# Report to the $21^{st}$ CCTF: The uncertainties of [UTC-UTC($k$)]

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## 1 Introduction

In this report the new algorithm for the calculation of the uncertainties of [UTC-UTC($k$)] as published in Section 1 of Circular T is presented. This is the first step of a major revision of the algorithm for the time links generation. Indeed the BIPM Time Department is planning an important change on the time links generation. Until now the system makes use of non-redundant time links based on GNSS techniques and on TWSTFT. The idea is the use of least square technique for obtaining the time links solution by using as basic ingredients all available measurements of time links and solving a redundant system. By applying the least square technique to the calculation of time links the related uncertainty is obtained too. This work will be developed in two steps demanding important changes in the algorithm, in the results analysis and in the software. The Circular T will evolve in consequence. The first step of this work, described in this report, consists in the development of the algorithm with its application to the evaluation of the uncertainty of [UTC-UTC($k$)]. The implementation of this part of the algorithm on the Circular T is planned in the next few months. Details on the algorithm are presented with the indication of the differences with respect to the current method used for the calculation.

## 2 The basic idea

The time measurements available to compare clocks are evolving in a significant way. For the clock comparison, we dispose until now of two independent techniques, the GNSS (until now GPS and GLONASS satellite systems) and TWSTFT. In the next future other satellite systems such as GALILEO, BEIDOU etc. will be probably in operation and they could be used also as time transfer systems in UTC calculation. The TWSTFT already provides time measurements that are not used by the BIPM considering that only the measurement with respect to PTB are used. The current method in use at BIPM is based on a non-redundant system with only one technique for time links or a combination of two (TWSTFT and GPS PPP [1] or GPS and GLONASS [2]), where comparison to a unique pivot laboratory (PTB) are conducted. Following the Recommendation 4 of CCTF 2012 [3] asking to all TAI participating laboratories to supply data from 3 GNSS receivers traceable to their local realization of UTC, many additional measurements are provided by laboratories but are not used for TAI generation. The main idea is to introduce a redundant time link system to make full benefit of the available measurements. The solution is obtained by solving the system with the least square technique. This is a significant evolution in time link calculation implying important changes in term of algorithms and result checks. For this reason we decided to proceed step by step to control each details for achieving a robust solution. The first step of the planned work, described in this report, consists in the development of the algorithm for redundant time links. In particular the effects on the uncertainty evaluation of [UTC-UTC($k$)] are studied and reported. In the second step the redundant time links are introduced in UTC calculation.

# 3 The uncertainty of [UTC-UTC($k$)]

The uncertainties of [UTC-UTC($k$)], indicated by uA and uB in the Section 1 of Circular T [4], depend mainly on the link uncertainty, declared in Section 5 of Circular T, and on the weight of the laboratory in the UTC calculation. With the evolution of the time link techniques (introduction of GPS PPP in UTC calculation and abandon of the less performing GPS SC technique) and the consequent improvement of the link uncertainties we observed three main problems on the current algorithm [5, 6] used for the uncertainty evaluation: 1) the uncertainties uA and uB for the pivot laboratory PTB are underestimated and unrealistic; 2) the uncertainties of all laboratories are strictly correlated to the USNO uncertainty (this is due also to its weight in UTC calculation); 3) if the value of the uncalibrated laboratories (until now arbitrarily fixed to 20 ns) is increased to a more realistic value all the laboratories will be affected by the uncertainties of uncalibrated links. The common problem of the current method, until now perfectly satisfactory for the national laboratories participating to UTC calculation, is the absence of correlations in the estimation of uncertainties. By using a particular participant of UTC (UTC(PTB) at present) as pivot it is not possible to exactly share the uncertainty between the PTB, that should correlate to all the laboratories, and the corresponding laboratory. In this kind of formalism the evaluation of the amount of correlations is not trivial. However with the introduction of redundant time links a different approach to the problem is considered [7]. The starting point is to decouple the uncertainty due to measurement noise to those characterizing the bias, indicated correspondingly by uStb and uCal in Section 5 of Circular T. Secondly to consider for GNSS time links an auxiliary time scale (ATS) as pivot. For the TWSTFT time links the PTB laboratory will remain the pivot even if in the case of redundant time links all the possible TWSTFT measurements should be considered. The consequences of this new approach and formalism, explained in details further in the text, are:

- for GNSS time links, the values of the calibration uncertainty of $L_k = (Lab_k - AST)$, uCal, are considered related only to the receiver of the laboratory indicated by "$Lab_k$" and not to the ATS. With this kind of formalism it is not necessary to evaluate correlations not easily valuables. There are no correlations involving the calibration uncertainty. The value of uCal for ATS is not relevant for the calculation.

- TWSTFT links are considered, as in the current version of the algorithm, with respect to PTB or in general with respect to a time scale participating to the calculation of UTC. In this case the correlations cannot be evaluated and this component is missing.

- the GNSS time links, all related to ATS time scale, are correlated in term of measurement uncertainty (uStb component). This correlation can be evaluated because the ATS is external to UTC calculation. This is the correlation component included in the calculation for solving the underestimation of the uA value of the PTB with respect to its weight in UTC calculation.

In the next Section a short presentation of the theory, two small examples are presented to explain the new proposed method.

## 3.1 The theoretical evaluation of the uncertainty of [UTC-UTC($k$)]

In the case of GNSS techniques a link measurement is indicated as $L_k = (Lab_k - AST) + b_{Gk}$, where $b_{Gk}$ is the bias of the receiver. The uncertainties are decoupled correspondingly so that uStb (uncertainty due to the measurement) relates to $(Lab_k - AST)$ and uCal (calibration uncertainty) to $b_{Gk}$. For the TWSTFT the following formalism is considered: $L_{k,l} = (Lab_k - Lab_l) + b_{k,l}$ when the measurement is related to the laboratory $k$, and $L_{l,k} = (Lab_l - Lab_k) + b_{l,k}$ for the laboratory $l$. The solution, indicated by $(UTC - Lab_k) + b_k$ with the associated uncertainty is obtained in two steps:

- In the first step, using this formalism, the solution for $(UTC - Lab_k)$ is obtained by solving the system AX = L where L is composed by elements above described, A the design matrix, and X, the solution, composed by $UTC - Lab_k$, $UTC - AST$ and the bias $b_k$. For obtaining the solution the weights attributed to the laboratories in the calculation of UTC are reported in the last line of the matrix A. The variances and covariances matrix, indicated by $S_L$ are composed by the link uncertainties and the correlation coefficients. The solution is obtained in the least square sense:

$$X = (A^T \times S_L^{-1} \times A)^{-1} \times A^T \times S_L^{-1} \times L \tag{1}$$

with the variance and covariance matrix:

$$S_X = (A^T \times S_L^{-1} \times A)^{-1} \tag{2}$$

The key point is to observe that, in the diagonal of $S_X$ where the uncertainties of $UTC - lab_k$ are reported, the uncertainty components are already decoupled.

In the first part of the vector the uncertainties correspond to the measurement uncertainty (uStb for the links) and the second part to the calibration uncertainty (uCal). With the second step the combined uncertainties of [UTC-UTC($k$)] are found.

- In the second step the final solution is found by solving in the least square sense the system CY=X, where X is the solution obtained at the first step and C is the design matrix combining the uncertainties. The components of Y (the final solution), are indicated as follow: $UTC - Lab_k + b_k$, $UTC - AST$ and the bias $b_k$. The solution $Y$ and the corresponding uncertainty $S_Y$ are obtained:

$$Y = (C^T \times S_X^{-1} \times C)^{-1} \times C^T \times S_X^{-1} \times X \tag{3}$$

with the variance and covariance matrix:

$$S_Y = (C^T \times S_X^{-1} \times C)^{-1} \tag{4}$$

The diagonal of the matrix $S_Y$ reports the uncertainty of [UTC-UTC($k$)]. In the next subsection two examples are reported giving details on the form of the matrices and the formalism.

## 3.2 Example 1 - Non redundant Links

In this example the case of non-redundant time links is considered. Only four links with the corresponding biases are considered. In this case the matrix L is:

$$L = \begin{bmatrix} Lab_1 - ATS + b_{G1} \\ Lab_2 - ATS + b_{G2} \\ Lab_3 - ATS + b_{G3} \\ Lab_4 - ATS + b_{G4} \\ b_{G1} \\ b_{G2} \\ b_{G3} \\ b_{G4} \end{bmatrix}$$

The variance and covariance matrix indicated by $S_L$ is:

$$S_L = \begin{bmatrix} u^2_{Stb(Lab_1)} + u^2_{Cal(b_1)} & u^2_{ATS} & u^2_{ATS} & u^2_{ATS} & u^2_{Cal(b_1)} & 0 & 0 & 0 \\ u^2_{ATS} & u^2_{Stb(Lab_2)} + u^2_{Cal(b_2)} & u^2_{ATS} & u^2_{ATS} & 0 & u^2_{Cal(b_2)} & 0 & 0 \\ u^2_{ATS} & u^2_{ATS} & u^2_{Stb(Lab_3)} + u^2_{Cal(b_3)} & u^2_{ATS} & 0 & 0 & u^2_{Cal(b_3)} & 0 \\ u^2_{ATS} & u^2_{ATS} & u^2_{ATS} & u^2_{Stb(Lab_4)} + u^2_{Cal(b_4)} & 0 & 0 & 0 & u^2_{Cal(b_4)} \\ u^2_{Cal(b_1)} & 0 & 0 & 0 & u^2_{Cal(b_1)} & 0 & 0 & 0 \\ 0 & u^2_{Cal(b_2)} & 0 & 0 & 0 & u^2_{Cal(b_2)} & 0 & 0 \\ 0 & 0 & u^2_{Cal(b_3)} & 0 & 0 & 0 & u^2_{Cal(b_3)} & 0 \\ 0 & 0 & 0 & u^2_{Cal(b_4)} & 0 & 0 & 0 & u^2_{Cal(b_4)} \end{bmatrix}$$

The system is solved in two steps as explained in the section 3.1; in the first step we find the solution concerning the measurement and the bias separately and in the second we obtain the complete solution. The system is indicated by AX=L where A is the design matrix and X the solution where the results are reported decoupled in term of measurement and bias uncertainties. The solution is found in the least square sense by applying the equations 1 and 2.

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ w_{Lab_1} & w_{Lab_2} & w_{Lab_3} & w_{Lab_4} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} UTC - Lab_1 \\ UTC - Lab_2 \\ UTC - Lab_3 \\ UTC - Lab_4 \\ UTC - ATS \\ b_{G1} \\ b_{G2} \\ b_{G3} \\ b_{G4} \end{bmatrix}$$

The matrix A reports the weights of the laboratories participating to the calculation of UTC for applying the time scale equation [5, 6]. The total solution is obtained by solving the system CY=X in least square sense by applying 3 and 4, where C and Y are:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} UTC - Lab_1 + b_1 \\ UTC - Lab_2 + b_2 \\ UTC - Lab_3 + b_3 \\ UTC - Lab_4 + b_4 \\ UTC - ATS \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

### 3.2.1 Numerical examples

The ensemble of links presented in subsection 3.2 is taken in to consideration. In this case the example is very similar to the case considered in the current algorithm but that AST is the pivot instead of PTB. Different cases for the parameter values are considered to show how they affect the solution. The statistical uncertainty uStab is fixed equal to 0.3 ns for all the laboratories. The weights of the laboratories are chosen as follow: $w_{Lab_1} = 0.25$, $w_{Lab_2} = 0$, $w_{Lab_3} = 0.35$, $w_{Lab_4} = 0.4$. In the first case all uCal are put equal to 1 ns and the value for the ATS uncertainty, constituting the correlations, will be considered equal to 0 ns, 0.03 ns, and 0.1 ns respectively. In this report the component of uncertainty called uA in Circular T is indicated as uf, the uncertainty relating to frequency measurements. Table 1 reports the results relating to u, the total uncertainty and uf, the measurement uncertainty.

| | uATS=0 | uATS=0.03 ns | uATS=0.1 ns |
|---|---|---|---|
| u (UTC-$Lab_1$) | 1.0373 | 1.0370 | 1.0332 |
| u (UTC-$Lab_2$) | 1.0588 | 1.0582 | 1.0524 |
| u (UTC-$Lab_3$) | 1.0286 | 1.0283 | 1.0255 |
| u (UTC-$Lab_4$) | 1.0242 | 1.0240 | 1.0216 |
| u (UTC-ATS) | 0.1762 | 0.1779 | 0.1939 |
| uf (UTC-$Lab_1$) | 0.2758 | 0.2744 | 0.2600 |
| uf (UTC-$Lab_2$) | 0.3479 | 0.3462 | 0.3280 |
| uf (UTC-$Lab_3$) | 0.2409 | 0.2397 | 0.2272 |
| uf (UTC-$Lab_4$) | 0.2215 | 0.2204 | 0.2088 |

Table 1. The total and the measurement uncertainties varying with ATS uncertainty values.

The results show the impact of the correlations on the evaluation of the uncertainties; the case of absence of correlations is also considered. It is clear that by increasing the value of correlation the value of the uncertainties decrease for all the laboratories except for AST. The role of the weights in the uncertainty evaluation is enhanced in this sample case; laboratories with bigger weights are affected by a smaller uncertainties. In Table 2 different values for the calibration uncertainty are tested to estimate the effect on the final results. In this example the statistical uncertainty uStab is fixed equal to 0.3 ns for all the laboratories, the weights of the laboratories are: $w_{Lab_1} = 0.25$, $w_{Lab_2} = 0$, $w_{Lab_3} = 0.35$, $w_{Lab_4} = 0.4$. All the values for uCal are put equal to 1 ns except for the laboratory indicated in Table 2, the value for the ATS uncertainty, giving the correlation, is fixed equal to 0.03 ns.

| | $uCal_1 = 200$ | $uCal_2 = 200$ | $uCal_4 = 200$ |
|---|---|---|---|
| u (UTC-$Lab_1$) | 199.8645 | 1.0370 | 1.0370 |
| u (UTC-$Lab_2$) | 1.0582 | 200.0003 | 1.0582 |
| u (UTC-$Lab_3$) | 1.0283 | 1.0283 | 1.0283 |
| u (UTC-$Lab_4$) | 1.0240 | 1.0240 | 199.8645 |
| u (UTC-ATS) | 0.1779 | 0.1779 | 0.1779 |
| uf (UTC-$Lab_1$) | 0.2744 | 0.2744 | 0.2744 |
| uf (UTC-$Lab_2$) | 0.3462 | 0.3462 | 0.3462 |
| uf (UTC-$Lab_3$) | 0.2397 | 0.2397 | 0.2397 |
| uf (UTC-$Lab_4$) | 0.2204 | 0.2204 | 0.2204 |

Table 2. The total and measurement uncertainties varying with the calibration uncertainty values in the case of non redundant links.

The results show that in the case of a laboratory with non calibrated equipment when a very big value is attributed to uCal the propagation don't affect the other laboratories. The total uncertainty depends on the weight of the laboratory and on its own uncertainty.

## 3.3 Example 2 - Redundant Links

In this subsection the case of redundant links is considered, the same ensemble of laboratories of the previous example is considered by adding one TWSTFT redundant link.

$$L = \begin{bmatrix} Lab_1 - ATS + b_{G1} \\ Lab_1 - Lab_4 + b_{1,4} \\ Lab_2 - ATS + b_{G2} \\ Lab_3 - ATS + b_{G3} \\ Lab_4 - ATS + b_{G4} \\ b_{G1} \\ b_{1,4} \\ b_{G2} \\ b_{G3} \\ b_{G4} \end{bmatrix}$$

In this example there is one more bias then in the previous case, that of the TWSTFT link added to the ensemble of the links. The variance and covariance matrix indicated by $S_L$ is:

$$S_L = \begin{bmatrix} u^2_{Stb(Lab_1)} + u^2_{Cal(b_1)} & 0 & u^2_{ATS} & u^2_{ATS} & u^2_{ATS} & u^2_{Cal(b_1)} & 0 & 0 & 0 & 0 \\ 0 & u^2_{Stb(Lab_{1,4})} + u^2_{Cal(b_{1,4})} & 0 & 0 & 0 & 0 & u^2_{Cal(b_{1,4})} & 0 & 0 & 0 \\ u^2_{ATS} & 0 & u^2_{Stb(Lab_2)} + u^2_{Cal(b_2)} & u^2_{ATS} & u^2_{ATS} & 0 & 0 & u^2_{Cal(b_2)} & 0 & 0 \\ u^2_{ATS} & 0 & u^2_{ATS} & u^2_{Stb(Lab_3)} + u^2_{Cal(b_3)} & u^2_{ATS} & 0 & 0 & 0 & u^2_{Cal(b_3)} & 0 \\ u^2_{ATS} & 0 & u^2_{ATS} & u^2_{ATS} & u^2_{Stb(Lab_4)} + u^2_{Cal(b_4)} & 0 & 0 & 0 & 0 & u^2_{Cal(b_4)} \\ u^2_{Cal(b_1)} & 0 & 0 & 0 & 0 & u^2_{Cal(b_1)} & 0 & 0 & 0 & 0 \\ 0 & u^2_{Cal(b_{1,4})} & 0 & 0 & 0 & 0 & u^2_{Cal(b_{1,4})} & 0 & 0 & 0 \\ 0 & 0 & u^2_{Cal(b_2)} & 0 & 0 & 0 & 0 & u^2_{Cal(b_2)} & 0 & 0 \\ 0 & 0 & 0 & u^2_{Cal(b_3)} & 0 & 0 & 0 & 0 & u^2_{Cal(b_3)} & 0 \\ 0 & 0 & 0 & 0 & u^2_{Cal(b_4)} & 0 & 0 & 0 & 0 & u^2_{Cal(b_4)} \end{bmatrix}$$

The system indicated by AX=L where A is the design matrix and X the solution:

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ w_{Lab_1} & w_{Lab_2} & w_{Lab_3} & w_{Lab_4} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} UTC - Lab_1 \\ UTC - Lab_2 \\ UTC - Lab_3 \\ UTC - Lab_4 \\ UTC - ATS \\ b_{G1} \\ b_{1,4} \\ b_{G2} \\ b_{G3} \\ b_{G4} \end{bmatrix}$$

The solution X and the related variance and covariance matrix $S_X$ are obtained by the least square fit applying the equations 1 and 2. In this case the second step for obtaining the final solution is different from the previous example because of the presence of a redundant link. The system is indicated by CY=X where C and Y are:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} UTC - Lab_1 + b_1 \\ UTC - Lab_2 + b_2 \\ UTC - Lab_3 + b_3 \\ UTC - Lab_4 + b_4 \\ UTC - ATS \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

In the case of redundant links the bias of laboratories are calculated in the least square sense and weighted with respect to their uncertainty. The solution Y and the matrix of variance and covariance $S_Y$ are obtained in the least square sense by the equations 3 and 4.

### 3.3.1 Numerical Examples

A numerical example of the sample considered in subsection 3.3 is presented. In this case the value for ATS uncertainty is equal to 0.03 ns. The statistical uncertainty uStab is fixed equal to 0.3 ns for all the laboratories, the weights of the laboratories are: $w_{Lab_1} = 0.25$, $w_{Lab_2} = 0$, $w_{Lab_3} = 0.35$, $w_{Lab_4} = 0.4$ as in the previous section. Different values for uCal uncertainties are considered to show its impact on the final results. All the values are put equal to 5 ns except those indicated in Table 3.

We can conclude from Table 3 that uncertainty values are optimized in case of redundant links and big values of uncertainties do not affect the other values. The algorithm is robust to the different changes tested.

# 4 Application of the new algorithm to a real case

In this section we apply the method presented in Section 3.1 to the real case of UTC calculation. Two different cases will be considered to estimate the role of redundant time

| | $uCal_{1,4} = 1$ | $uCal_{1,4} = 200$ | $uCal_2 = 200$ | $uCal_3 = 200$ |
|---|---|---|---|---|
| u (UTC-$Lab_1$) | 0.9988 | 5.0021 | 3.5406 | 3.5406 |
| u (UTC-$Lab_2$) | 5.0119 | 5.0119 | 200.0003 | 5.0119 |
| u (UTC-$Lab_3$) | 5.0057 | 5.0057 | 5.0057 | 199.8588 |
| u (UTC-$Lab_4$) | 5.0027 | 5.0027 | 5.0027 | 5.0027 |
| u (UTC-ATS) | 0.1760 | 0.1760 | 0.1760 | 0.1760 |
| uf (UTC-$Lab_1$) | 0.1901 | 0.1901 | 0.1901 | 0.1901 |
| uf (UTC-$Lab_2$) | 0.3452 | 0.3452 | 0.3452 | 0.3452 |
| uf (UTC-$Lab_3$) | 0.2383 | 0.2383 | 0.2383 | 0.2383 |
| uf (UTC-$Lab_4$) | 0.1648 | 0.1648 | 0.1648 | 0.1648 |

Table 3. The total and measurement uncertainties varying with calibration uncertainty values in case of redundant links.

links in UTC calculation in term of uncertainty evaluation. This kind of evaluation is not recommended with the current formalism because of the difficulty in the estimation of the correlations. Two different examples are considered, in the first case the only difference with the current method is the use of the auxiliary time scale instead of PTB as pivot and the corresponding addition of the correlations. In the second example the redundant time links with TWSTFT are added. The difference between the current method and that proposed in this document when a big value is associated to non calibrated equipment is also presented.

## 4.1 Current status of time links with pivot attributed to an auxiliary time scale

This example uses the current links in UTC using the ATS as pivot in the case of GNSS equipment, and using PTB for the TWSTFT links. The IGRT time scale is considered as the ATS and the correlation is set equal to 0.03 ns. This value is chosen based on experience. The values of the link uncertainties are those reported in Section 5 of Circular T. The PTB continues to be a particular case because is used as pivot for the TWSTFT.

Table 4 shows the solutions for uf and u for several laboratories where the weights and link uncertainties are those in Circular T calculation. The value of the total uncertainty of PTB is very small due to the fact that is pivot for the TWSTFT technique.

### 4.1.1 Comparison between the current and new method in the case of uncalibrated equipment with a big value of uCal

200 ns is affected to USNO calibration uncertainty to show the difference between the current and the proposed methods. As in the previous example the ensemble of laboratories in UTC calculation is considered, IGRT is the pivot for GNSS time links

| Lab | Link | Weight (% ) | uCal (ns) | uStb (ns) | uf (ns) | u (ns) |
|---|---|---|---|---|---|---|
| IT | TWGPPP | 0.03 | 1.0 | 0.3 | 0.26 | 1.03 |
| USNO | TWGPPP | 0.33 | 1.0 | 0.3 | 0.2 | 1.02 |
| NIST | TWGPPP | 0.06 | 1.5 | 0.3 | 0.25 | 1.52 |
| OP | TWGPPP | 0.04 | 1.0 | 0.3 | 0.26 | 1.03 |
| APL | GPSPPP | 0.03 | 11.2 | 0.3 | 0.3 | 11.2 |
| AUS | GPSPPP | 0.0017 | 5.8 | 0.3 | 0.3 | 5.81 |
| CAO | GPS MC | 0.00 | 20.0 | 8.0 | 8.0 | 21.54 |
| SG | GPS P3 | 0.01 | 5.8 | 0.7 | 0.69 | 5.84 |
| SMD | GPSPPP | 0.0016 | 7.3 | 0.3 | 0.3 | 7.31 |
| MBM | GPS MC | 0.00 | 20.0 | 5.0 | 5.0 | 20.62 |
| PTB | - | 0.02 | - | - | 0.15 | 0.38 |

Table 4. uf and u for several laboratories depending on weights and link uncertainties as in Circular T calculation.

and the PTB for the TWSTFT technique.

| Lab | Link | uCal (ns) | $u_{old}$ (ns) | $u_{new}$ (ns) |
|---|---|---|---|---|
| IT | TWGPPP | 1.0 | 66.9 | 1.03 |
| USNO | TWGPPP | 200.0 | 133.1 | 181.02 |
| NIST | TWGPPP | 1.5 | 66.9 | 1.52 |
| OP | TWGPPP | 1.0 | 66.9 | 1.03 |
| APL | GPSPPP | 11.2 | 67.8 | 11.2 |
| AUS | GPSPPP | 5.8 | 67.2 | 5.81 |
| CAO | GPS MC | 20.0 | 69.8 | 21.54 |
| SG | GPS P3 | 5.8 | 67.1 | 5.84 |
| SMD | GPSPPP | 7.3 | 67.3 | 7.31 |
| MBM | GPS MC | 20.0 | 69.8 | 20.62 |
| PTB | - | - | 66.9 | 0.40 |

Table 5. The uncertainty obtained with the current and the new method with USNO uncertainty uCal set to 200 ns.

The results reported in Table 5 show how the algorithm is robust when a big value is associated to uCal. Even if USNO has more than 30% of the total weight, its uncertainty do not affect the other participating laboratories.

### 4.2 Redundant TWSTFT time links and pivot attributed to an auxiliary time scale for GNSS

In this example all redundant TWSTFT links are considered for UTC calculation. Table 6 reports the results for uf and u. In this case all the redundant links (IT, USNO,

| Lab | Link | Weight (% ) | uCal (ns) | uStb (ns) | uf (ns) | u (ns) |
|---|---|---|---|---|---|---|
| IT | - | 0.03 | - | - | 0.14 | 0.34 |
| USNO | - | 0.33 | - | - | 0.13 | 0.34 |
| NIST | - | 0.06 | - | - | 0.14 | 0.35 |
| OP | - | 0.04 | - | - | 0.14 | 0.35 |
| APL | GPSPPP | 0.03 | 11.2 | 0.3 | 0.27 | 11.2 |
| AUS | GPSPPP | 0.0017 | 5.8 | 0.3 | 0.28 | 5.81 |
| CAO | GPS MC | 0.00 | 20.0 | 8.0 | 8.0 | 21.54 |
| SG | GPS P3 | 0.01 | 5.8 | 0.7 | 0.69 | 5.84 |
| SMD | GPSPPP | 0.0016 | 7.3 | 0.3 | 0.28 | 7.31 |
| MBM | GPS MC | 0.00 | 20.0 | 5.0 | 5.0 | 20.62 |
| PTB | - | 0.02 | - | - | 0.13 | 0.37 |

Table 6. uf and u for several laboratories depending on weights and link uncertainties in case of redundant TWSTFT time links.

NIST and OP in the table) are considered with uStb equal to 0.3 ns and uCal equal to 1 ns. The algorithm, in this case, succeed in giving the same uncertainty to all the laboratories with redundant time links.

## 5 Conclusions and Perspectives

This document presents a proposal for a new algorithm for the calculation of the uncertainties of [UTC-UTC($k$)] reported in Section 1 of Circular T. The algorithm succeeds in solving the problem of correlations not easily valuables. With respect to the current version of the algorithm two major changes are envisaged; the pivot for GNSS time links is an auxiliary time scale instead of the PTB and correlations are added. With these actions current problems related to frequency measurement uncertainty, called uf, and to the total uncertainty, called u, are solved. It is planned to introduce the new algorithm in the computation of UTC within a few months, and to modify in consequence the information in Section 5 of Circular T. The future development of this work is the introduction of redundant links in UTC calculation, starting from TWSTFT measurements.

## 6 Acknowledgments

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