} + \hat{\sigma}^2_{\beta},$$

$$\hat{\sigma}^2_{\alpha} = \sum_d \hat{p}(d)\hat{\sigma}^2_{\hat{a}_0(d),\text{ran}},$$

$$\hat{\sigma}^2_{\beta} = \sum_d \hat{p}(d)(\hat{a}_0(d) - \hat{\bar{a}}_0)^2.$$



- **cross-validation method to estimate selection probability of each model** (Kevin, Coakley et. al., *Metrologia* 54 204 (2017))
- **select the optimal polynomial model and bandwidth by minimizing the uncertainty that accounts for both random and systematic effects**

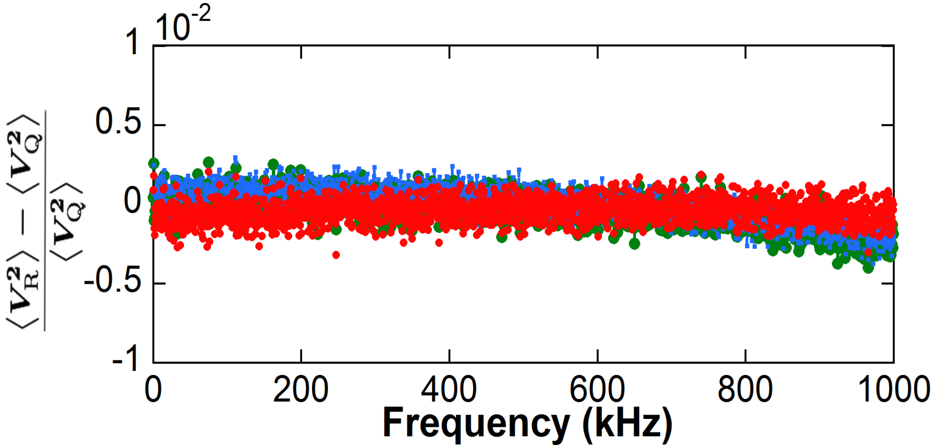
Measurement results

	2015	2017	
k_B /10^{-23} J/K	1.3806513	1.3806497	
$u_r(k_B)$ / 10^{-6}	3.9	2.7	
Component			Correlation
Statistical	3.2	2.4	0
Correction Model ambiguity	1.8	1.0	0
Fitting bandwidth	NA	0.6	0
Dielectric losses	1.0	0.2	0
EMI	0.4	0.4	0
Nonlinearity	0.1	0.1	1
R	0.5/0.2	0.2	1
TPW	0.4	0.4	1
QVNS waveform	0.1	0.1	1

JNT Development and Measurements at NIST

Recent system development at NIST:

- Input impedance adjustment to match imaginary and real part of connections to the Quantum Voltage Noise Source and the sense resistor in the Triple-point cell.

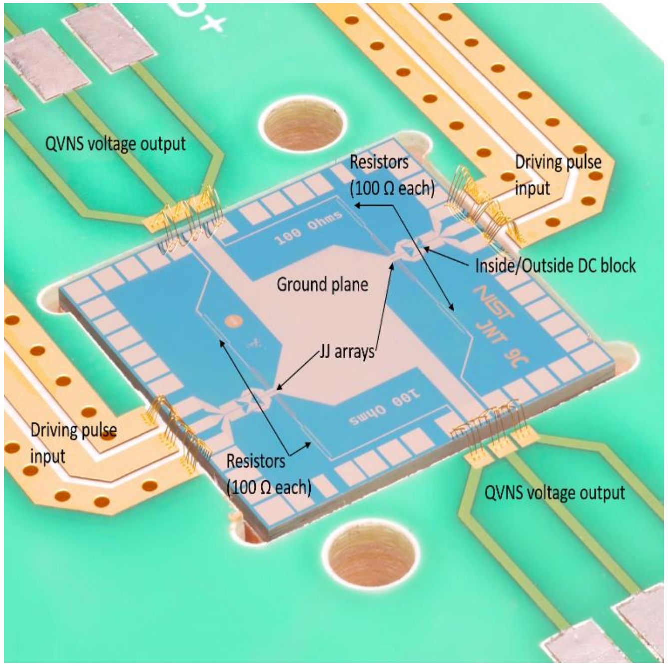


- ChA auto-corr
- ChB auto-corr
- Cross-corr

- Switchable signal and grounding path at the input of the amplifiers to eliminate effects of floating amplifiers and decrease switching transients.
- Improved system wiring to reach differential crosstalk suppression of more than 110dB.
- Battery operation of the electronics with switchable sets for continuous operation.

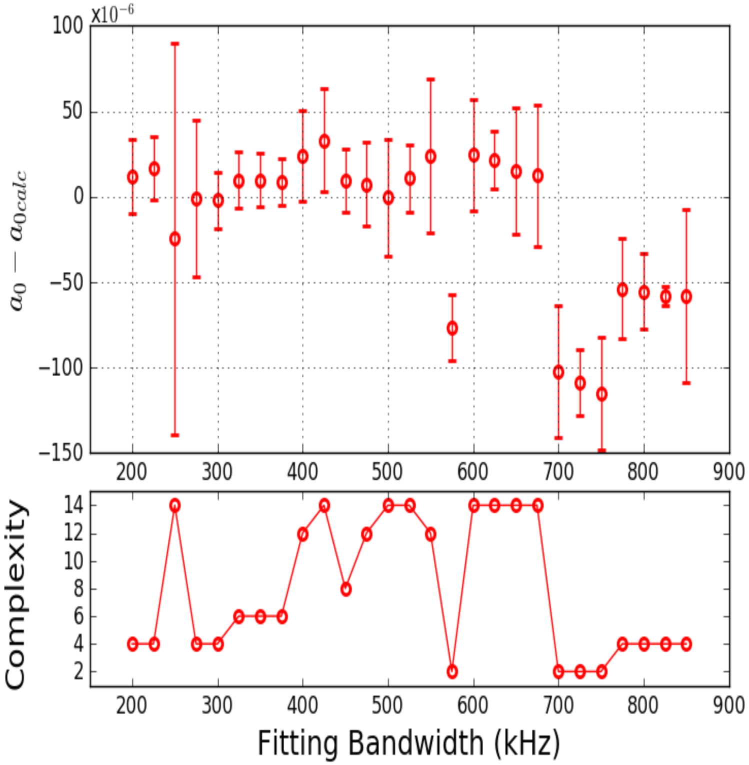
JNT Development and Measurements at NIST

- New QVNS chip with inner/outer DC block on chip:



- Measurements electronics, battery power supplies, JNT chip and analysis software has been made available to NIM within the cooperation agreement between NIST and NIM.

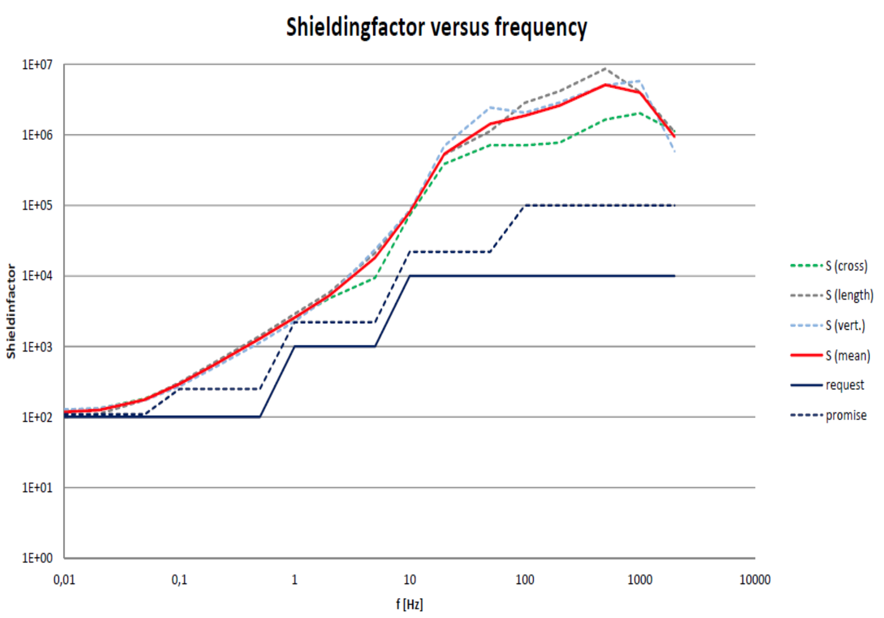
- Data analysis using a bootstrap method to automatically select the bandwidth-dependent order of even-order polynomial fit to JJ/R ratio measurements.



Example for 11 day measurement in noisy environment,

JNT Development and Measurements at NIST

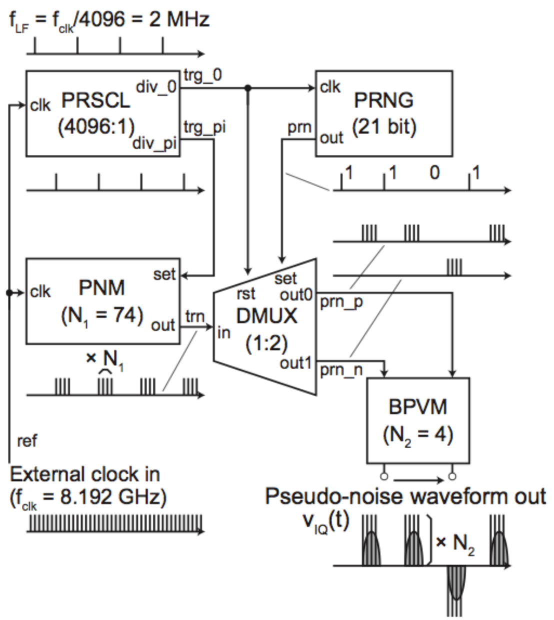
- The JNT project at NIST suffered from EMI problems since a high-quality shielded room was not available.
- Since early 2017 the measurement system could be placed into a high quality magnetically shielded room with excellent low frequency shielding and rf-shielding of typically 100 dB at 1 MHz and above.

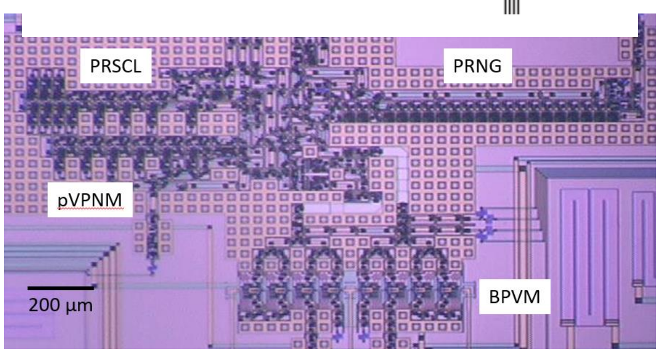


- **Since February 2017 measurements are being performed with the aim to present a value for Boltzmann's constant *k* with an uncertainty of less that 5 ppm to support the results at NIM in the NIST/NIM cooperation.**

Current status of JNT at NMIJ

Integrated QVNS

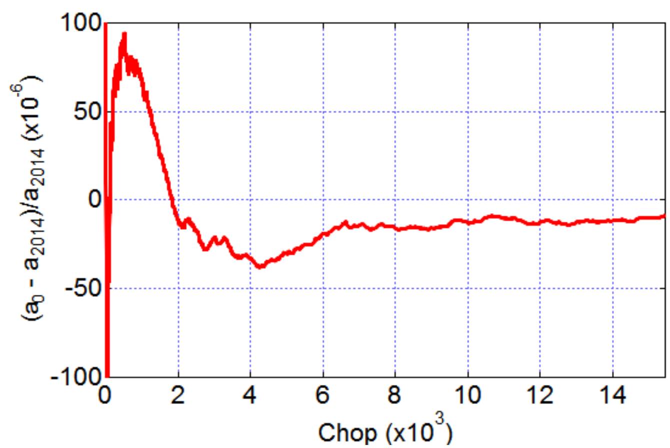


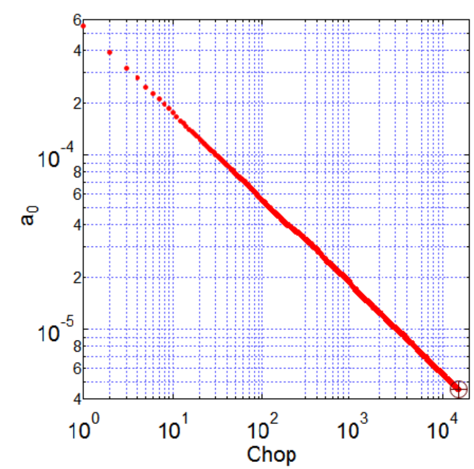


Offset from k_{2014} : - 9.1 ppm

Combined uncertainty : 4.6 ppm

Accumulated for 41 days





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3 Electronic measurement of kB

4 **Conclusion**

Summary

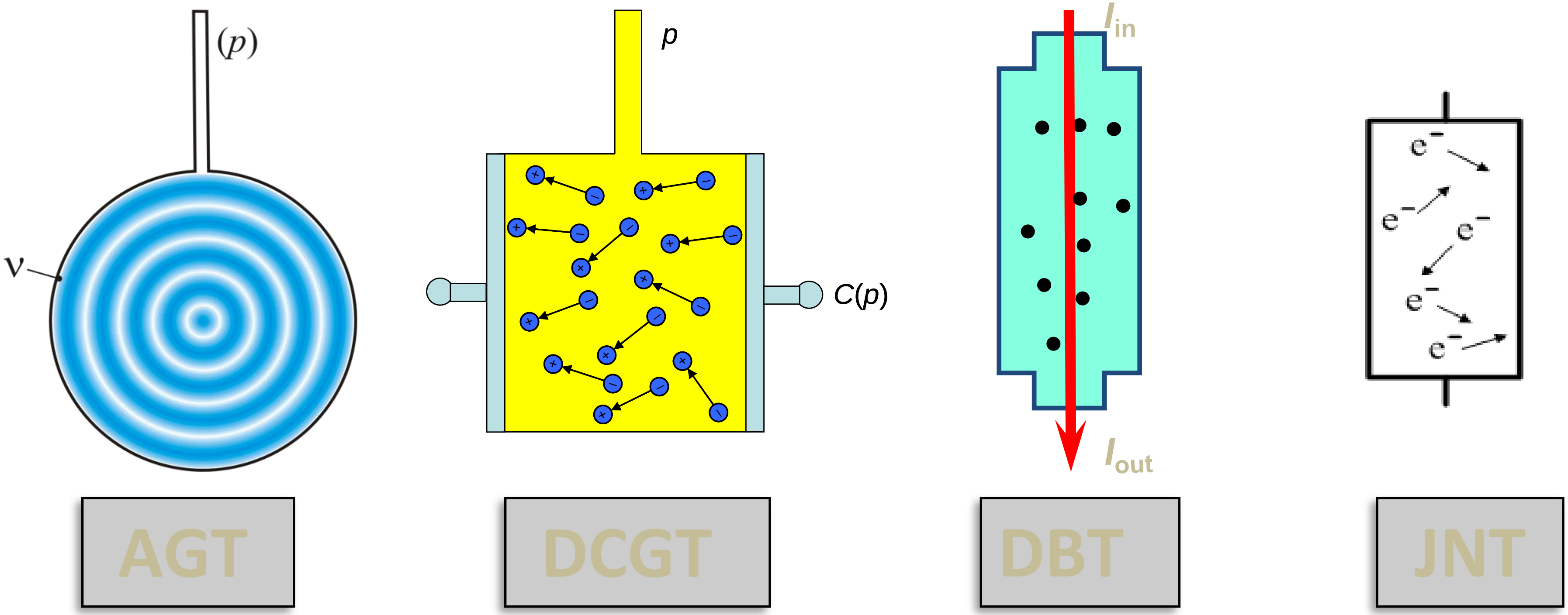
- **Technology breakthroughs (switch correlator, ADC, QVNS) made it possible to realize absolute measurement by Johnson noise thermometry**

- **We measured the Boltzmann constant with a relative uncertainty of 2.7 ppm that meet the second requirements of CCT to proceed the kelvin redefinition**

- **Purely electronic approach, provide strong assurance that there are no major systematic errors in the k determination by primary gas thermometry**

Thanks for your attention!

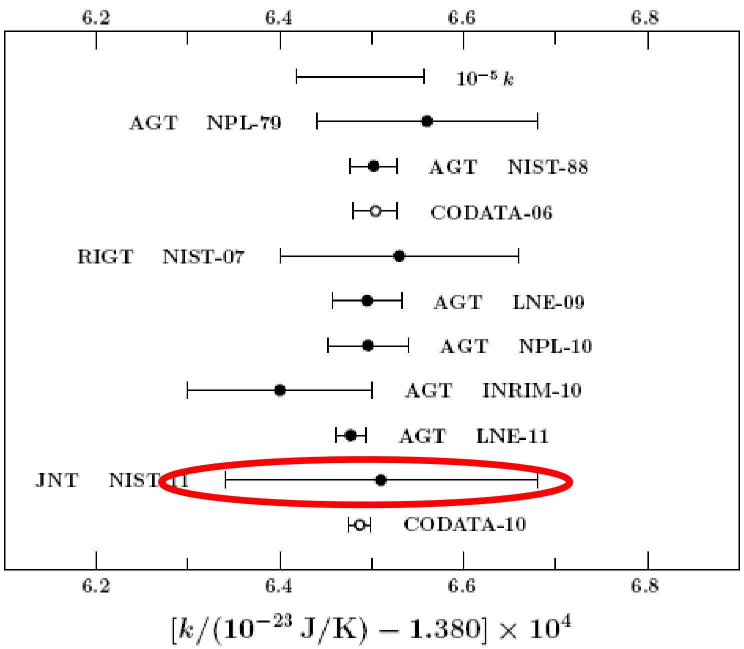
Primary thermometry



- **primary gas thermometry limited by non-ideal properties of real gas**
- **JNT uses electron gas**
- **pure electronic approach, attracting increasing interest**

Joachim Fischer, Metrologia 52 S364, 2015

Electronic measurement of k_B



CODATA 2010 k_B input data

- **NIST reported first electronic measurement of k_B with $u_r = 12.1 \times 10^{-6}$**
- **CCT required at least two methods with $u_r < 3\times10^{-6}$ to redefine the kelvin**
- **NIST, NIM, NMIJ, pursuing even lower uncertainty**

Mohr et al., *Rev. Mod. Phys.* 84 1527 (2012)

Benz et al., *Metrologia* 48 142 (2011)