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Present Status of SI Values of K_J and R_K

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The 1998 Committee on Data for Science and Technology (CODATA) adjustment of the values of the fundamental physical constants was completed in July 1999. Carried out by the authors under the auspices of the CODATA Task Group on Fundamental Constants, it took into account all relevant data available through 1998 December 31. A lengthy paper that gives the 1998 CODATA recommended values and describes in detail the data and their treatment is widely available [1,2].

The 1998 recommended values of the Josephson constant K_J (assumed equal to $2e/h$) and von Klitzing constant R_K (assumed equal to $h/e^2 = \mu_0 c/2\alpha$, where μ_0 is the magnetic constant, c is the velocity of light in vacuum, and α is the fine-structure constant) are

$$K_J = K_{J-90}\left[1 - 0.43(3.9) \times 10^{-8}\right] \tag{1}$$

$$R_K = R_{K-90}\left[1 + 2.22(37) \times 10^{-8}\right]. \tag{2}$$

Here $K_{J-90} = 483.597.9$ GHz/V exactly and $R_{K-90} = 25812.807\ \Omega$ exactly are the conventional values of K_J and R_K adopted by the CIPM in 1988 for the purpose of basing a representation of the volt and of the ohm on the Josephson effect (JE) and on the quantum Hall effect (QHE), respectively, starting 1990 January 1.

The standard uncertainty relative to the (SI) volt assigned by the CIPM to a "perfectly" realized (i.e., no experimental uncertainty) representation of the volt based on the JE and K_{J-90} is $u = 0.4\ \mu$V, corresponding to a relative standard uncertainty $u_r = 40 \times 10^{-8}$, and the standard uncertainty relative to the (SI) ohm assigned by the CIPM to a "perfectly" realized representation of the ohm based on the QHE and R_{K-90} is $u = 0.2\ \mu\Omega$, corresponding to $u_r = 20 \times 10^{-8}$. The conventional values of K_J and R_K and these uncertainties were deduced by the CCEM (then the CCE) in 1988 June from the data available by 1988 June 15.

Based on Eq. (2) and a review of the relevant R_K data as it existed at the time, the 22nd CCEM in September 2000 concluded that the assigned standard uncertainty u of a "perfectly" realized ohm representation based on the QHE and

R_{K-90} could be reduced by a factor of two to 0.1 μΩ, corresponding to a relative standard uncertainty $u_r = 10 \times 10^{-8}$. On the other hand, the 22nd CCEM decided that, in spite of Eq. (1), it was premature to reduce the value $u = 0.4$ μV assigned by the CIPM in 1988 to a "perfectly" realized volt representation based on the JE and K_{J-90}.

Early in 2002, again under the auspices of the CODATA Task Group on Fundamental Constants, the authors began work on the next CODATA adjustment of the values of the constants. This adjustment is to be called the 2002 CODATA adjustment and the closing date for data to be considered for possible inclusion in the adjustment will be 2002 December 31. The 2002 CODATA recommended values are expected to be posted on the Web site of the NIST Fundamental Constants Data Center at http://physics.nist.gov/cuu by mid-summer 2003, and the detailed written description of the 2002 adjustment should be available by the end of 2003.

The data currently available relevant to determining the values of K_J and R_K in SI units are graphically compared in the two attached figures. Although the K_J Figure hints at the possibility that the 2002 recommended value of K_J might differ significantly from the 1998 value, and hence that practical representations of the volt based on the JE and K_{J-90} may not be as consistent with the SI as predicted by the 1998 CODATA adjustment, additional watt-balance and Avogadro constant reults will be necessary to resolve the apparent difference between the value of K_J implied by these two methods. (Fortunately the 0.4 μV standard uncertainty assigned in 1998* by the CIPM as discussed above still appears "safe.") Such data are not likely to be available this year, although a result from the NPL watt balance and from the NIST watt balance by 2002 December 31 is not completely out of the question. The R_K Figure, on the other hand, indicates that the 1998 recommended value of R_K, and hence practical representations of the ohm based on the QHE and R_{K-90}, remain quite consistent with the SI.

References

[1] P. J. Mohr and B. N. Taylor, J. Phys. Chem Ref. Data **28**, 1713-1852 (1999).
[2] P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **72**, 351-495 (2000).

Notes on the Figures

The data given in the figures are based on the information available by 2002 August 26. Some values are preliminary and hence could change in the near future. Those values of R_K that depend on atomic physics and/or QED theory (i.e., the LAMPF–99 value of R_K inferred from the value of the fine-structure constant α derived from the muonium hyperfine splitting, and the UWash–87 value of R_K inferred from the value of α derived from the electron magnetic moment anomaly) are based on the relevant theory as it existed on the above date. In those cases where inexactly known constants are required to obtain K_J or R_K from the quantity actually measured, the 1998 CODATA recommended values for these constants are used. However, in each such case the uncertainty

* Note (BNT): This should read 1988.

of the measured quantity significantly exceeds the combined uncertainty of the required constants. It is therefore reasonable to expect that any future changes in the recommended values of these constants will not have a significant impact on the figures.

K_J Figure. The values are given in order of increasing standard uncertainty starting from the bottom of the figure. The two vertical dotted lines symmetric about the origin on the bottom scale indicate the relative standard uncertainty assigned by the CIPM in 1998* to a "perfectly" realized Josephson effect volt representation. The year given with the laboratory abbreviations is the year of publication.

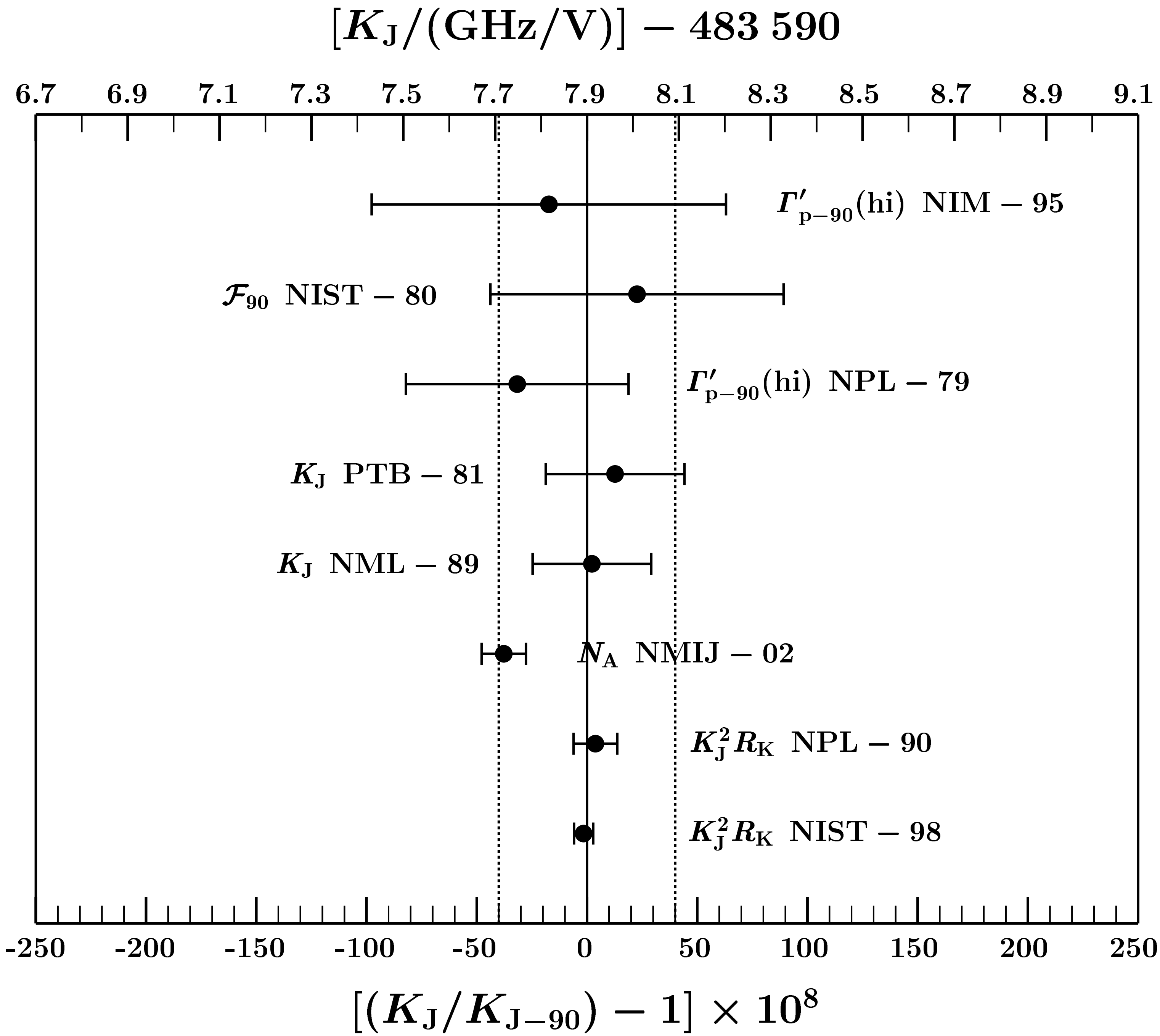
The NIST–98 and NPL–90 values labled $K_\mathrm{J}^2 R_\mathrm{K}$ are moving-coil watt balance results. $\left[\text{Recall } K_\mathrm{J}^2 R_\mathrm{K} = 4/h \text{ and } K_\mathrm{J} = (8\alpha/\mu_0 ch)^{1/2}\right]$. The NMIJ–02 value of K_J is based on a value of the Planck constant h obtained indirectly from a measured value of the Avogadro constant N_A, which is representative of other values of N_A also obtained using the silicon XRCD (x-ray crystal density) method. However, all such values are based on the same Institute for Reference Materials and Measurements (IRMM) determination of the molar mass of a particular sample of silicon, called WASO 17. [Recall that $h = cA_\mathrm{r}(\mathrm{e})M_\mathrm{u}\alpha^2/2R_\infty N_\mathrm{A}$, where $A_\mathrm{r}(\mathrm{e})$ is the relative atomic mass of the electron, $M_\mathrm{u} = 10^{-3}$ kg/mol is the molar mass constant, and R_∞ is the Rydberg constant.] The NML–89 and PTB–91 values labled K_J were measured directly and hence do not require the values of any other fundamental constants. The NPL–79 and NIM–95 values of K_J are based on values of h obtained indirectly from measurements of the proton gyromagnetic ratio by the high field method. Similarily, the NIST–80 value of K_J is based on a value of h obtained indirectly from a measurement of the Faraday constant.

R_K Figure. The first paragraph of the note on the K_J Figure applies to the R_K Figure, except that the two vertical dotted lines indicate the relative standard uncertainty assigned in September 2000 by the 22nd CCEM to a "perfectly" realized quantum hall effect representation of the ohm. (As pointed out above, the 22nd CCEM reduced the original 0.2 $\mu\Omega$ uncertainty assigned by the CIPM in 1988 to 0.1 $\mu\Omega$.)

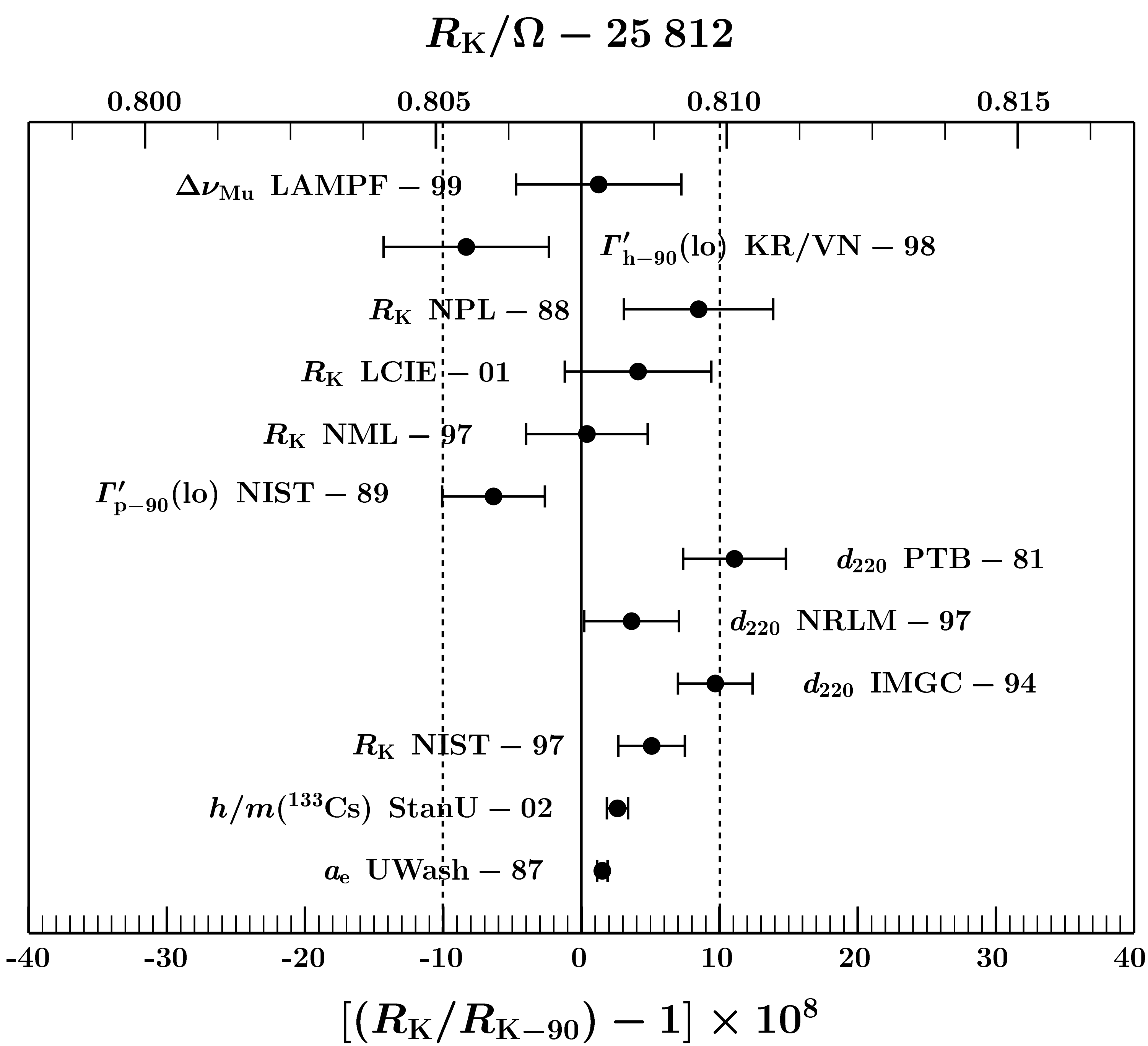
The University of Washington 1987 (UWASH–87) result for R_K was obtained via the relation $R_\mathrm{K} = \mu_0 c/2\alpha$ given above using the value of the fine-structure constant α implied by the experimental value of the electron magnetic moment anomaly a_e and its theoretical expression calculated from QED. The 2002 Stanford University (StanU–02) result for R_K is a preliminary result obtained from the value of α inferred from the atom-interferometric measurement of $h/m(^{133}\mathrm{Cs})$. (Recall $\alpha = \left[2R_\infty A_\mathrm{r}(\mathrm{X})/cA_\mathrm{r}(\mathrm{e})\right]^{1/2}\left[h/m(\mathrm{X})\right]^{1/2}$, where $A_\mathrm{r}(\mathrm{X})$ is the relative atomic mass of atom X and $m(\mathrm{X})$ is its mass.) The four values labeled R_K are all based on direct calculable capacitor measurements. The IMGC–94, NRLM–97, and PTB–81 values of R_K were obtained from the values of α deduced from independent d_{220} silicon lattice spacing measurements

* Note (BNT): This should read 1988

but from the same PTB–99 result for $h/m_{\mathrm{n}}d_{220}$, where m_{n} is the mass of the neutron. The NIST–89 result for R_{K} follows from the value of α inferred from the determination of the proton gyromagnetic ratio by the low-field method. The KRISS/VNIIM 1998 (KR/VN–98) result for R_{K} follows similarly from the determination of the gyromagnetic ratio of the helion (nucleus of the ^{3}He atom) by the low-field method. The LAMPF–99 result for R_{K} was obtained from the value of α deduced from the muonium ground-state hyperfine splitting experimentally determined at the Los Alamos Meson Physics Facility (LAMPF) and the theoretical expression for the splitting based on atomic physics and QED.



$[K_J/(\mathrm{GHz/V})] - 483\,590$
6.7
6.9
7.1
7.3
7.5
7.7
7.9
8.1
8.3
8.5
8.7
8.9
9.1
$\Gamma'_{p-90}(\mathrm{hi})$ NIM − 95
$\mathcal{F}_{90}$ NIST − 80
$\Gamma'_{p-90}(\mathrm{hi})$ NPL − 79
K_J PTB − 81
K_J NML − 89
N_A NMIJ − 02
$K_J^2 R_K$ NPL − 90
$K_J^2 R_K$ NIST − 98
-250
-200
-150
-100
-50
0
50
100
150
200
250
$[(K_J/K_{J-90}) - 1] \times 10^8$



$R_K/\Omega - 25\,812$
0.800
0.805
0.810
0.815
$\Delta\nu_{Mu}$ LAMPF − 99
Γ'_{h-90}(lo) KR/VN − 98
R_K NPL − 88
R_K LCIE − 01
R_K NML − 97
Γ'_{p-90}(lo) NIST − 89
d_{220} PTB − 81
d_{220} NRLM − 97
d_{220} IMGC − 94
R_K NIST − 97
$h/m(^{133}Cs)$ StanU − 02
a_e UWash − 87
-40
-30
-20
-10
0
10
20
30
40
$[(R_K/R_{K-90}) - 1] \times 10^8$