KBTM

KB Mechanism and Alignment

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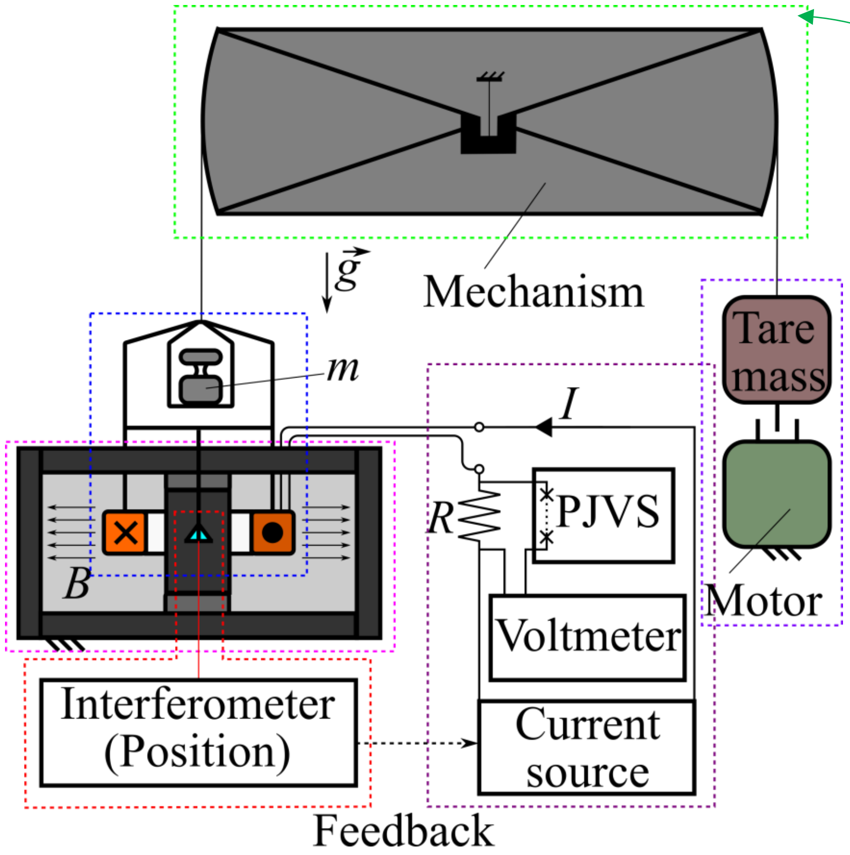
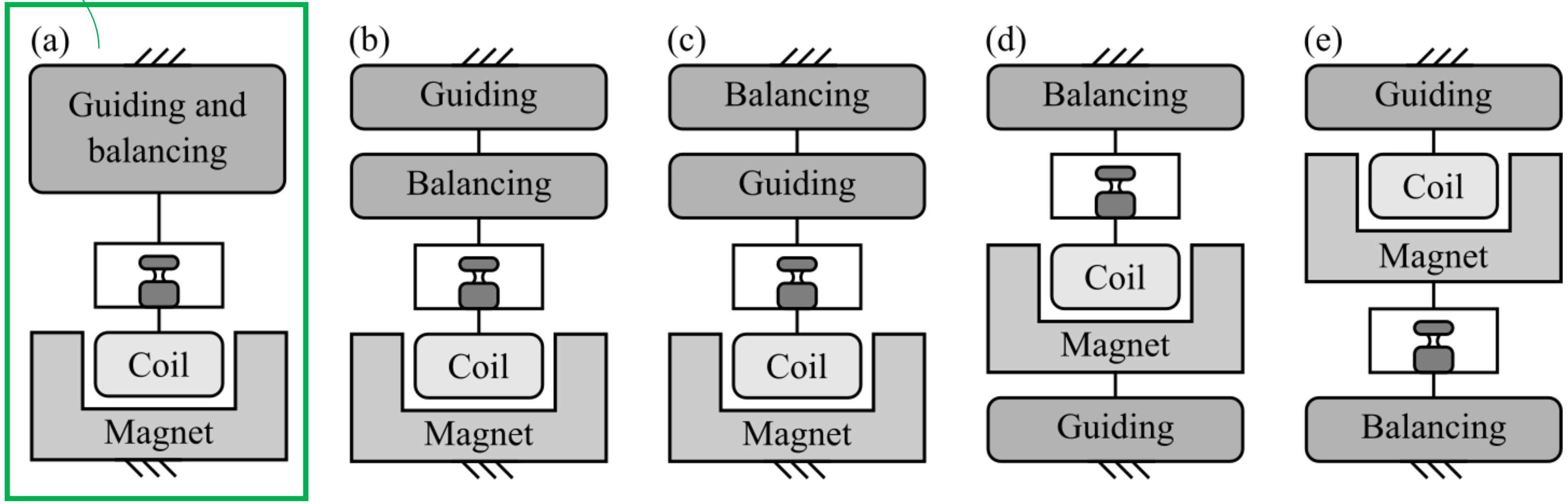
NIST

Kibble Balance Technical Meeting
special session “knowledge transfer”
20 November 2025, BIPM

State of the Art



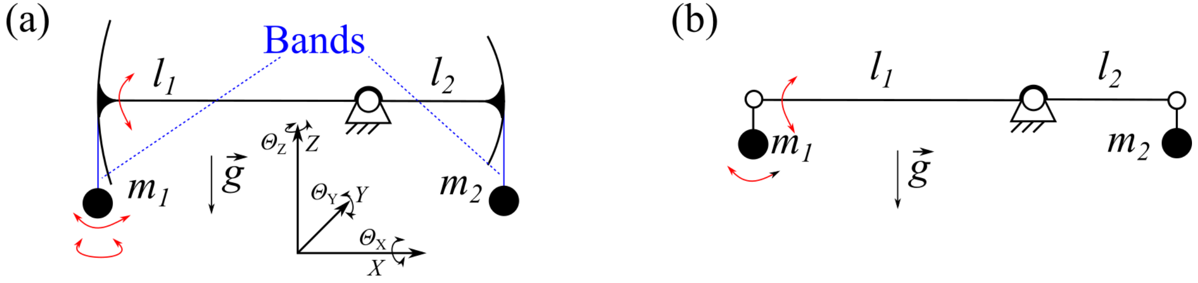


Institutes				
NIST, NRC, NPL, PTB/TU Ilmenau, MSL	KRISS, METAS	LNE,	BIPM	NIM, UME, Tsinghua University	TU Ilmenau

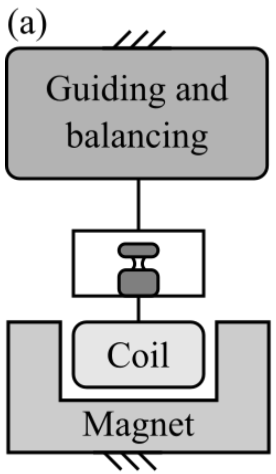
→ Option (a) favored for theorist/experimenter:

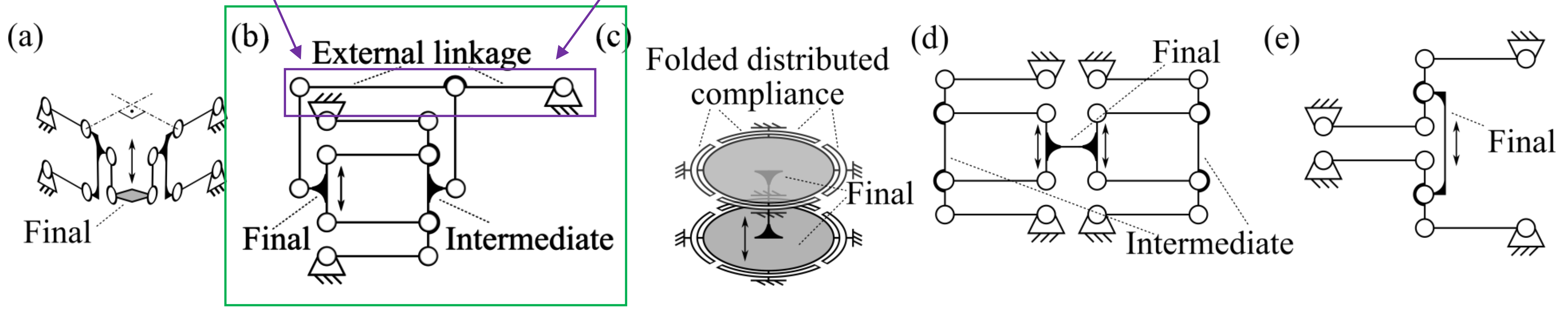
- Same hardware in both modes of operation: cancellation of common biases possible
- Reduced number of parts and sources for failure
- More simple to operate and maintain

Kinematic structure with degree of freedom equal to 1



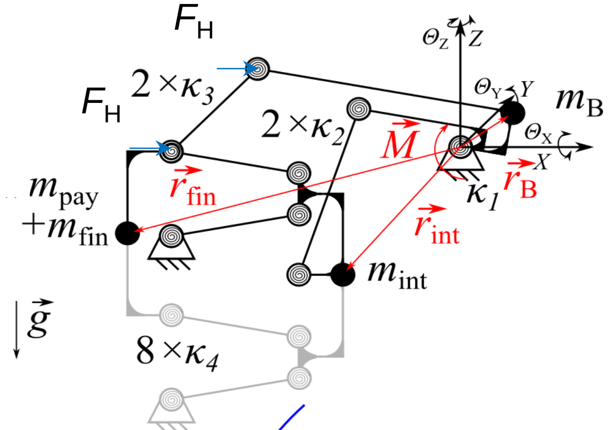
$$l_1 m_1 = l_2 m_2$$

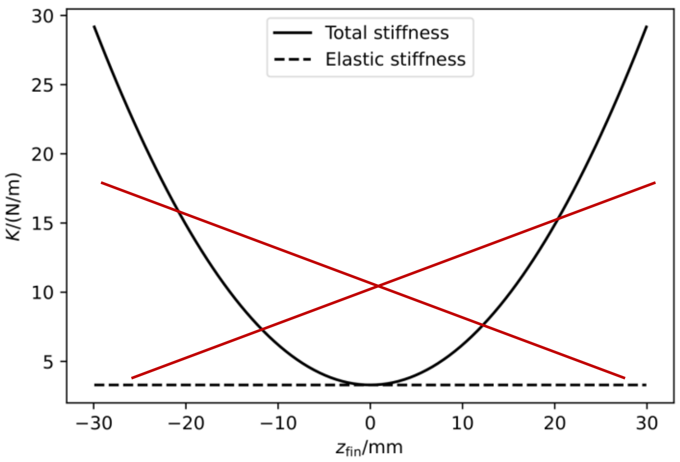




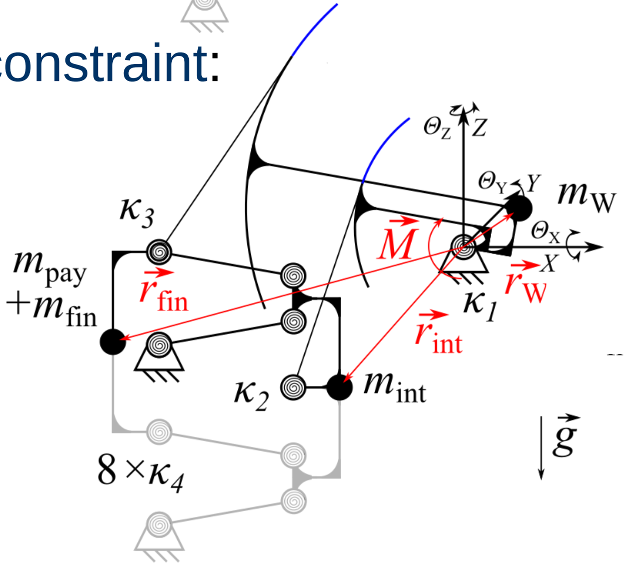
Rigid body models, Lagrange method

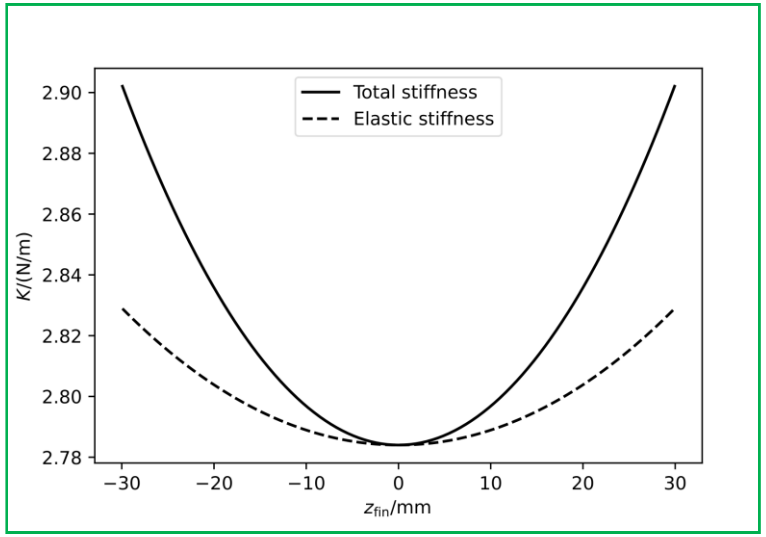
- Beam-constraint:





- Wheel-constraint:

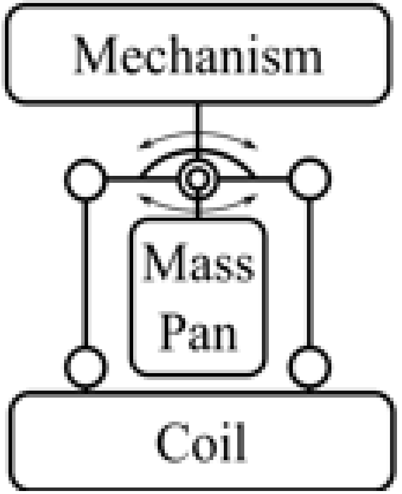


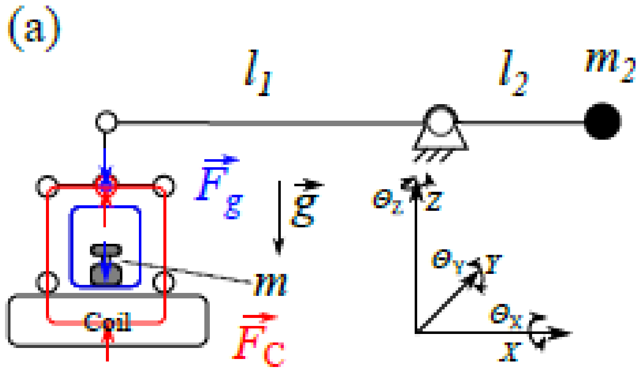
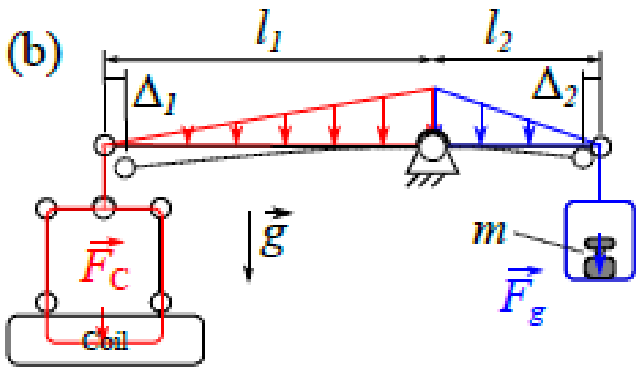


Assumptions:

1. Ideally stiff links
2. Ideal rotation pivots
3. Gravity/mass load on
4. All geometric nonlinearities considered

Comparison of forces/velocity of the mass



	Direct force compensation, $q = 1$	Force compensation through lever arm, $q \neq 1$
Simplified drawing		
Lever deformation	No force change on lever due to test mass	Depending on structural stiffness of links
State of the art	Proven to work with required accuracy	Not yet proven to work with required accuracy
Force comparison	Full weight, F_g, must be generated with actuator force, F_C	Actuator force, F_C, may be attenuated by lever arm ratio
Comment	–	Coil location changes with respect to magnet (magnet not shown) by Δ_1, load to central pivot changes when placing or removing the mass, m

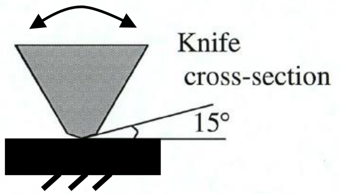
$$mgl_2 = BLIl_1 \qquad BL = \frac{U}{v_c}$$

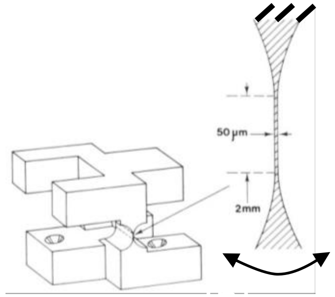
$$mgv_c\frac{l_2}{l_1} = UI$$

$$mgv_m = UI$$

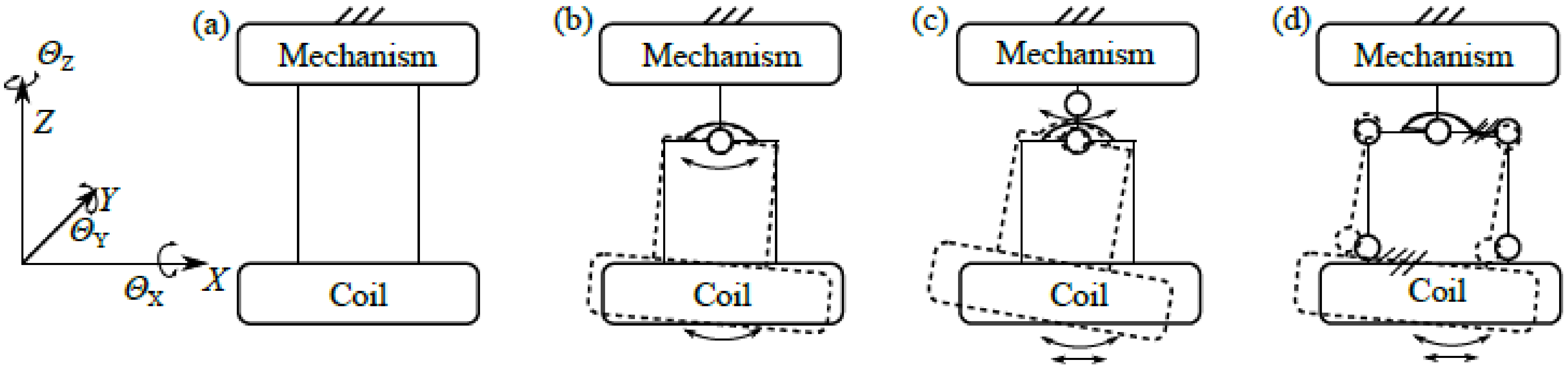
Pivot: knife edge vs flexure

Knife edge pivots	Flexure pivots
Most Kibble Balances made according to option (a)	Kibble Balances according to options (b) – (e)
High admissable loads (tens of kilograms)	Often negligible hysteresis: elastic deformation
Large deflections possible (tens of degrees)	Repeatable motion
Substantial hysteresis due to plastic deformation	Anelastic aftereffect present
Anelastic aftereffect present but often negligible	Careful design necessary
Relative sliding possible	Flexures have elastic stiffness





Coil suspension

Rigid	Single gimbal	Serial gimbal	Gimbal and parallelogram
			
So far accuracy $> 10^{-6}$ (tabletop instruments)	High accuracy Kibble Balances for laboratory environment		
Simplicity; no coil swing	Sensitive to parasitic forces and torques to the coil; adjustment of magnetic axes of coil and magnet in and about X and Y direction necessary but possible		
			Clearer separation of torques and forces to the coil
Abbe error difficult to adjust/measure; sensitive to off-axis loads	Low-frequency oscillation mode of the coil < 2 Hz; oscillation should be damped		

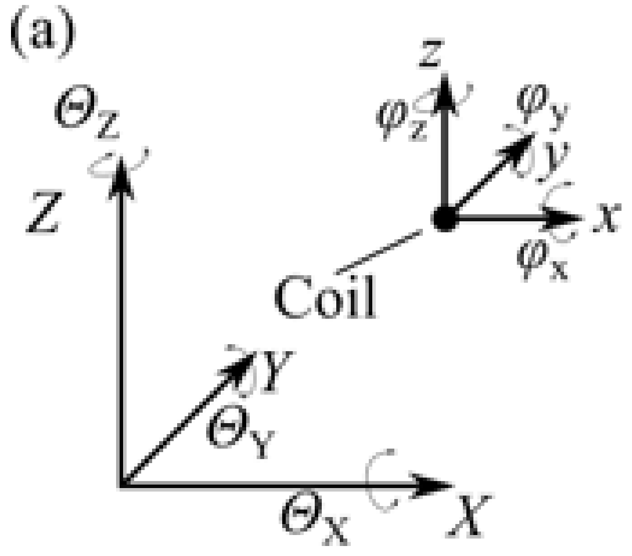
Equivalence of mechanical and electrical power in 6-dimensional case

$$\boldsymbol{VI = F_z\dot{z}\left(1 + \frac{F_x}{F_z}\frac{\dot{x}}{\dot{z}} + \frac{F_y}{F_z}\frac{\dot{y}}{\dot{z}} + \frac{M_x}{F_z}\frac{\dot{\varphi}_x}{\dot{z}} + \frac{M_y}{F_z}\frac{\dot{\varphi}_y}{\dot{z}} + \frac{M_z}{F_z}\frac{\dot{\varphi}_z}{\dot{z}}\right).}$$

Bias terms: $\boldsymbol{\frac{F_x}{F_z}\frac{\dot{x}}{\dot{z}} + \frac{F_y}{F_z}\frac{\dot{y}}{\dot{z}} + \frac{M_x}{F_z}\frac{\dot{\varphi}_x}{\dot{z}} + \frac{M_y}{F_z}\frac{\dot{\varphi}_y}{\dot{z}} + \frac{M_z}{F_z}\frac{\dot{\varphi}_z}{\dot{z}}}$ **→ need to be characterized.**

If $\frac{F_x}{F_z} = 10^{-5}$, and $\frac{\dot{x}}{\dot{z}} = 10^{-4}$

then $\frac{F_x}{F_z}\frac{\dot{x}}{\dot{z}} = 10^{-9}$



Kibble Robinson Theory (KRT)

Using the coupled coordinate ζ to describe the motion of the coil, we can write:

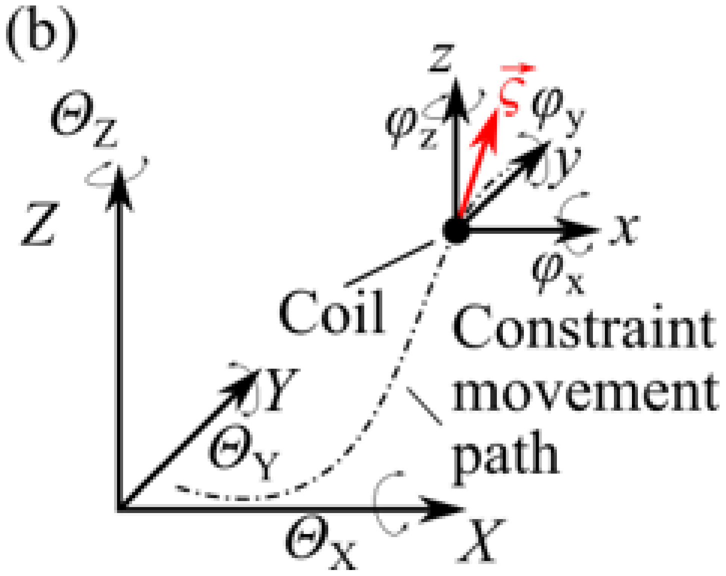
$V = -Nv_\zeta\left(\vec{\nabla}\Phi \cdot \partial_\zeta(x, y, z, \varphi_x, \varphi_y, \varphi_z)\right)$, **and**

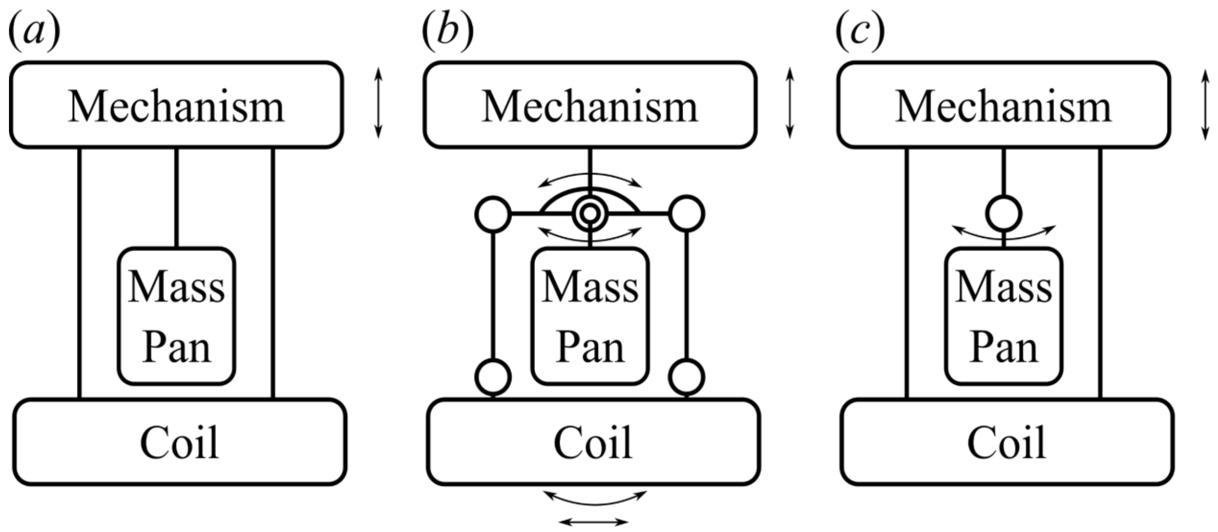
$F_\zeta = -IN\left(\vec{\nabla}\Phi \cdot \partial_\zeta(x, y, z, \varphi_x, \varphi_y, \varphi_z)\right)$.

Consolidating the last two equations yields for the force of the coil depending on the induced voltage and velocity, which can be measured in a velocity mode:

$$F_\zeta = \frac{V}{v_\zeta} I,$$

which describes the force exerted by the coil depending on the measured induced voltage in a single axis ζ.



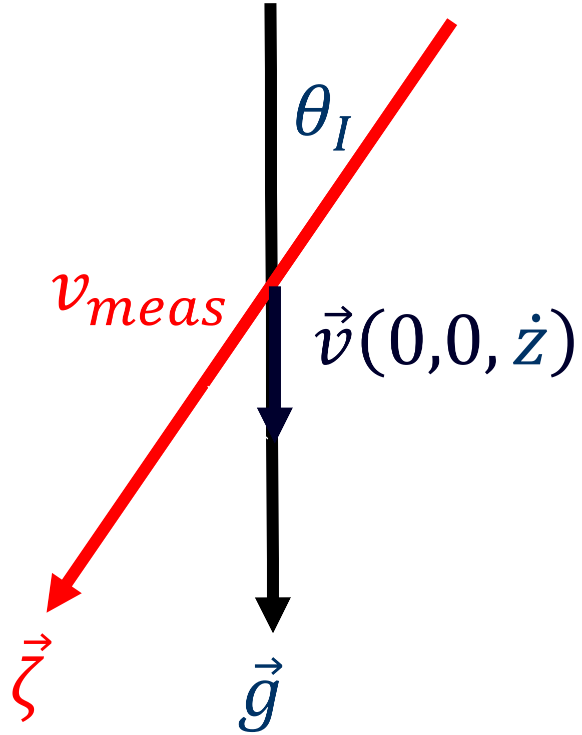


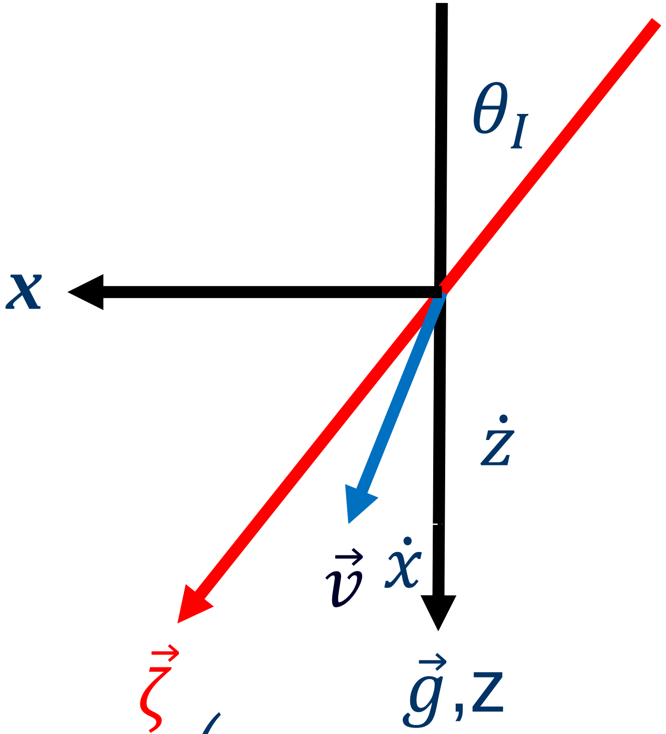
Obeys KRT?	Yes	No	Yes
Applications (as of present)	Commercial tabletop balances and tabletop Kibble balances	Laboratory-grade Kibble balances	Mass comparators
Advantages	No gimbals, stiff suspension	Proper alignment and characterization of biases possible	Allows one to adjust location of mass pan gimbal to minimize off-axis forces in weighing
Disadvantages	Sensitive to alignment and off-axis forces due to mass placement	Suspension modes affect measurement	Coil suspension and balance mechanism must be stiff against off-axis forces from coil

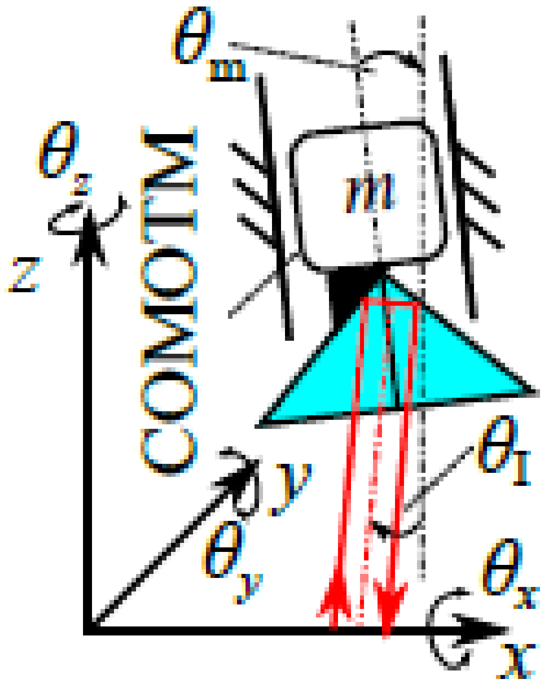
But…

You still need to consider errors that are present in the velocity measurement.

Cosine, Sine errors (OT)







$$\dot{z}_{meas} = \dot{z}cos\theta_I$$

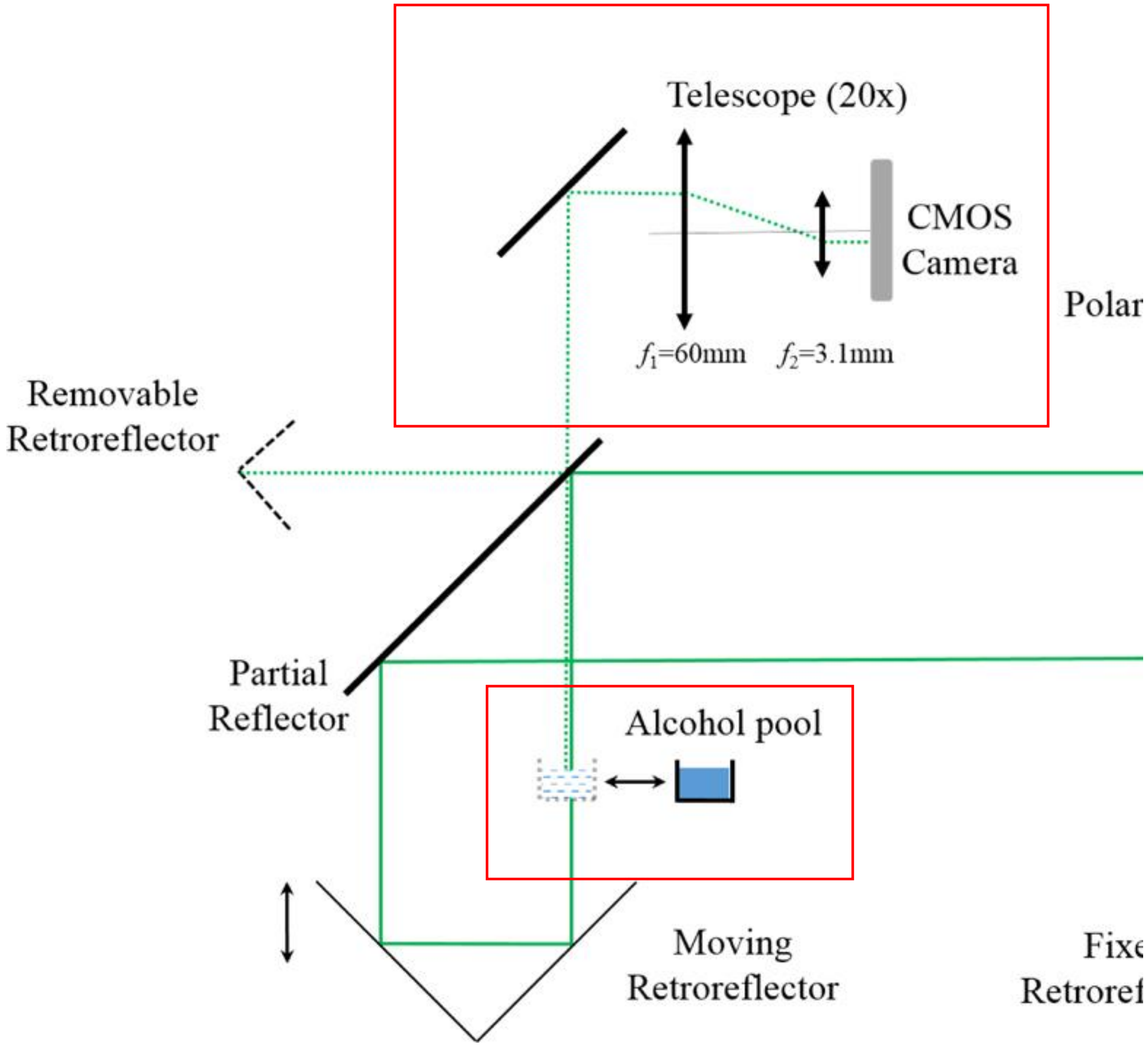
$$\dot{z}_{meas} = \dot{z}\left(cos\theta_I + sin\theta_I\frac{\dot{x}}{\dot{z}}\right)$$

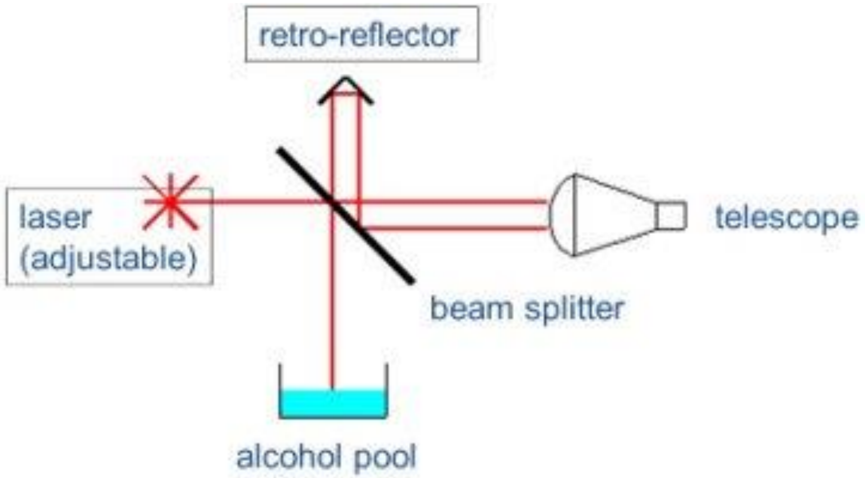
$$\theta_I = 45\mu rad, \frac{\dot{x}}{\dot{z}} = \frac{x}{z} = \frac{1\mu m}{10mm}$$

$$\frac{\dot{z}_{meas}}{\dot{z}} - 1 = -\frac{{\theta_I}^2}{2} + \theta_I\frac{\dot{x}}{\dot{z}}$$

$$\frac{\dot{z}_{meas}}{\dot{z}} - 1 = (-10^{-9} + 4.5 \times 10^{-9})$$

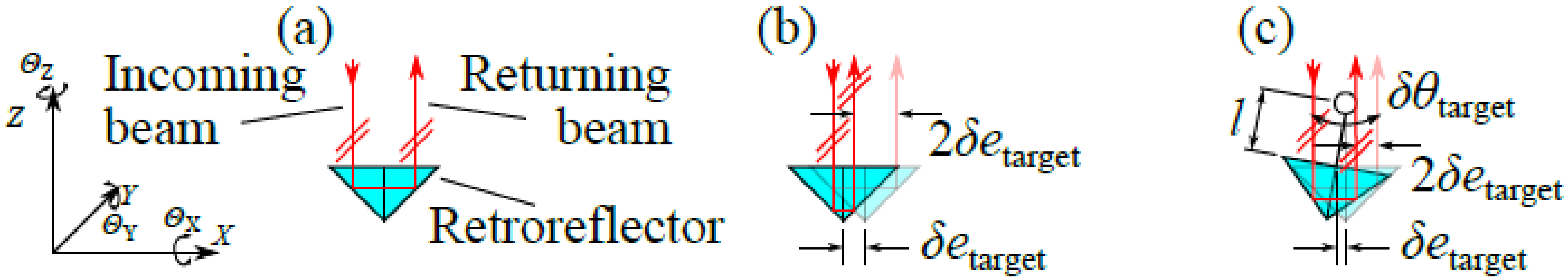
Reference liquid mirror or transfer horizontal mirror





METAS

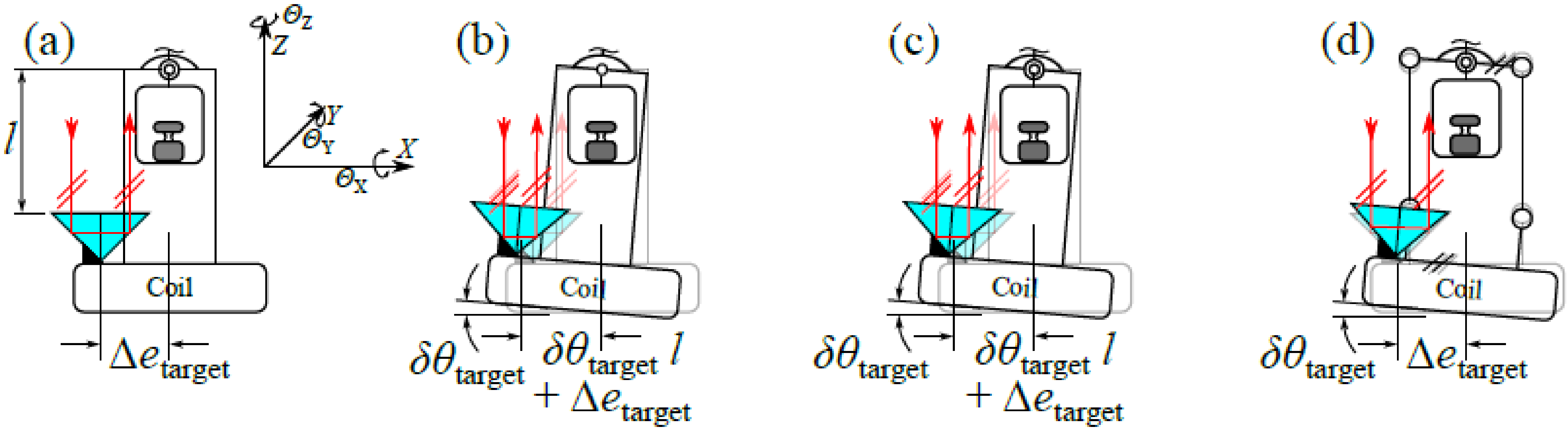
Beam shear error



$$\frac{\Delta m_{\mathrm{BS}}}{m} = k_{\mathrm{BS}} \left(\frac{\mathrm{d}(\delta e_{\text{target}})}{\mathrm{d}z_{\text{fin}}} + \frac{\mathrm{d}(\delta \theta_{\text{target}} l)}{\mathrm{d}z_{\text{fin}}} \right), \text{ with } k_{\mathrm{BS}} = \frac{\lambda}{10} \frac{2}{d_{\text{beam}}},$$

$$\frac{x}{z} = \frac{1\mu m}{10mm}, \lambda = 633nm, d_{beam} = 3mm \Rightarrow beam\ shear\ error = 4.2 \times 10^{-9}$$

Abbe offset with different suspension

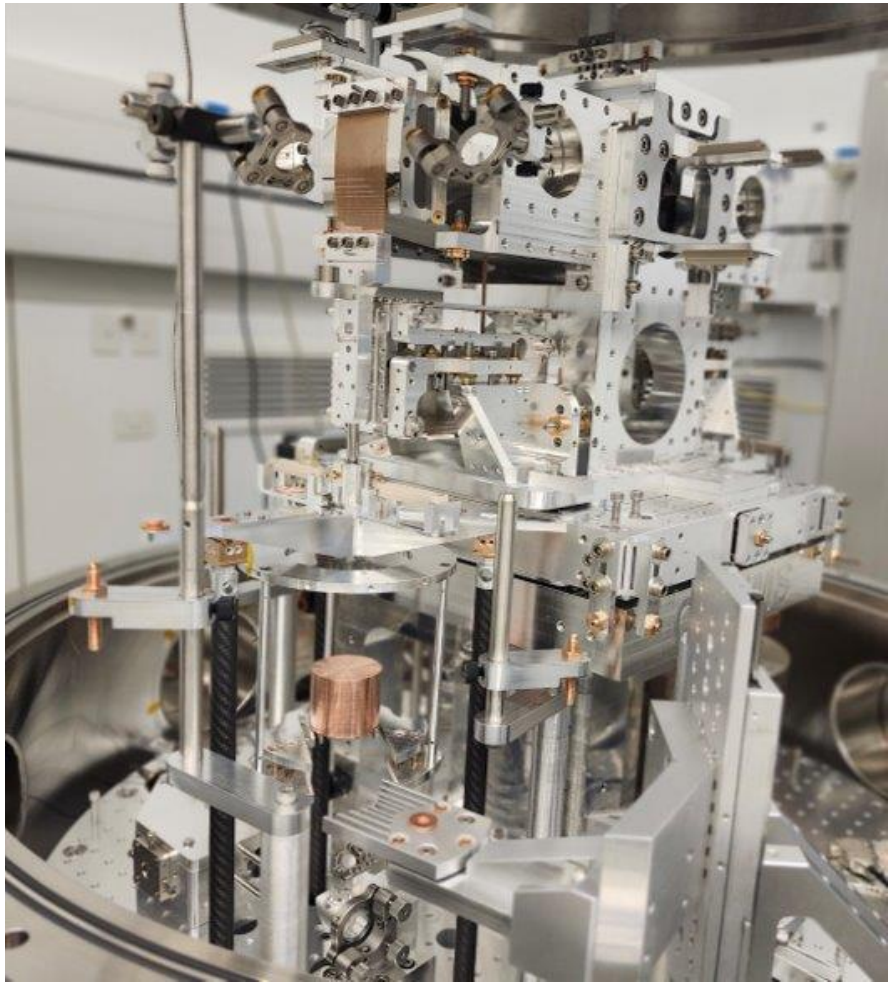


$$\frac{\Delta m_{\text{Abbe}}}{m} = \frac{\mathrm{d}(\delta\theta_{\text{target}})}{\mathrm{d}z_{\text{fin}}}\left(\Delta e_{\text{target}} + \delta\theta_{\text{target}}\, l\right).$$

$$\frac{d(\delta\theta_{target})}{dz_{fin}} = \frac{10\mu rad}{10mm}, \Delta e_{target} = 200\mu m \Rightarrow Abbe\ offset\ bias = 200 \times 10^{-9}$$

"You are not alone" by M.J.

GitHub in 2026



References

Lorenz Keck, Flexure-Based Mechanism for a Kibble Balance, PhD Thesis, *University of Ilmenau,* DOI 10.22032/dbt.63554

Lorenz Keck et al., Thoughts on the Kibble-Robinson Theory, *Metrologia, Volume 62, Number 2*, **DOI** 10.1088/1681-7575/adc30e

Darine Haddad et al., A precise instrument to determine the Planck constant, and the future kilogram, *Rev. Sci. Instrum.* 87, 061301 (2016), https://doi.org/10.1063/1.4953825

Thank you.

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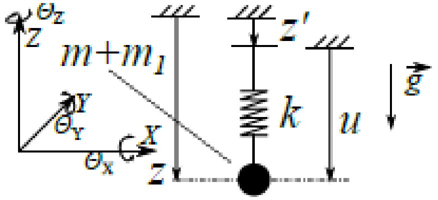
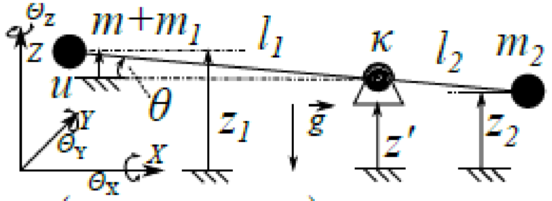
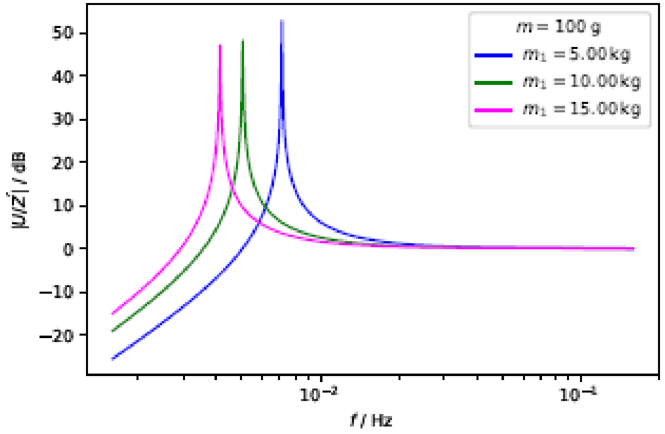
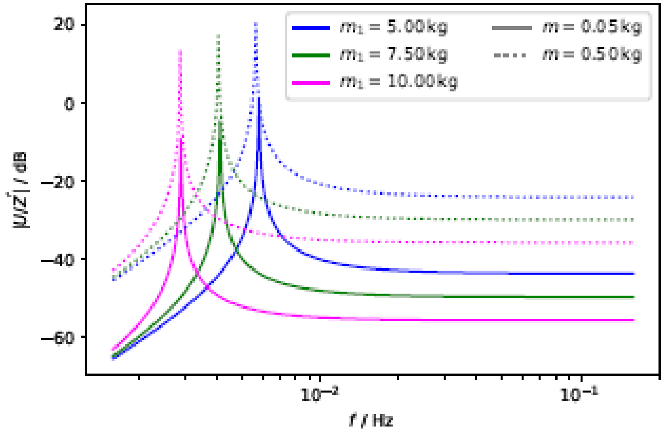
Seismometer vs lever arm balance

Category	Active	Passive
Pros	Significant reduction of mechanical parts	Static, thermal, and dynamic symmetry
Cons	Noise and drift in the active balancing, sensitive to ground vibrations in the vertical direction	Larger and more complex mechanical structure necessary, central pivot additionally loaded with counter weight
Simplified drawing		
Transfer function	$\frac{U}{Z'} = \frac{-s^2}{\frac{k}{m+m_1} + s^2}$	$\frac{U}{Z'} = \frac{s^2\left(m_2 l_2 l_1 - (m_1+m)\, l_1^2\right)}{s^2\left((m_1+m)\, l_1^2 + m_2 l_2^2\right) + \kappa}$
Bode diagram		
Assumptions	Open loop, small deflections, systems in equilibrium	
	m_1 is mass of payload, $k = 0.01\,\mathrm{N\,m^{-1}}$	$m_2 = 15\,\mathrm{kg} - m_1$, $l_1 = 0.25\,\mathrm{m}$, $l_2 = l_1 \frac{m_1}{m_2}$, $\kappa = k l_1^2 = 0.63\,\mathrm{N\,mm\,rad^{-1}}$
Conclusion	Low resonance frequency → sensitive to external vibrations in vertical direction	Effective attenuation of external vibrations in vertical direction [93]

Optical target location in weighing and velocity