A revised way of fixing an upper limit to clock weights in TAI computation

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1. Introduction

The free atomic time scale EAL (échelle atomique libre), from which TAI is derived by frequency steering, is obtained as a weighted average of a large number of free-running and independent atomic clocks spread worldwide. The computation uses the algorithm ALGOS, which is optimized for long-term stability.

Since January 1998, a modified form of the algorithm has been used for the calculation of TAI. This algorithm is based on the same defining equations as ALGOS [1], but includes two changes:

(a) reduction of the time interval T of the computation from two months to one month [2];

(b) application of an upper limit to the relative weights, ω_{MAX}, instead of to the absolute weights, p_{MAX}, attributed to the contributing clocks [3, 4].

These changes are described in more detail in [5] and [6].

The value $\omega_{\mathrm{MAX}} = 7.00 \times 10^{-3}$ was fixed at the time of the first computation (January 1998) to ensure continuity of the time scale. It corresponds to the value of the maximum relative weight assigned to clocks in the EAL computation (with $p_{\mathrm{MAX}} = 2500$) in the 60 day interval November/December 1997. Our analysis of the distribution of the relative weights attributed to clocks shows, however, that this value is no longer appropriate.

The underlying aim of this study is to improve the stability of EAL and hence of TAI. To achieve this we need to take best advantage from the HP 5071A clocks and also from the hydrogen masers, by discriminating efficiently between them. Fixing an appropriate upper limit to the clocks weights in the EAL computation can ensure this.

As the choice of ω_{MAX} is empirical, we could simply update its value, but, as we demonstrate that fixing ω_{MAX} to a constant could lead to a situation where the weights ω_i attributed to clocks are not normalized.

To avoid such a situation, we suggest making ω_{MAX} a function of the number, N, of clocks participating in TAI. A relation such as $\omega_{\mathrm{MAX}} = A/N$ is proposed*,* where A is an empirical constant.

Three test time scales (E2, E25 and E3) have been established over 2.5 years using real clock data. They are based on the algorithm ALGOS and use the values 2.0, 2.5 and 3.0, respectively, for the constant A. By comparing the distributions of the resulting relative weights attributed, we show that E2, E25 and E3 allow a much better discrimination between clocks than does EAL. As they rely more heavily on the very best clocks, the time scales E2, E25 and E3, are thus more stable than EAL. We conclude that the stability of EAL, and hence that of TAI, can be improved.

2. The upper limit of relative weights in EAL computation

The computational process used to apply an upper limit to relative weights in ALGOS is described in [4]. The most important feature of this process is that it does not independently assign a weight to each clock; instead, the set of clocks is treated globally.

The classical variance of the frequency of clock H_i relative to EAL [$\sigma_i^2\,(12,T)$, computed from twelve consecutive 30 day samples] plays an important part in the computation of its relative weight, ω_i, and therefore has an effect on the stability of EAL. This variance $\sigma_i^2\,(12,T)$ (calculated over twelve consecutive samples of duration T) is an estimate of the zero-dead-time sample variance $\sigma_i^2\,(12,T,T)$ [7, 8] of the frequencies of clock H_i relative to EAL, and is linked to the usual Allan variance $\sigma_{y_i}^2\,(T)$ by the relation:

$$\sigma_i^2\,(12,T,T) = B_1 \sigma_{y_i}^2\,(T) \qquad (1)$$

where $B_1 = 1.924$ in the case of flicker-frequency noise modulation over averaging times $T = 30$ d.

When EAL is computed, $\sigma_i^2\,(12,T)$ values of the clocks reaching ω_{MAX}, differ significantly. We define σ^2_{MIN} to be the largest of these variances, corresponding to the least stable clock attributed ω_{MAX}.

Figure 1 shows the distribution of relative weights, $\{\omega_i,\ i = 1, N\}$, attributed to clocks in the EAL computation. The given values are the means calculated over 2.5 years (from January 1998 until June 2000). The number of clocks reaching $\omega_{\mathrm{MAX}} = 7.00 \times 10^{-3}$ and falling within five other classes of relative weight ω_i are presented. The classes are defined below:

class I	$80\ \%\ \omega_{\mathrm{MAX}} \leq \omega_i$	$< \omega_{\mathrm{MAX}}$
class II	$60\ \%\ \omega_{\mathrm{MAX}} \leq \omega_i$	$< 80\ \%\ \omega_{\mathrm{MAX}}$
class III	$40\ \%\ \omega_{\mathrm{MAX}} \leq \omega_i$	$< 60\ \%\ \omega_{\mathrm{MAX}}$
class IV	$20\ \%\ \omega_{\mathrm{MAX}} \leq \omega_i$	$< 40\ \%\ \omega_{\mathrm{MAX}}$
class V	ω_i	$< 20\ \%\ \omega_{\mathrm{MAX}}$

We observe that 123 clocks reach ω_{MAX} and that there are less than 10 clocks within each class, apart from class V which includes 29 clocks. Fixing ω_{MAX} to 7.00×10^{-3} thus allows a large number of clocks (70 % of the total number) to reach this upper limit.

The clocks that contribute to the construction to EAL can be separated into three different clock types: hydrogen masers, HP 5071A clocks, and other caesium clocks. Figure 2 shows the distribution of relative weights for these three clock types. Again the same classes of relative weights are presented. For each clock type there are less than 5 units within each class, apart from class V which includes 3 HP 5071A clocks, 6 hydrogen masers and 20 other caesium clocks. Among the clocks reaching ω_{MAX}, we have: 93 HP 5071A clocks (83 % of the total number of such clocks), 17 hydrogen masers (60 % of the total number of hydrogen masers) and 31 % other caesium clocks (31 % of this group). These data are summarized in Table 1. It is clear that the discrimination is not efficient, especially for the HP 5071A clocks and the hydrogen masers.

3. Need to update ω_{MAX}

When attributing relative weights, efficient discrimination between clocks must be made in order that the resulting time scale relies most heavily upon a maximum number of the very best clocks. With $\omega_{\mathrm{MAX}} = 7.00 \times 10^{-3}$, however, clocks with $\sigma_i\,(12,T) = 15.9 \times 10^{-15}$ are given the same relative weight as clocks with $\sigma_i\,(12,T)$ values many times less than this. Figure 3, a histogram of the $\sigma_i(12,T)$ data, clearly shows that $\omega_{\mathrm{MAX}} = 7.00 \times 10^{-3}$ yields a value of σ_{MIN} that does not achieve the required discrimination. It is therefore necessary to update ω_{MAX} such that the corresponding σ_{MIN} value is small enough that such discrimination is possible.

Figures 4 and 5 show the relative frequency stabilities of some of the best HP 5071A clocks and hydrogen masers used in the establishment of TAI. On each figure, the indicated value of $\sigma_{y\mathrm{MIN}}(30)$, computed from (1), differs significantly from the plotted $\sigma_{y_i}\,(30)$ values.

4. Revised way of fixing an upper limit to clock weights

We propose not to fix ω_{MAX} to a given value, but to make ω_{MAX} a function of the number, N, of clocks contributing to the construction of TAI. A relation such as $\omega_{\mathrm{MAX}} = A/N$ is suggested, where A is an empirical constant. There is no fundamental difference between this function and a fixed value; in both cases the choice of ω_{MAX} is empirical. Nevertheless a fixed value of ω_{MAX} could lead, if N were small, to a situation where N_1 clocks reach ω_{MAX}, while the weights of the remaining $(N - N_1)$ clocks is zero. This situation is very unlikely to occur, but if it did, then the relative weights would not be normalized and the relation of definition of EAL would no longer be valid. Such a situation could not occur with the proposed alternative: if N decreased suddenly, then ω_{MAX} would increase so that discrimination among clocks would still be made and the normalization requirement fulfilled.

5. Tests on real data

In this section three time scales are considered and compared to EAL. Each is calculated by running the ALGOS algorithm on real clock data, using different values of $\omega_{\mathrm{MAX}} = A/N$. These time scales are E2, E25 and E3, with values of 2.0, 2.5 and 3.0, respectively, for the constant A.

The results for our set of clocks over the period from January 1998 to June 2000 are summarize in Table 1, where they are compared with EAL, and the properties of the test time scales are described in more detail below.

Table 1. Summary of the results of the four time scales EAL, E2, E25 and E3 calculated over the period January 1998 to June 2000. The values of ω_{MAX} and σ_{MIN} are indicated, along with the fraction of clocks attributed the maximum weighting.

Time scale	$10^3 \times \omega_{\mathrm{MAX}}$	$10^{15} \times \sigma_{\mathrm{MIN}}$	100 × Fraction of clocks reaching ω_{MAX}			
			Total	HP 5071A	Hydrogen masers	Other Cs clocks
EAL	7.00	15.9	70	83	60	31
E2	9.51	7.8	41	50	41	12
E25	11.89	5.8	27	33	36	7
E3	14.27	4.9	18	20	3	1

5.1 Time scale E2

Figures 6 and 7 show the distribution of relative weights attributed to clocks in the computation of E2. The results seem more satisfying than those obtained with EAL (cf. Fig. 1) in the following respects:

- it reduces reasonably the amount of clocks reaching this upper limit;
- it allows a given clock H_i, providing it does not show abnormal behaviour, to reach ω_{MAX} if its $\sigma_i(12,30)$ value is less than or equal to 7.8×10^{-15};
- it allows a reasonable discrimination between the clocks;
- it relies more heavily upon the best clocks.

5.2 Time scale E25

Figure 8 shows the distribution of relative weights, attributed to clocks in the computation of E25. The value σ_{MIN}, corresponding to the least stable clock attributed ω_{MAX}, is nearly three times smaller than that obtained for EAL (Table 1). The conditions for a given clock to reach the upper limit of relative weights are more severe here than for EAL or E2.

The distribution of relative weights for the three clock types defined in Section 2 is presented in Figure 9. We observe that an efficient discrimination among the clocks is made, and that the number of clocks at ω_{MAX} is still great enough to ensure the reliability of the time scale. Figure 10 shows a histogram of the standard deviation $\sigma_i(12,30)$ and the position of σ_{MIN}.

For the group of clocks considered, the time scale E25 has the following advantages over EAL:

- it substantially reduces the number of clocks attributed ω_{MAX};
- it allows a given clock H_i, providing it does not show abnormal behaviour, to reach ω_{MAX} when its $\sigma_i(12,30)$ value is less than or equal to 5.8×10^{-15};
- it allows efficient discrimination between the clocks;
- it relies more heavily upon the very best clocks.

5.3 Time scale E3

Figures 11 shows the distribution of relative weights attributed to clocks in the computation of E3, and Figure 12 shows the distribution of relative weights for the three different clock types. Only 18 % of the clocks are attributed the maximum relative weight, and this fraction is not enough large to ensure the reliability of TAI. Here the conditions required for a clock to obtain the maximum relative weight are too severe. Figure 13 shows a histogram of the standard deviations $\sigma_i(12,30)$ and the position of σ_{MIN}.

With the considered clock ensemble, E3

- reduces excessively the number of clocks attributed ω_{MAX};
- allows a given clock H_i, providing it does not show abnormal behaviour, to reach ω_{MAX} if its $\sigma_i(12,30)$ value is less than or equal to 4.9×10^{-15};
- provides severe discrimination between the clocks;
- relies more heavily upon a selection of the very best clocks than the other time scales considered here, but the small fraction selected is not large enough to ensure the reliability of the resulting time scale.

5.4 Stability of time scales EAL, E2, E25 and E3

The stabilities of the time scales EAL, E2, E25, and E3 have been compared to three of the best independent time scales in the world: those maintained at the NIST (Boulder, Colorado, USA), the BNM-LPTF (Paris, France) and the AMC (Colorado Spring, USA). Intrinsic values of $\sigma_y(\tau)$ computed using the *N*-cornered-hat technique are shown in Figure 14.

As expected, because the time scales E2, E25 and E3 allow more efficient discrimination between the clocks, they rely more heavily upon the very best units and are consequently more stable than EAL.

6. Conclusions and proposal

The fixed value 7×10^{-3} for the upper limit of clock weights ω_{MAX} is no longer appropriate because it does not allow efficient discrimination between the clocks. It is suggested that this fixed value be replaced by a function $\omega_{\mathrm{MAX}} = A / N$, where A is an empirical constant and N is the number of clocks participating in the construction of TAI. Such a function would avoid a possible problem described in Section 4 which could arise if an insufficient number of clocks were available.

The suggested function has been tested successfully, with $A = 2.0$, $A = 2.5$ and $A = 3.0$, using real clock data over 2.5 years, and the resulting time scales E2, E25 and E3 are more stable than EAL.

We therefore propose to adopt a function such as $\omega_{\mathrm{MAX}} = A/N$ to define the upper limit for clock weights in the algorithm ALGOS used to calculate TAI. Regarding the choice of the constant A, among the three tested values, A=3.0 must be set aside because it causes an excessive reduction in the number of clocks reaching ω_{MAX}. A more appropriate value seems to be $A = 2.5$; the corresponding time scale E25 allows a more efficient discrimination between clocks and shows a better stability. Adoption of this value for TAI computation would, however, change ω_{MAX} from 7.00×10^{-3} to 11.89×10^{-3}. Such a discontinuity could influence the behaviour of TAI and should be avoided. It would be preferable to move more gradually towards this value, for example by using $A = 2.0$ for one year and then moving to $A = 2.5$

We thus propose to set $A = 2$ from January 2001 until December 2001, and to set $A = 2.5$ from January 2002 onwards. In this way ω_{MAX} will change from 7.00×10^{-3} to a value ω'_{MAX} close to 9.51×10^{-3} at the beginning of January 2001, and then close to 11.89×10^{-3} in January 2002. Based on the experience of the BIPM Time Section, such small changes will not perturb the behaviour of TAI.

References

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[5] Thomas C., Azoubib J., A modified version of the TAI algorithm under test, *Report to the CCDS Working Group on TAI*, GT-TAI/95-9, 1995, 15p.

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Note

This Report has previously been submitted to the CCTF Working Group on TAI and widely diffused to its members. It has been supported by a large number of laboratories which participate in the construction of TAI. Consequently, the value of the maximum relative clock weight was set to $2/N$ in January 2001.

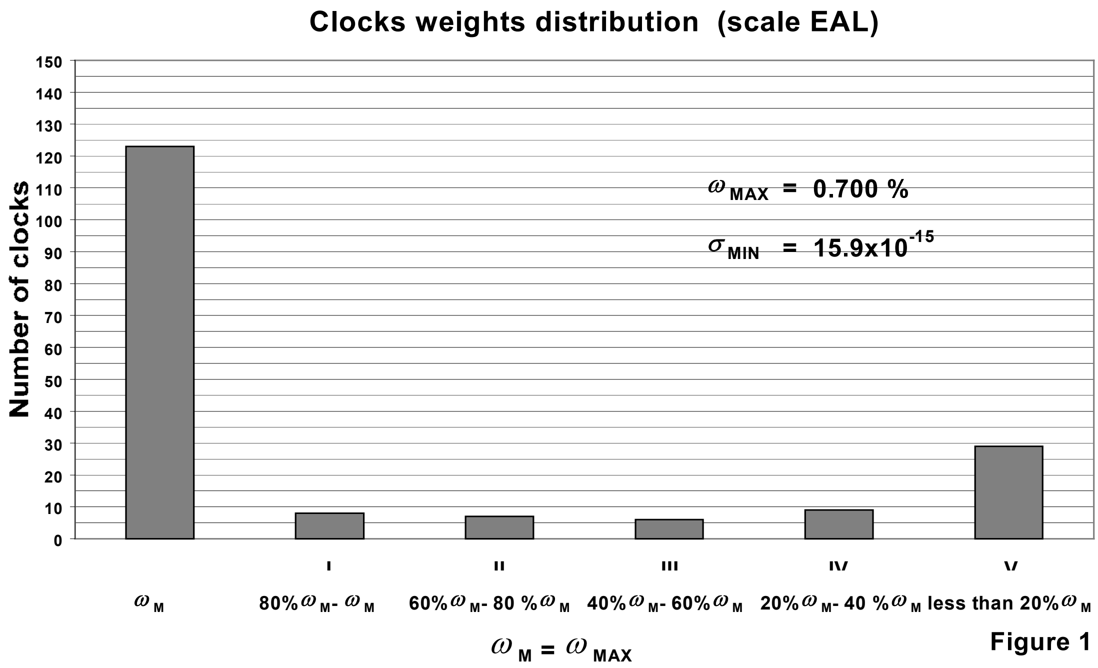

Figure 1

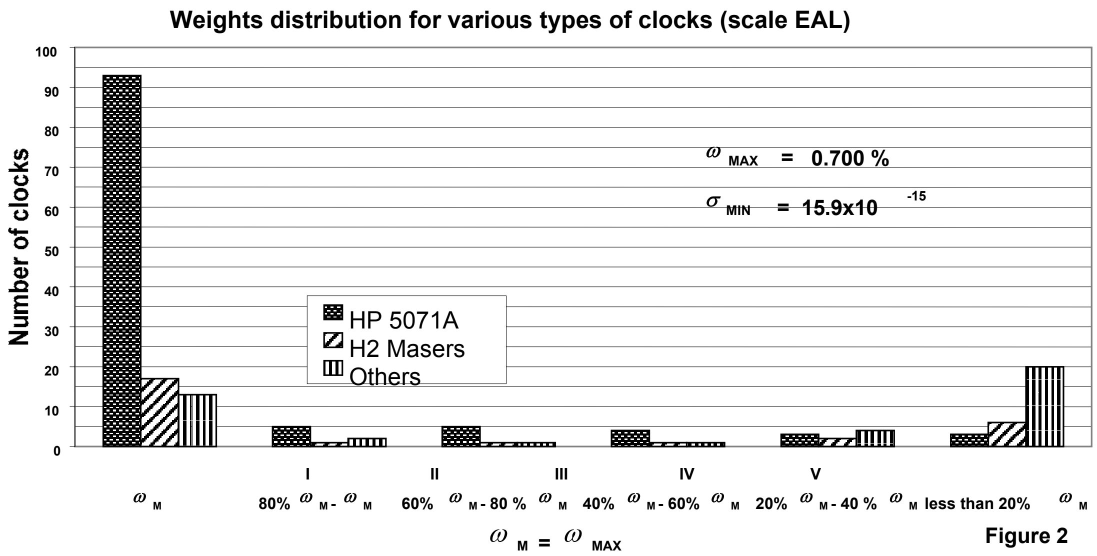

Figure 2

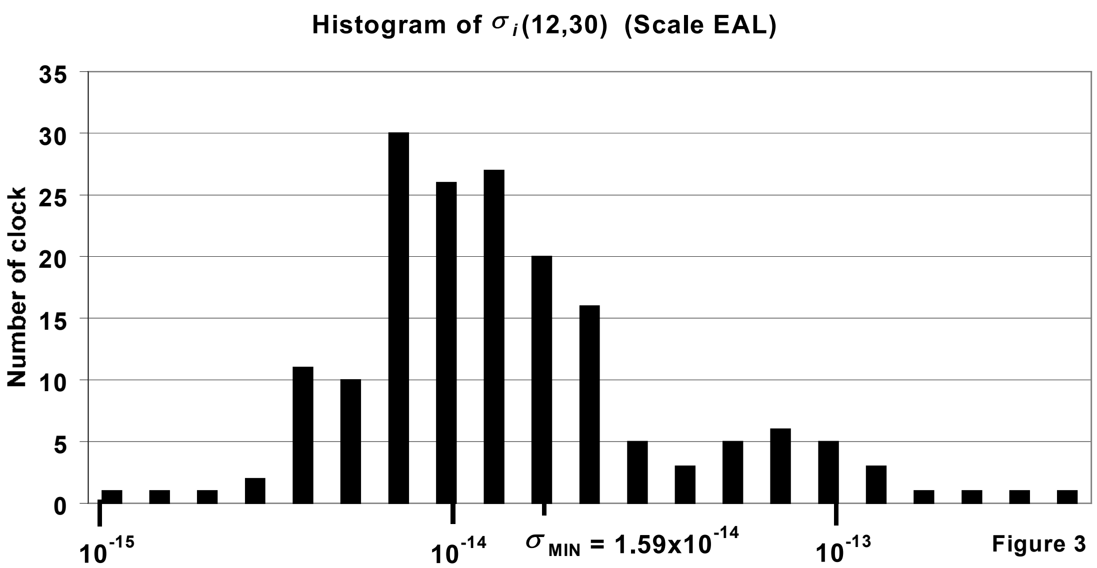

Figure 3

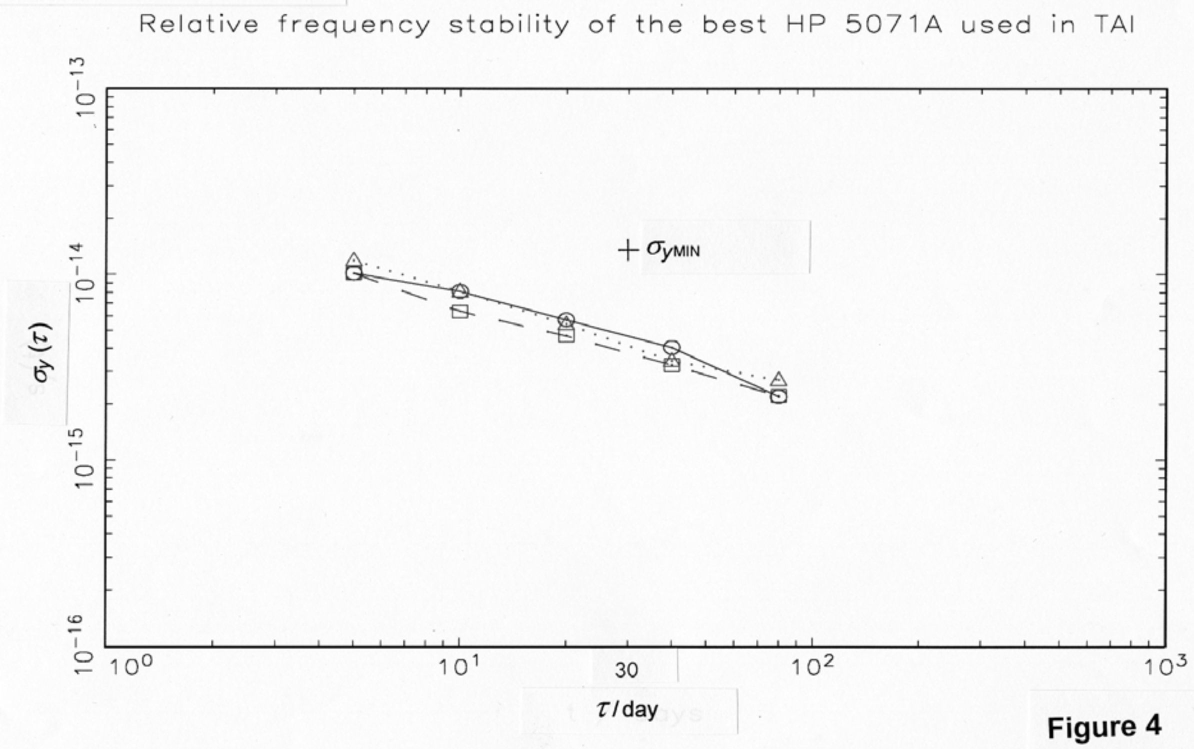

Figure 4

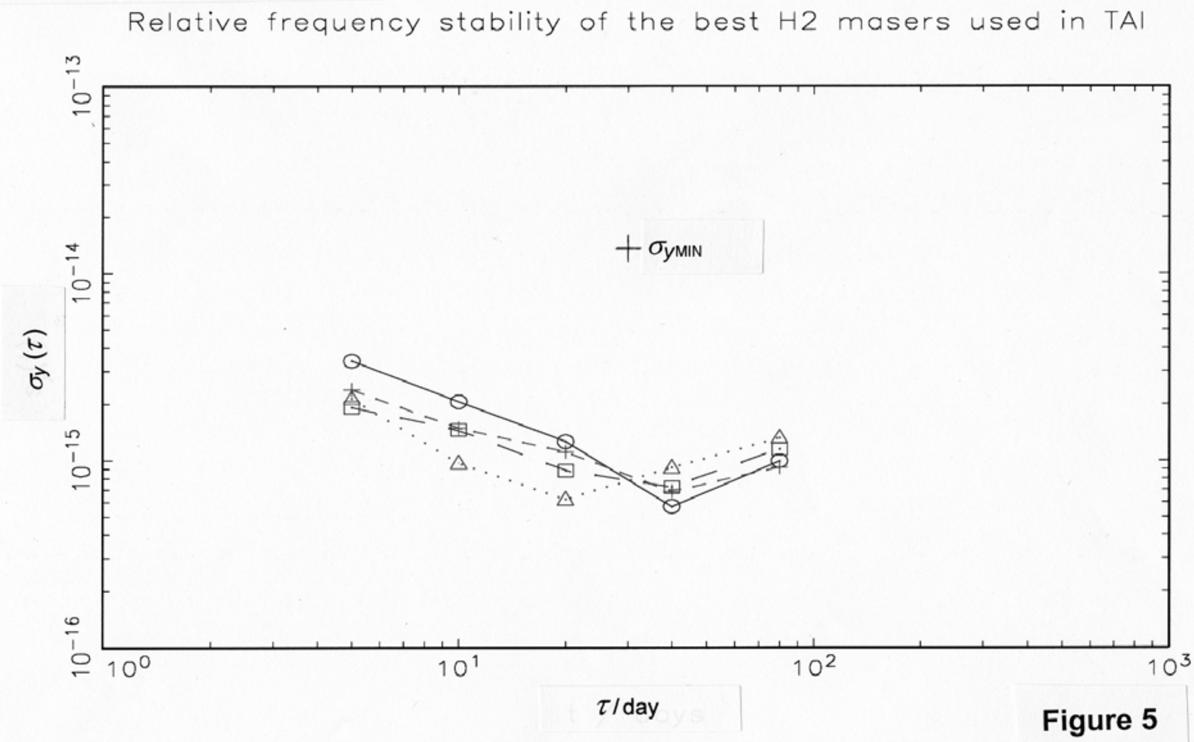

Figure 5

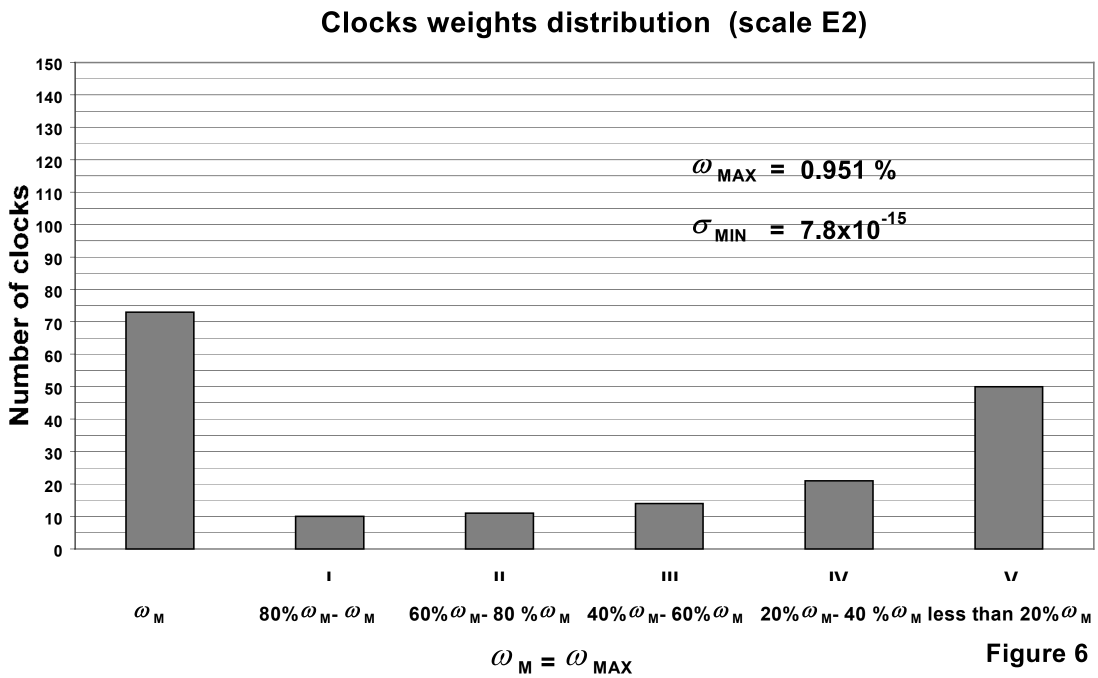

Figure 6

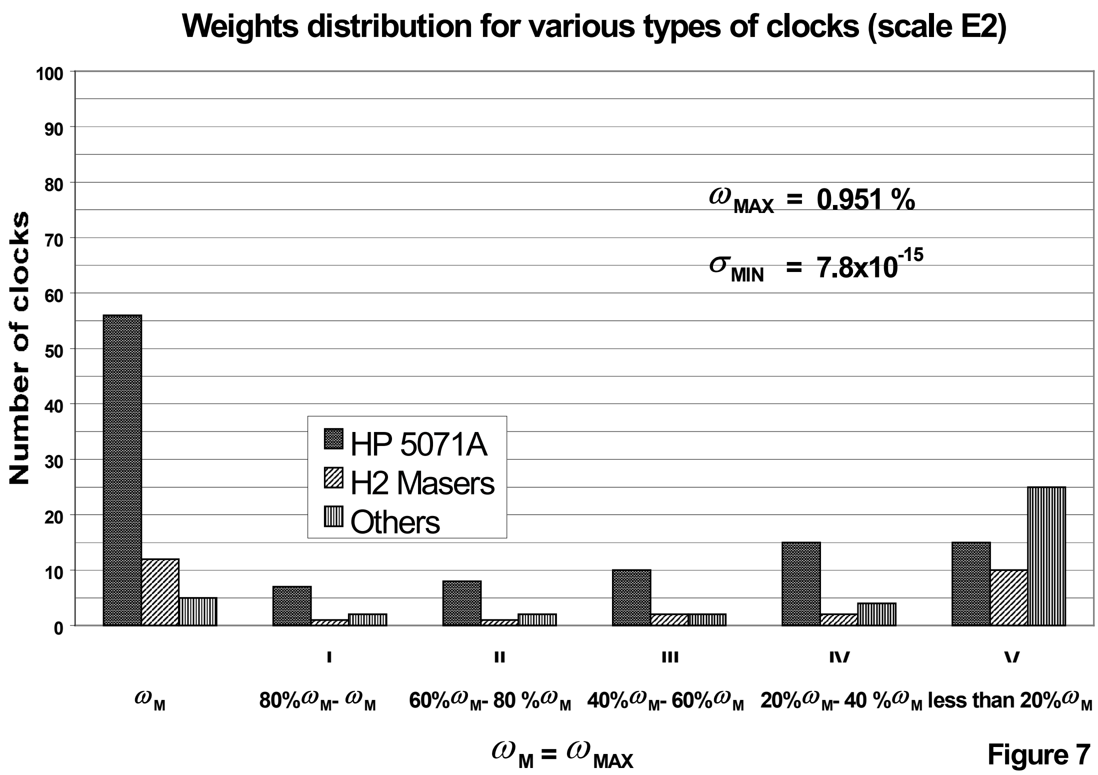

Figure 7

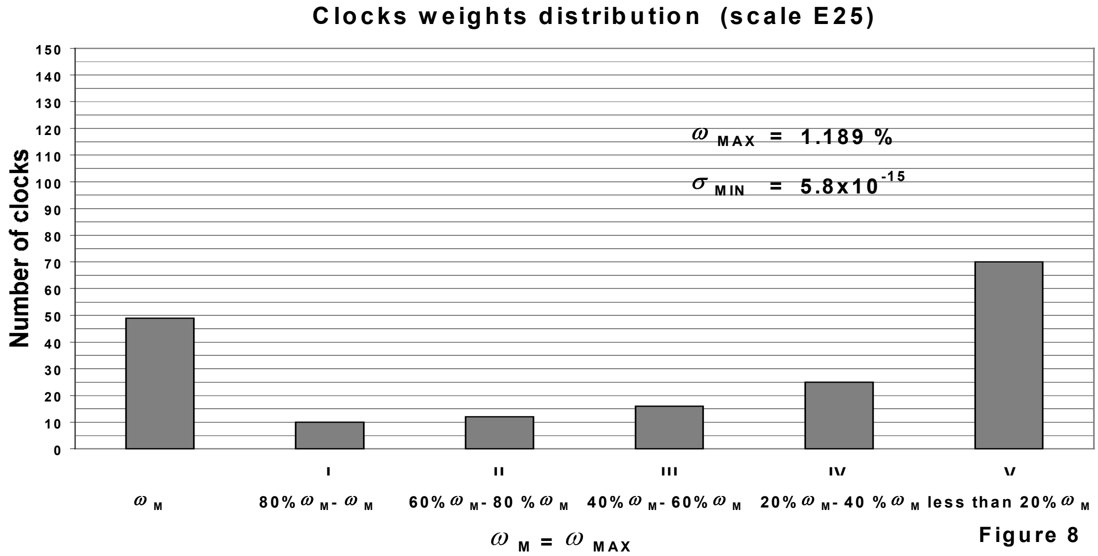

Figure 8

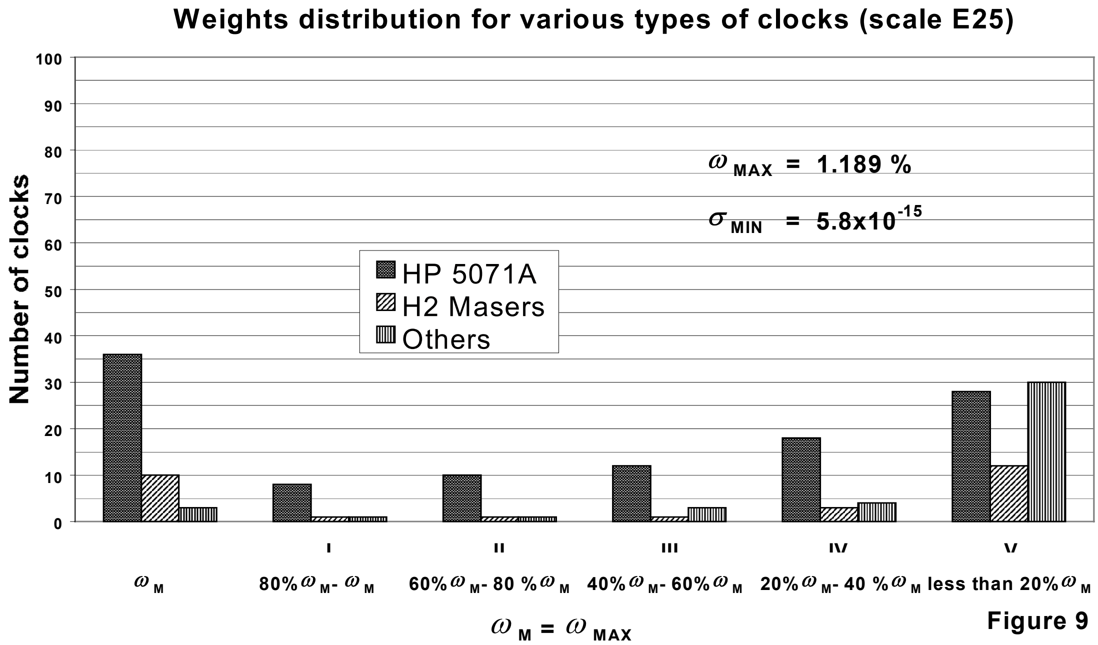

Figure 9

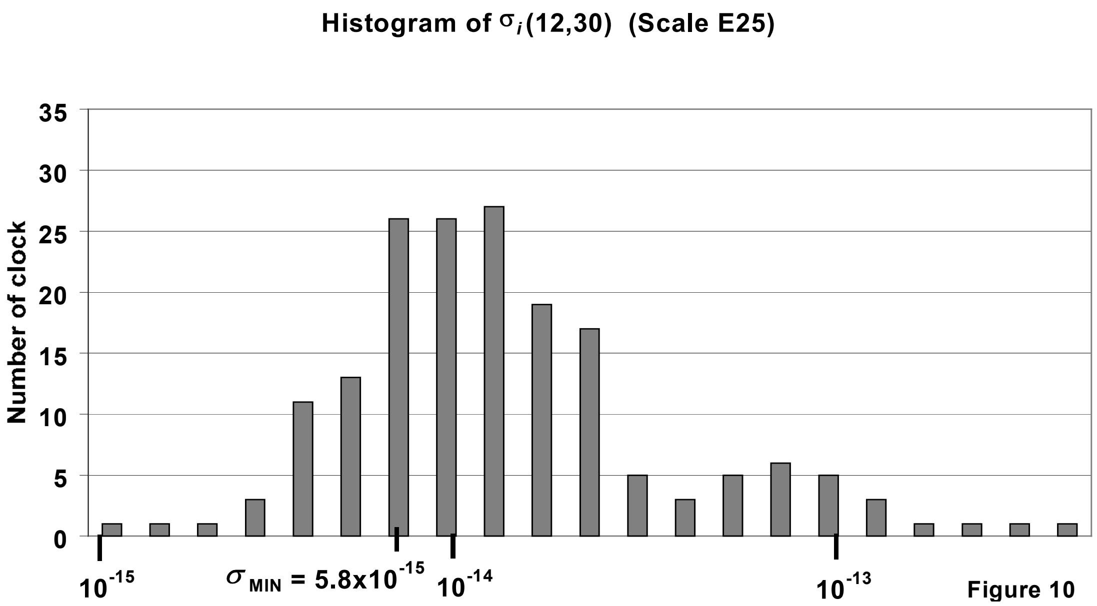

Figure 10

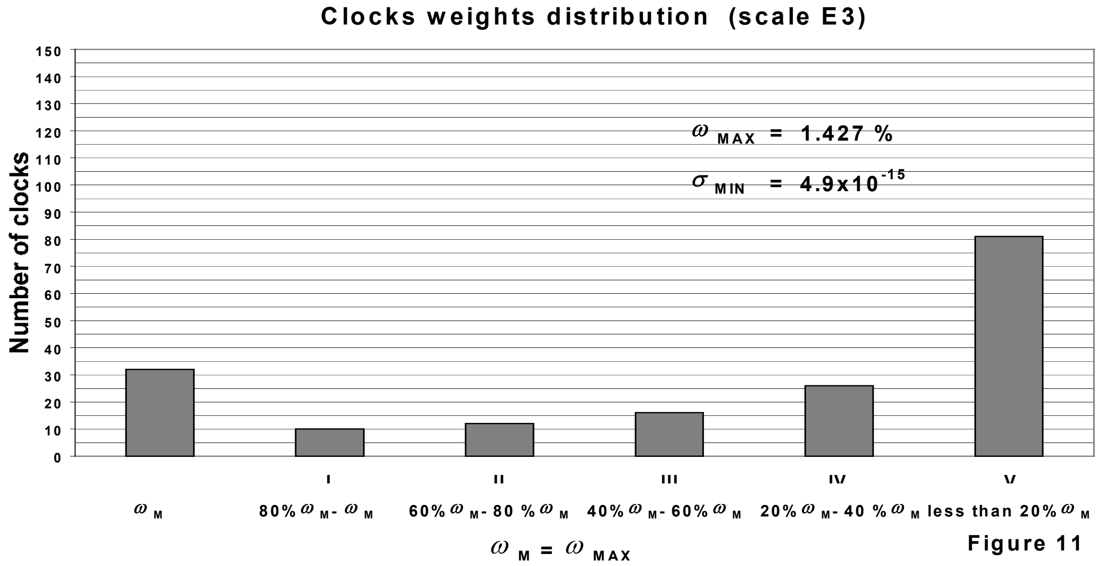

Figure 11

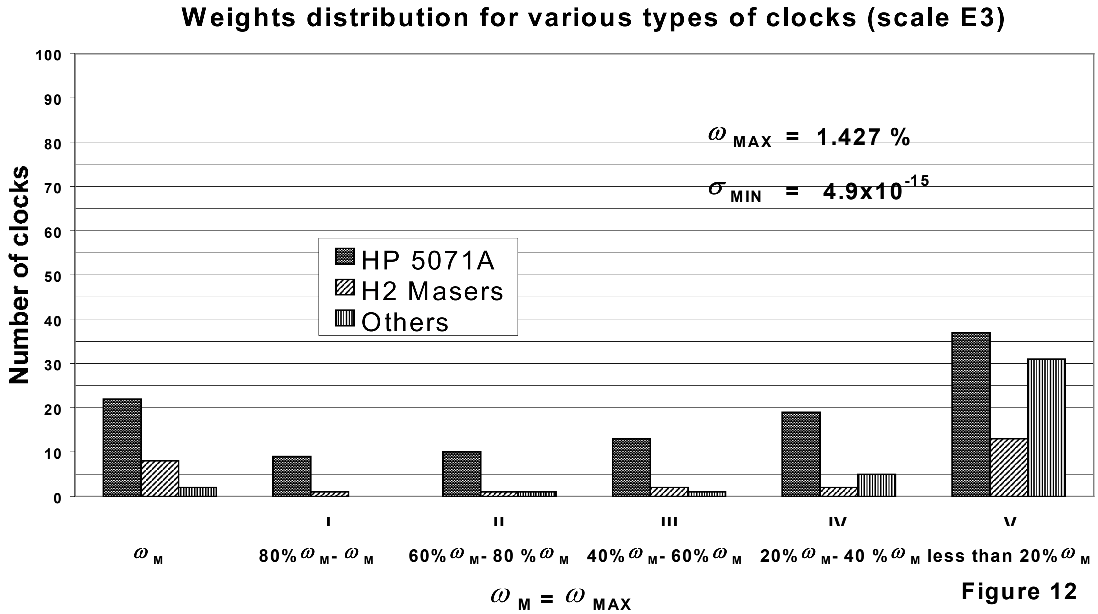

Figure 12

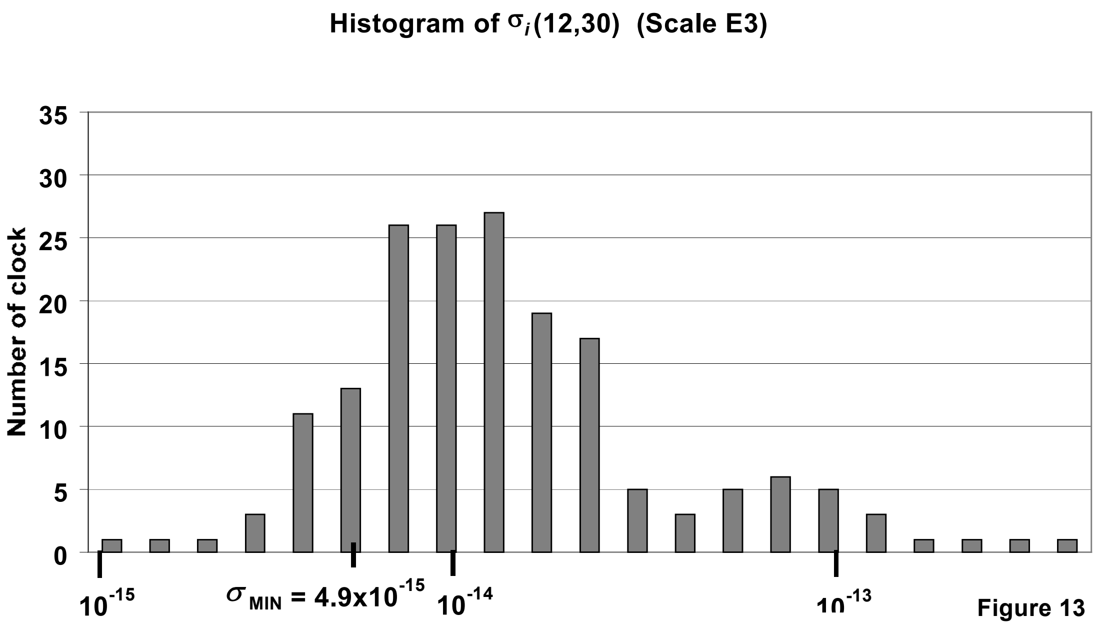

Figure 13

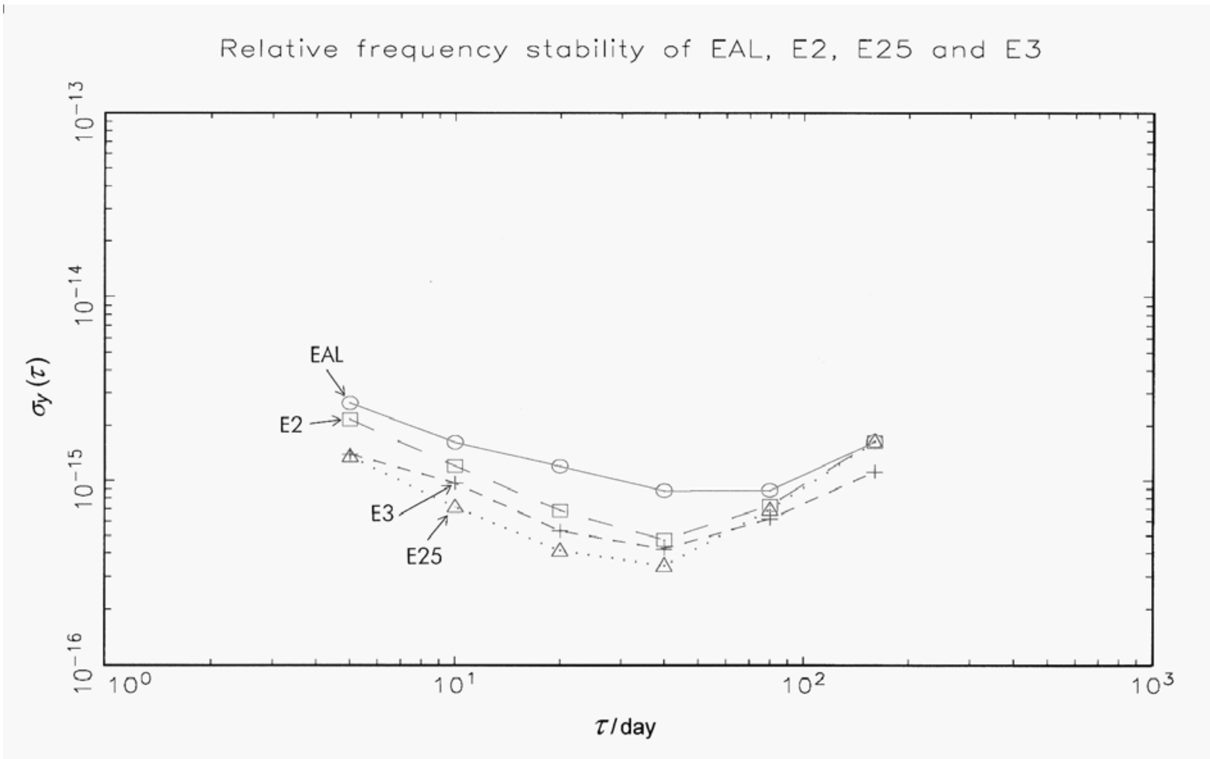
Relative frequency stability of EAL, E2, E25 and E3
$\sigma_y(\tau)$
10^{-13}
10^{-14}
10^{-15}
10^{-16}
EAL
E2
E3
E25
10^0
10^1
10^2
10^3
τ/day

Figure 14