

KBTM

Magnet/coil design & effects

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20 November 2025



**Kibble Balance Technical Meeting
special session “knowledge transfer”**

20 November 2025, BIPM

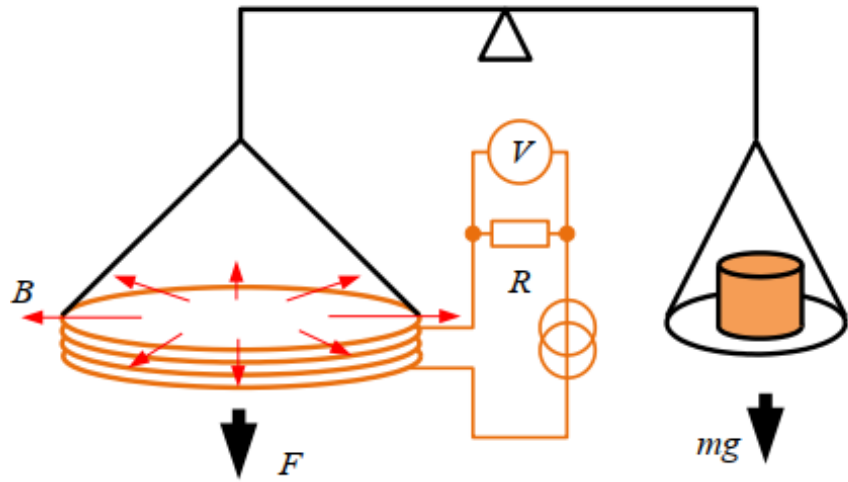
Outlines



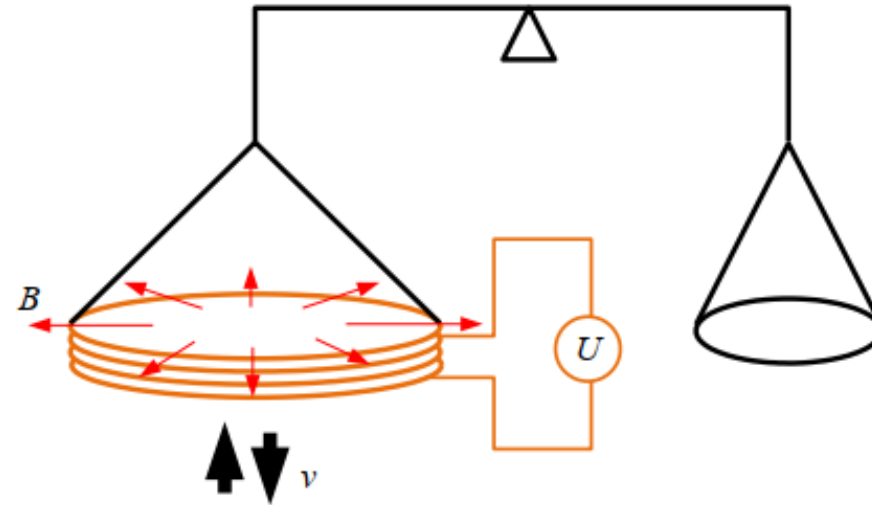
- 1. Magnetic basics in KB experiments**
2. A Brief history KB magnet-coil designs
3. How to design a KB magnet system
4. Evaluation performance of a KB magnet
5. Systematic effects and how to reduce
6. Summary & outlook

KB Magnetic basics:

Role of Bl in KBs



Weighing phase $mg = BlI$



$U = Blv$ Velocity phase

$$mgv = U \frac{V}{R}$$

$$m = \frac{UV}{gvR}$$

$$m = \frac{n_1 n_2 i f_1 f_2}{4gv} h$$

KB Magnetic basics:

A good Bl value helps



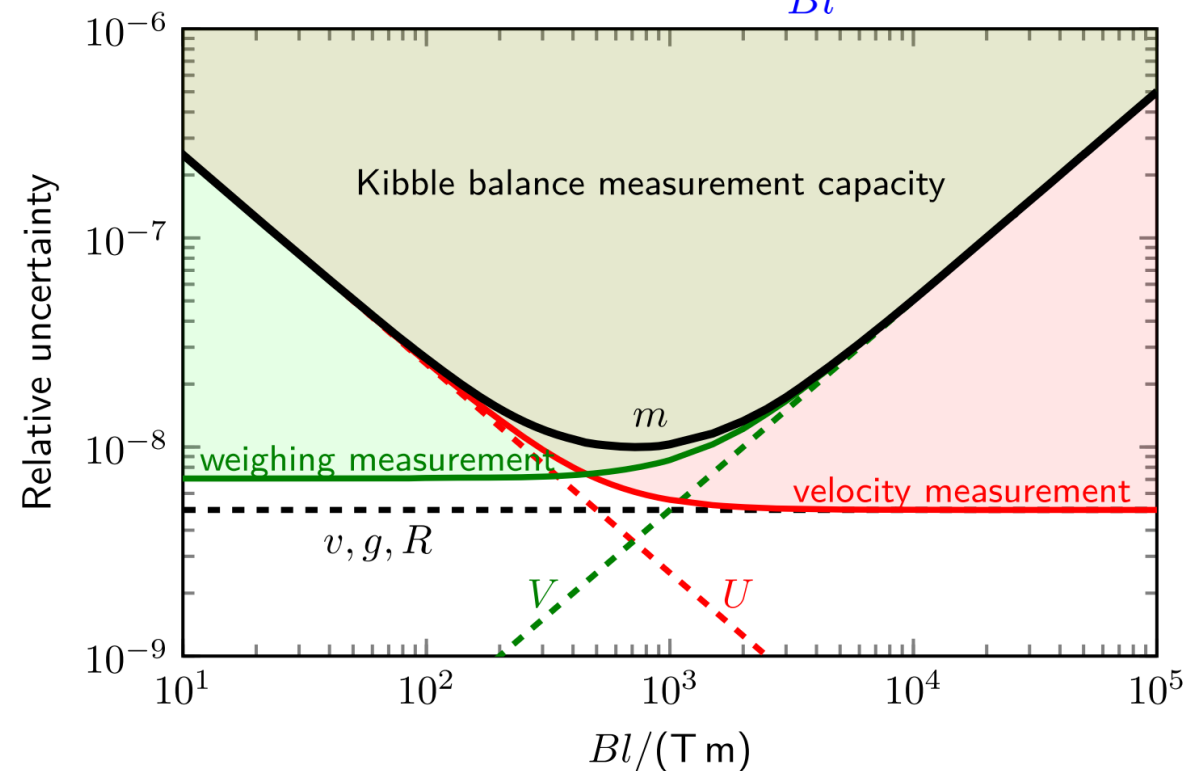
$$m = \frac{UV}{gvR} \rightarrow \left(\frac{\sigma_m}{m}\right)^2 = \left(\frac{\sigma_v}{v}\right)^2 + \left(\frac{\sigma_g}{g}\right)^2 + \left(\frac{\sigma_R}{R}\right)^2 + \left(\frac{\sigma_U}{U}\right)^2 + \left(\frac{\sigma_V}{V}\right)^2$$

$mg = BlI$

$U = Blv \quad V = \frac{mgR}{Bl}$

$$\left(\frac{\sigma_m}{m}\right)^2 = \left(\frac{\sigma_v}{v}\right)^2 + \left(\frac{\sigma_g}{g}\right)^2 + \left(\frac{\sigma_R}{R}\right)^2 + \left(\frac{1}{Bl}\right)^2 \left(\frac{\sigma_U}{v}\right)^2 + (Bl)^2 \left(\frac{\sigma_U}{mgR}\right)^2$$

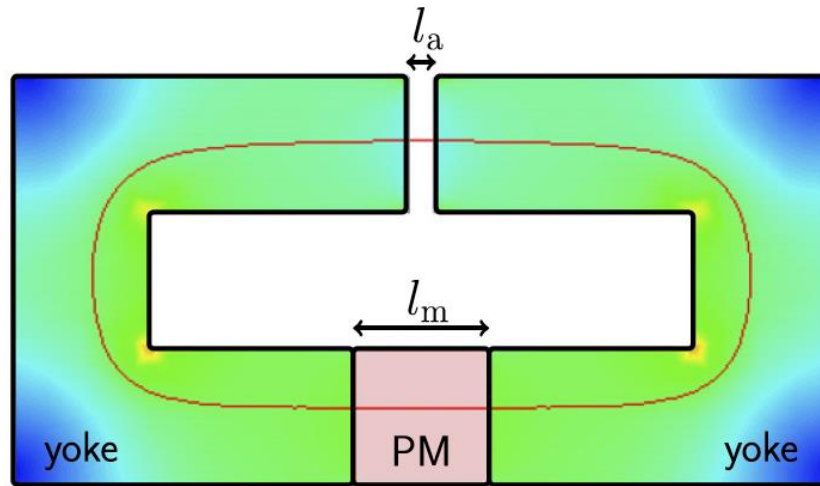
$$(Bl)_{\text{op}} = \sqrt{\frac{mgR}{v}}$$



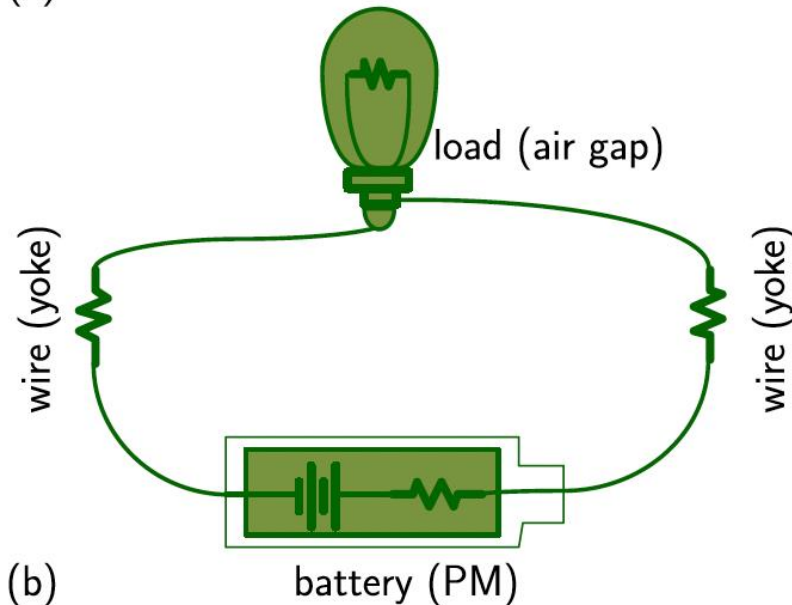
$\sigma_v/v = \sigma_g/g = \sigma_R/R = 5 \times 10^{-9}$, $\sigma_U = 5 \text{ nV}$, $v = 2 \text{ mm s}^{-1}$, $mgR = 1000 \text{ } \Omega\text{N}$.

KB Magnetic basics:

Magnetic Ohm's law



(a)



(b)

$$I = U/R$$

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}}$$

U the voltage electromotive force (EMF)
 $\mathcal{F}$ the magnetomotive force (MMF).

I the electrical current
 Φ the magnetic flux

R the electrical resistance
 $\mathcal{R}$ the magnetic reluctance

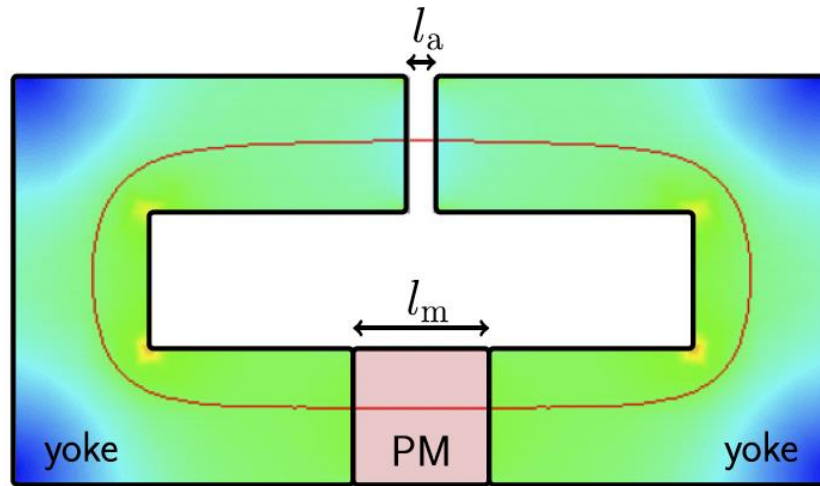
$$R_a = \frac{l_a}{S\mu_0},$$

$$R_y = \frac{l_y}{S\mu_0\mu_r},$$

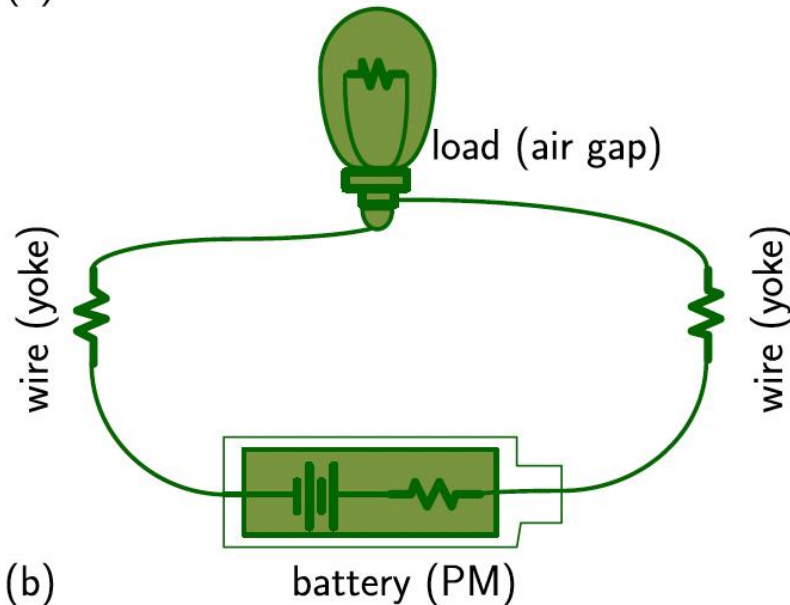
$$R_m = \frac{l_m}{S\mu_0\mu_m},$$

KB Magnetic basics:

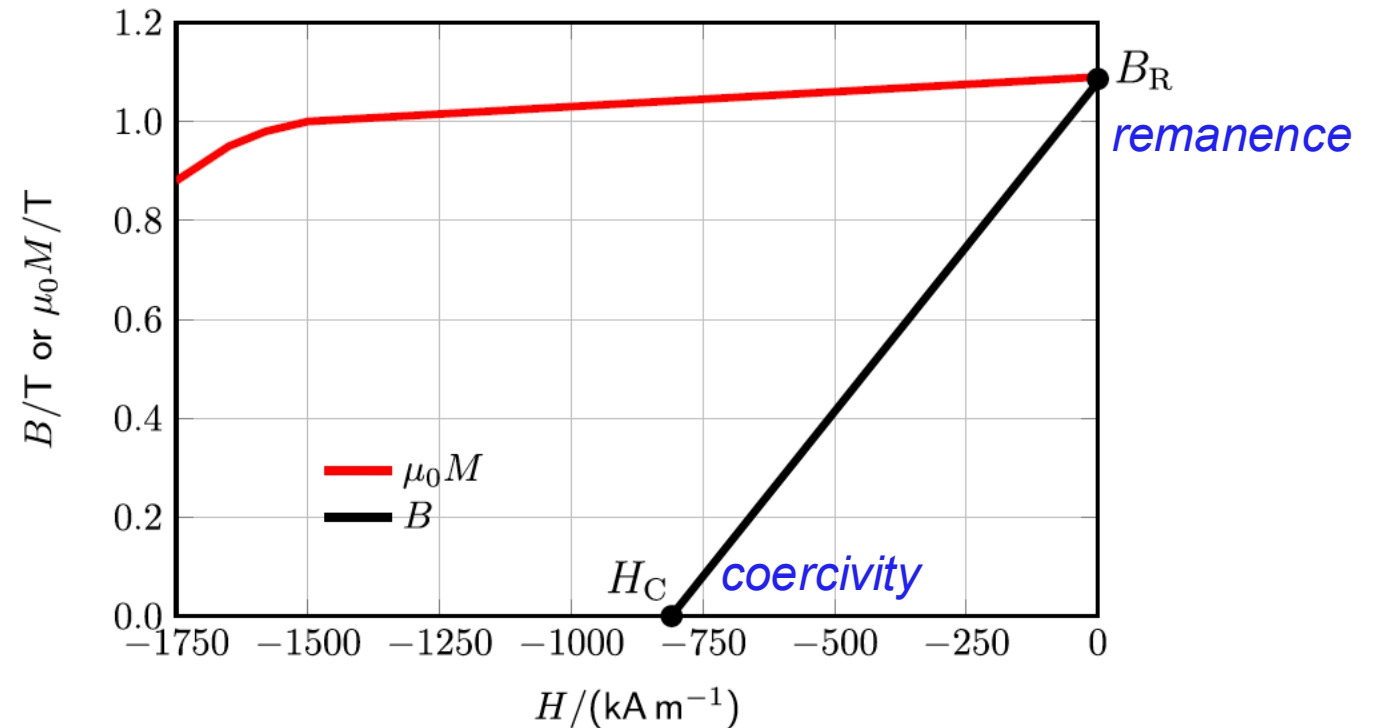
Magnetic Ohm's law



(a)



(b)



$$\Phi \left(\frac{l_m}{S\mu_m\mu_0} + \frac{l_y}{S\mu_0\mu_r} + \frac{l_a}{S\mu_0} \right) = \frac{B_R}{\mu_m\mu_0} l_m = -H_C l_m$$

Double the PM width, the field will not be doubled!

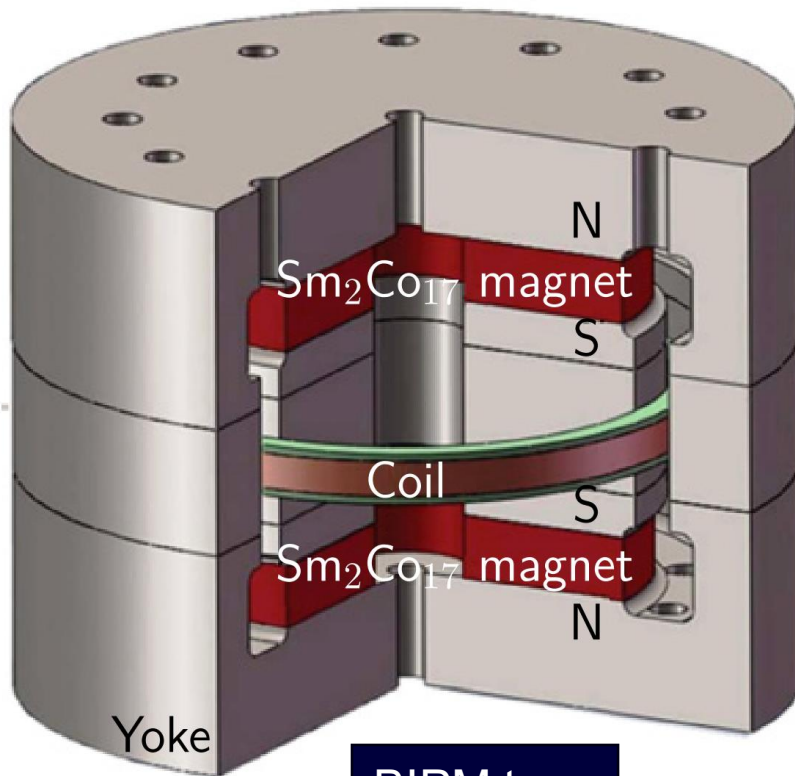
KB Magnetic basics:

What is Bl ?

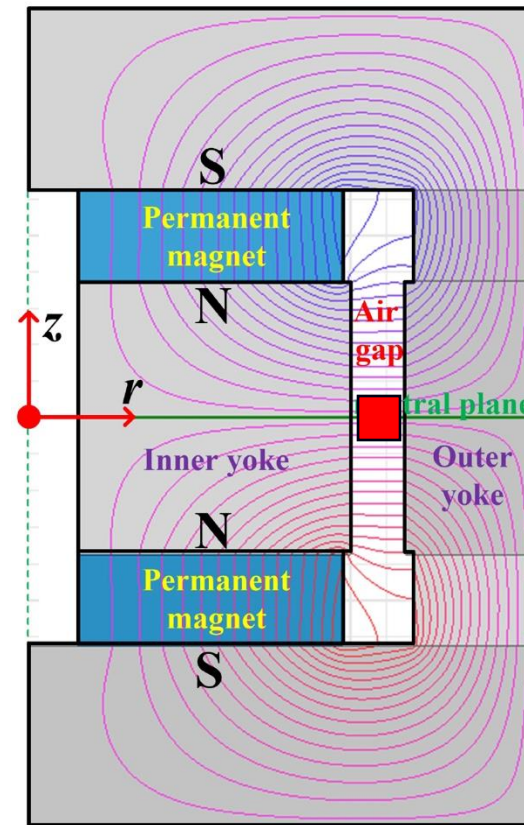
↑↓ BIPM

$$Bl = \frac{\partial \Phi}{\partial z} = \int (\mathbf{dl} \times \mathbf{B})_z$$

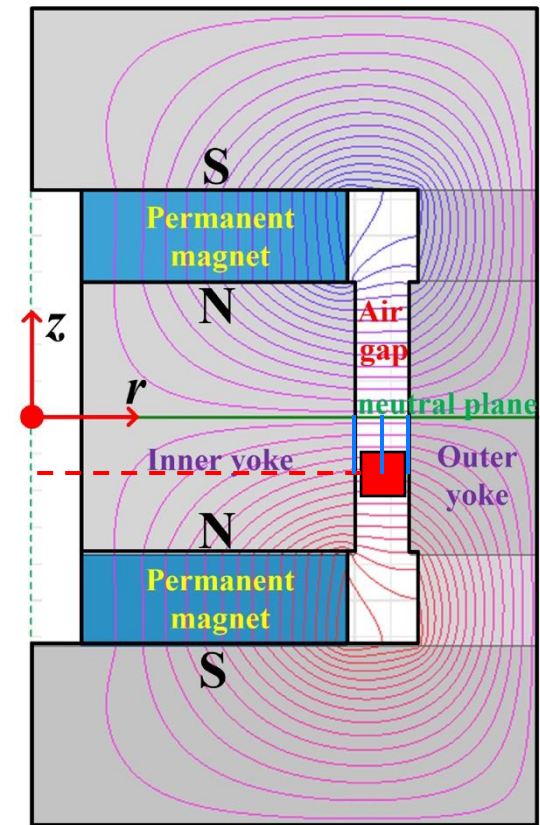
$$Bl : [\text{Tm}] = \frac{[\text{Tm}^2]}{[\text{m}]} : \frac{\partial \Phi}{\partial z}$$



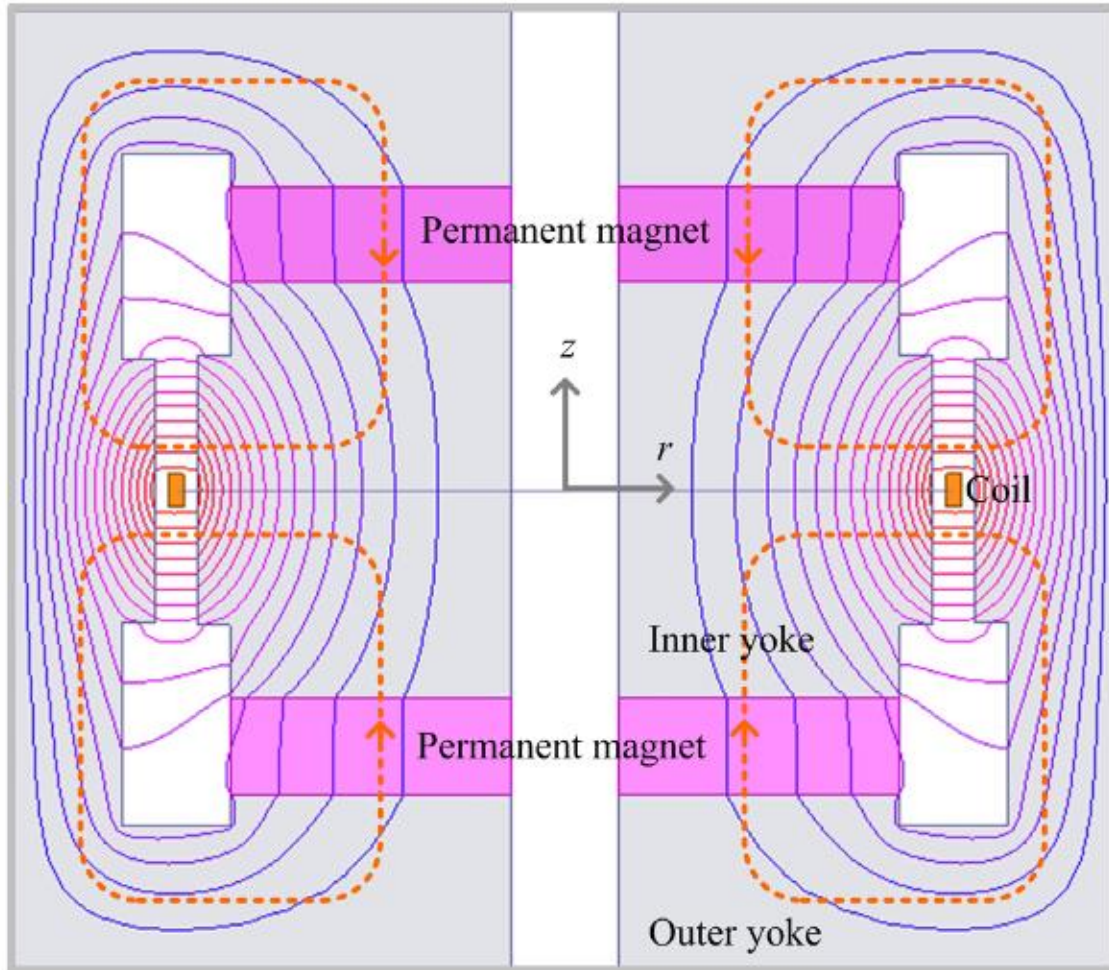
BIPM type



$$z = 0, \Phi = 0$$



$$\Phi(z) = Blz$$



*Magnetic Energy=flux linkage*current*

The energy of a bifilar coil system is

$$E = \frac{L}{2}I_1^2 + \frac{L}{2}I_2^2 + MI_1I_2$$

By the virtual principle , the force is

$$\Delta F = \frac{\partial E}{\partial z} = \frac{I^2}{2} \frac{\partial L}{\partial z}$$

*Note that a **current constant** system,
the sign should be positive!*

The current offers additional energy (2E)

Outlines

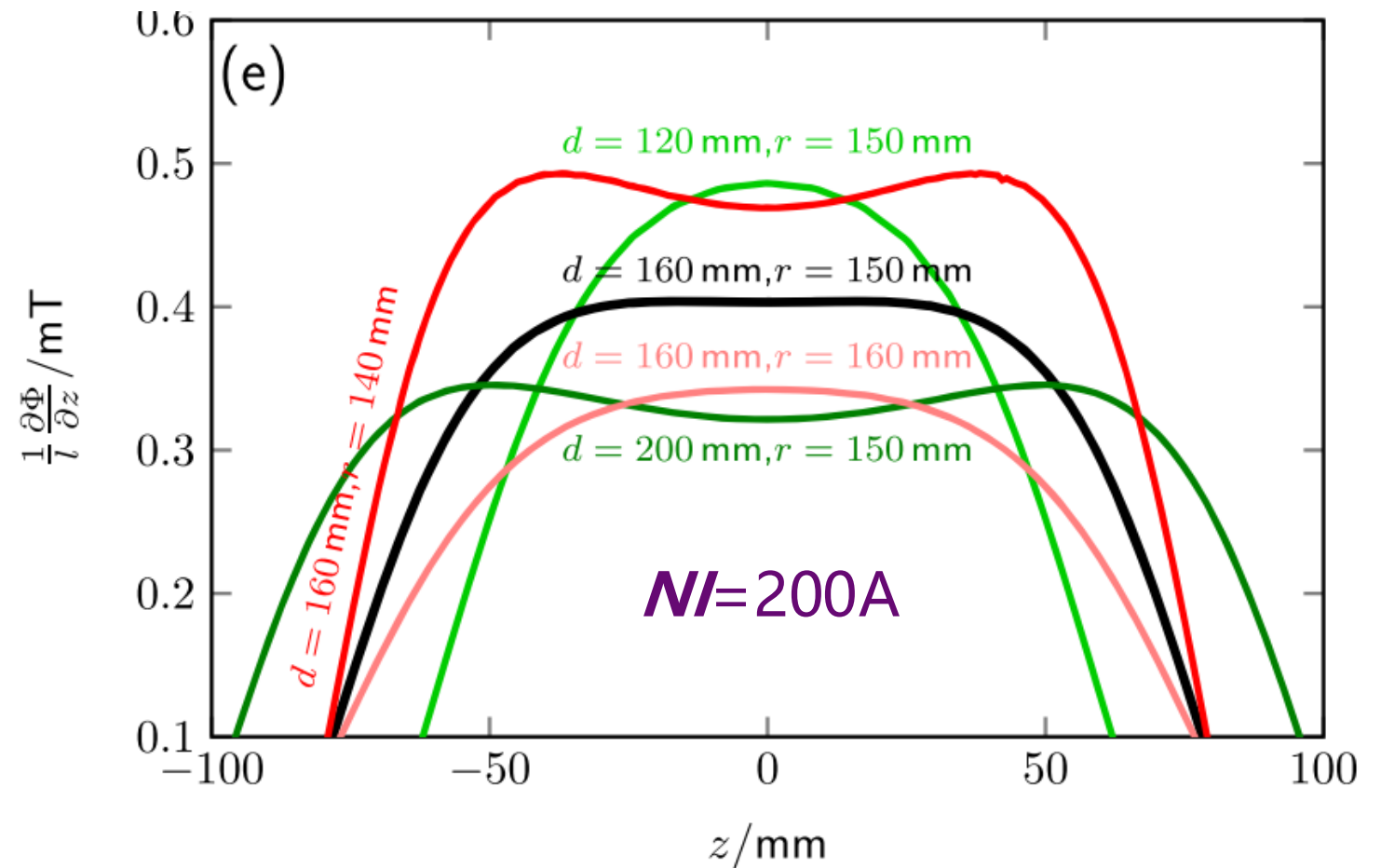
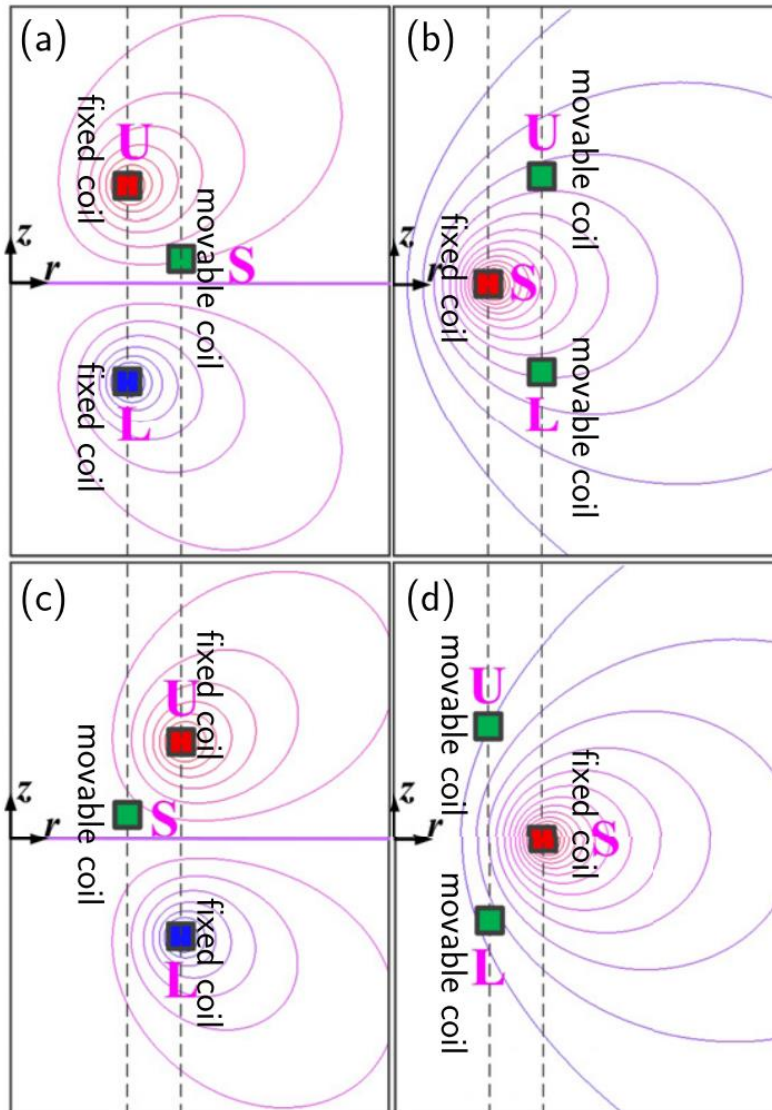


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KB magnet-coil history:

conventional coil system

↑↓ BIPM

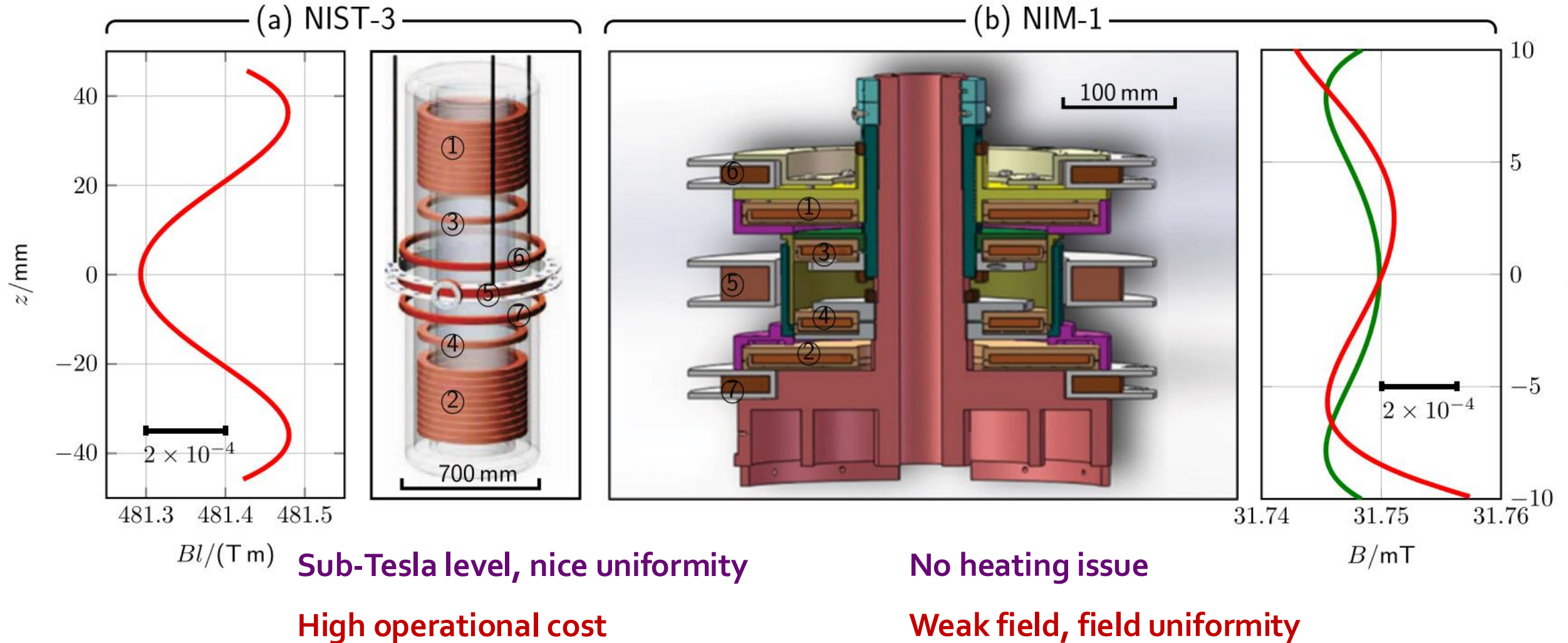


Used in the ampere balance experiment
Weak magnetic force or heating issue

KB magnet-coil history:

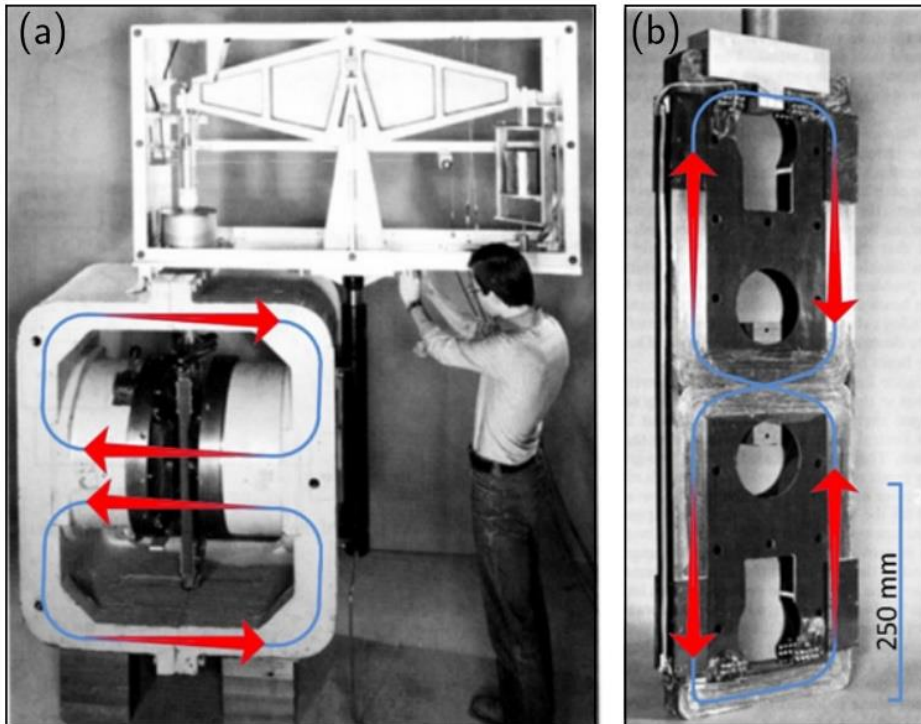
multi-coil magnet system

↑↓ BIPM



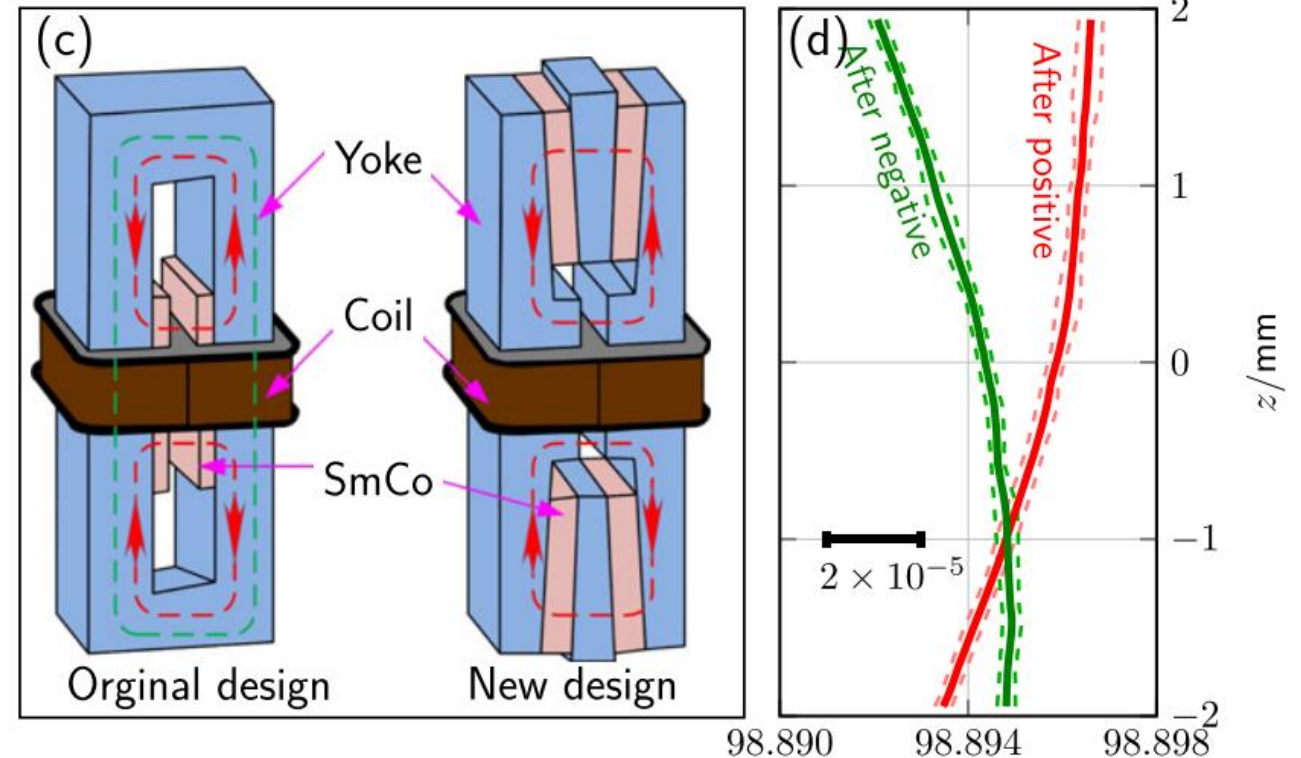
KB magnet-coil history:

Flat PM system



56 mm width, $0.3 \text{ m} \times 0.3 \text{ m}$ sectional area air gap.
0.68 T in the air gap center.
The magnet weighs 6000 kg, and the coil 30 kg.

Kibble B P et al 1990 Metrologia 27 173–92



Field edge effect; Need an 8-shape coil; $Bl/(T \cdot m)$
Large size or low efficiency compared to radial system;
Be careful with closed yoke path!

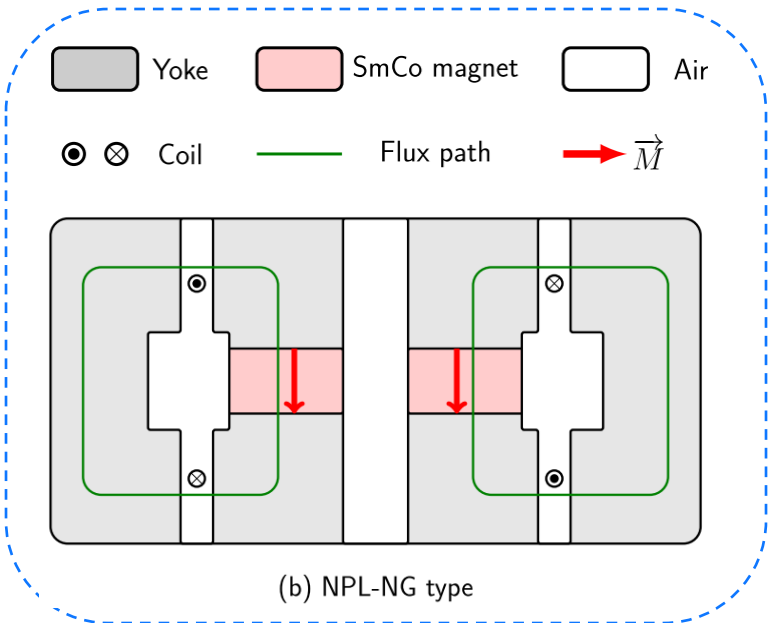
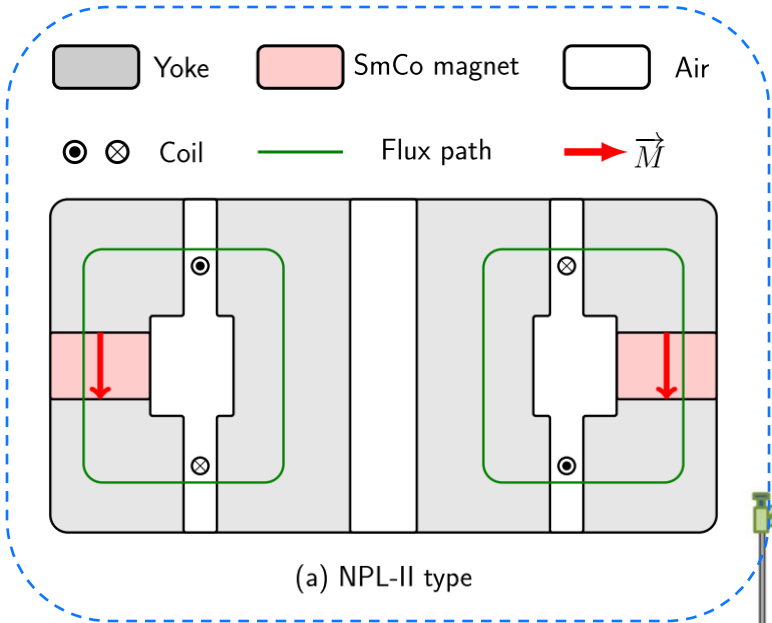
Eichenberger A et al 2011 Metrologia 48 133–41
Beer W et al 2003 IEEE TIM 52 626–30

KB magnet-coil history:

Radial PM system

 **BIPM**

Great merit: nice cancellation of electromagnetic noise!



First radial PM system

Symmetry: ★ ★ ★ ★ ★

Efficiency: ★ ★ ★ ★

Shielding: ★ ★ ★

Robinson I A 2009 IEEE TIM 58 936–41
Wood BM et al 2017 Metrologia 54 399–409
Robinson I A 2018 CPEM

NPL's next generation KB magnet

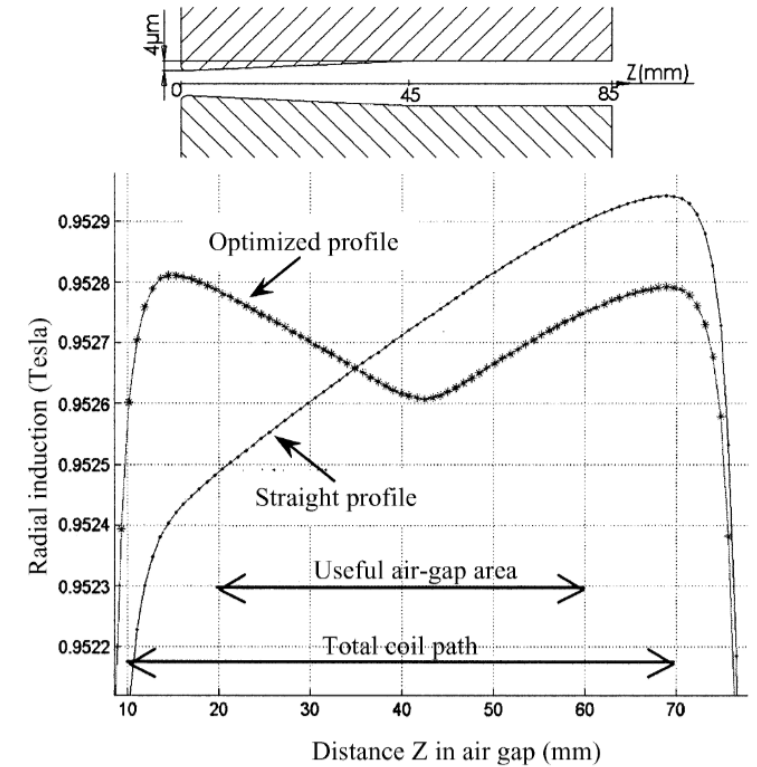
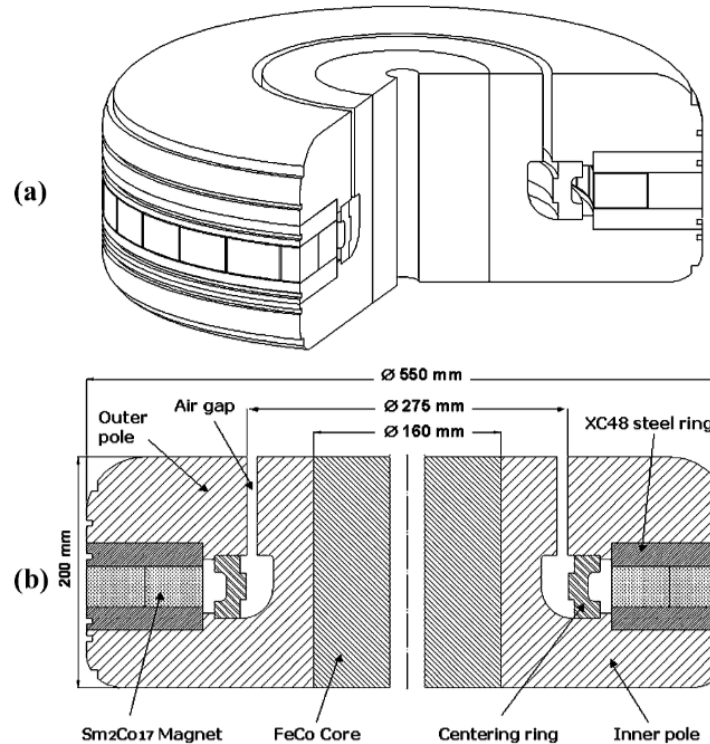
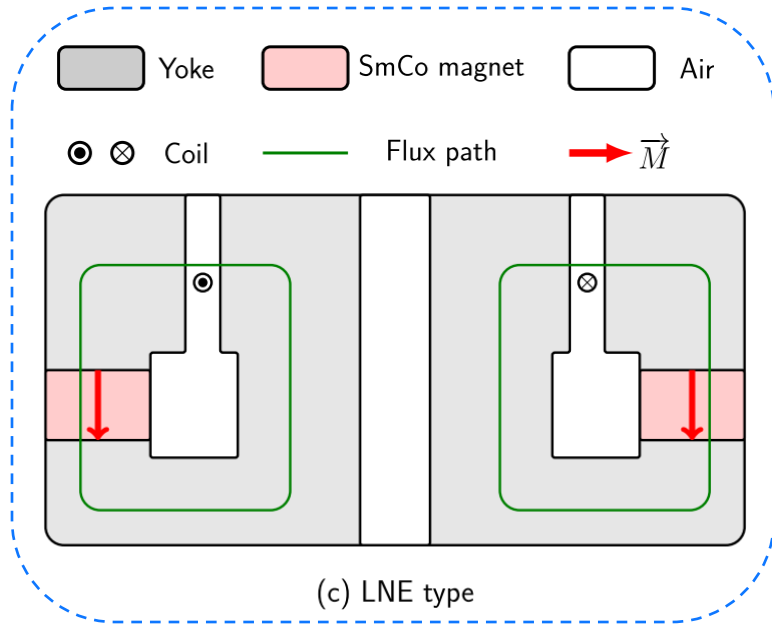
Symmetry: ★ ★ ★ ★ ★

Efficiency: ★ ★ ★

Shielding: ★ ★ ★ ★

KB magnet-coil history:

Radial PM system



PM system with the strongest field

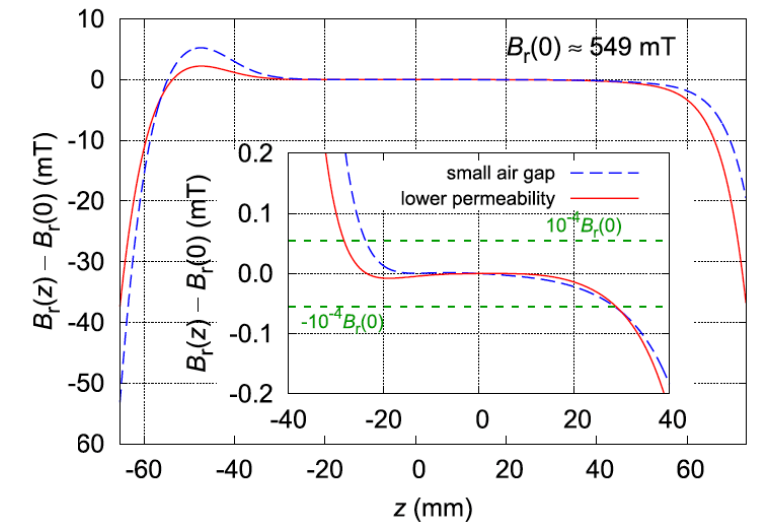
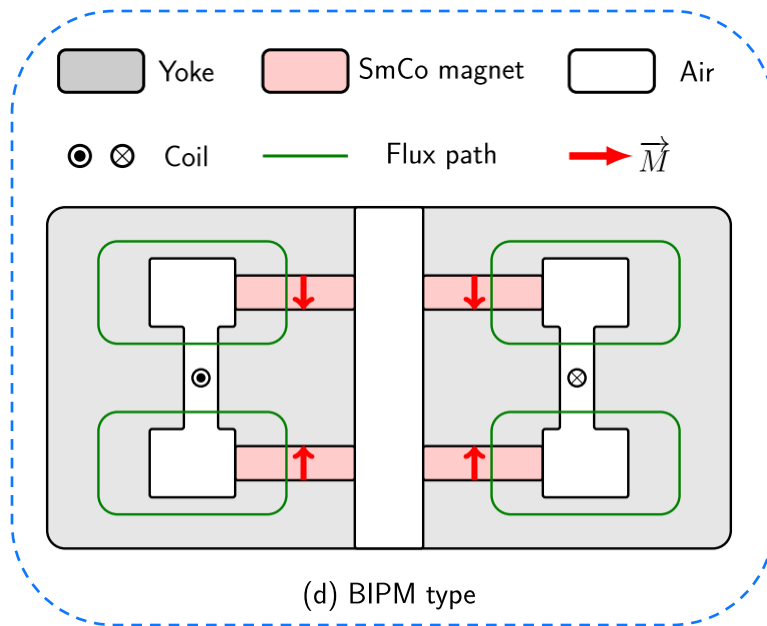
Symmetry: ★ ★ ★ ★

Efficiency: ★ ★ ★ ★

Shielding: ★ ★ ★

KB magnet-coil history:

Radial PM system

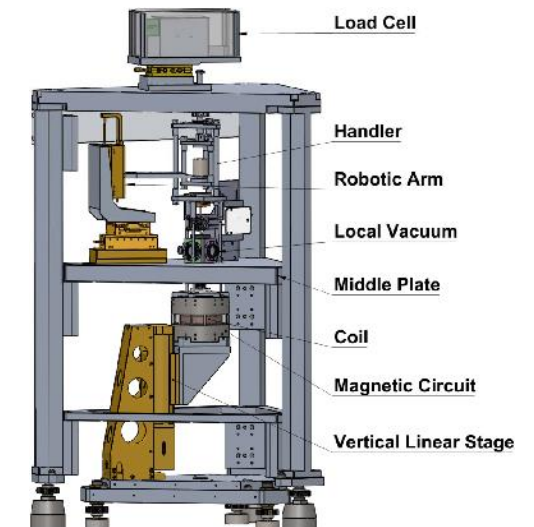
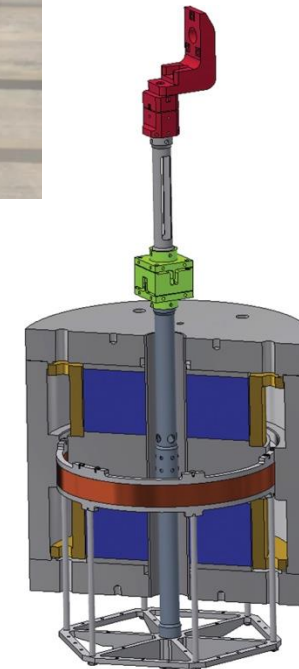
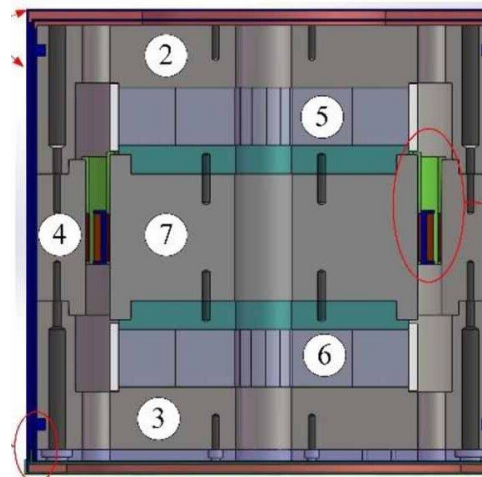


Most popular PM system

Symmetry: ★ ★ ★ ★ ★

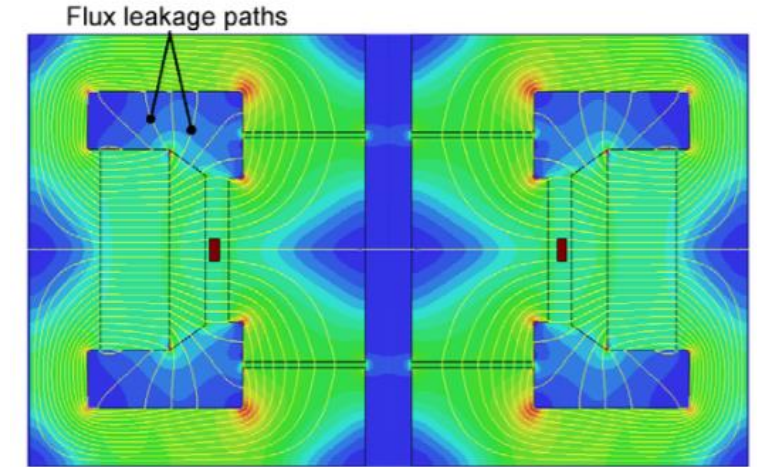
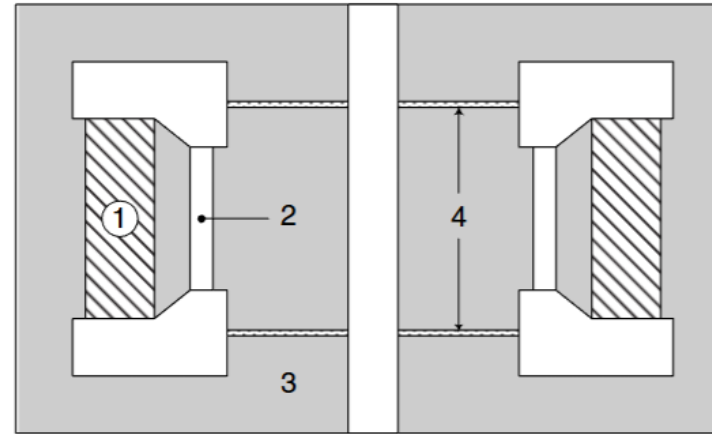
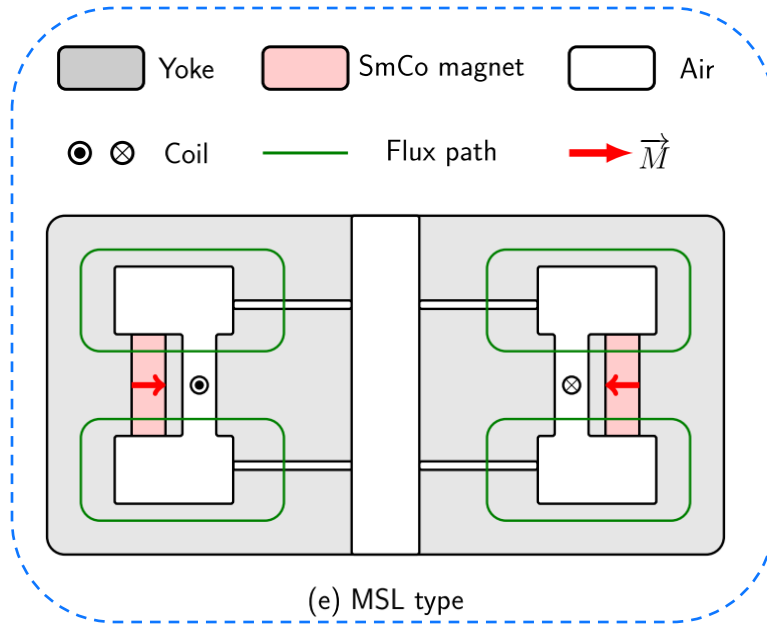
Efficiency: ★ ★ ★ ★ ★

Shielding: ★ ★ ★ ★ ★



KB magnet-coil history:

Radial PM system



Coil flux return, and additional air gap is required, which reduce the field strength at the coil position.

Installation may be not easy.

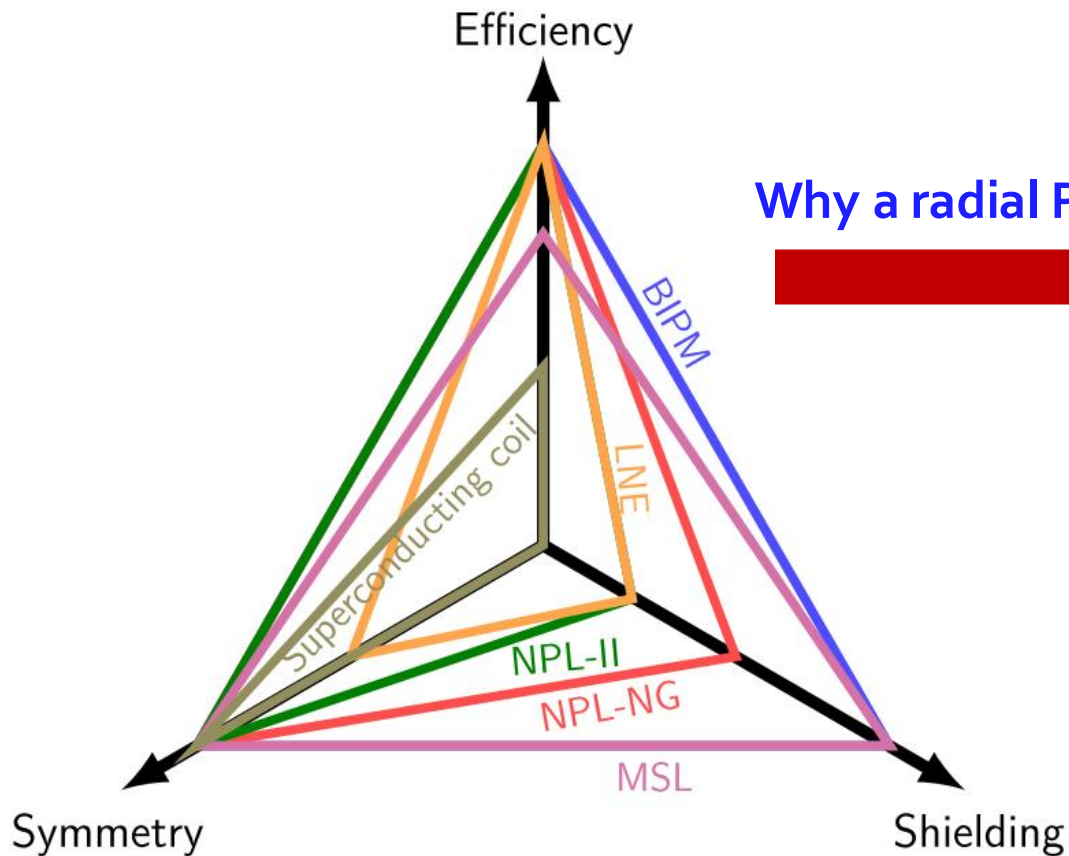
Symmetry: ★ ★ ★ ★ ★

Efficiency: ★ ★ ★

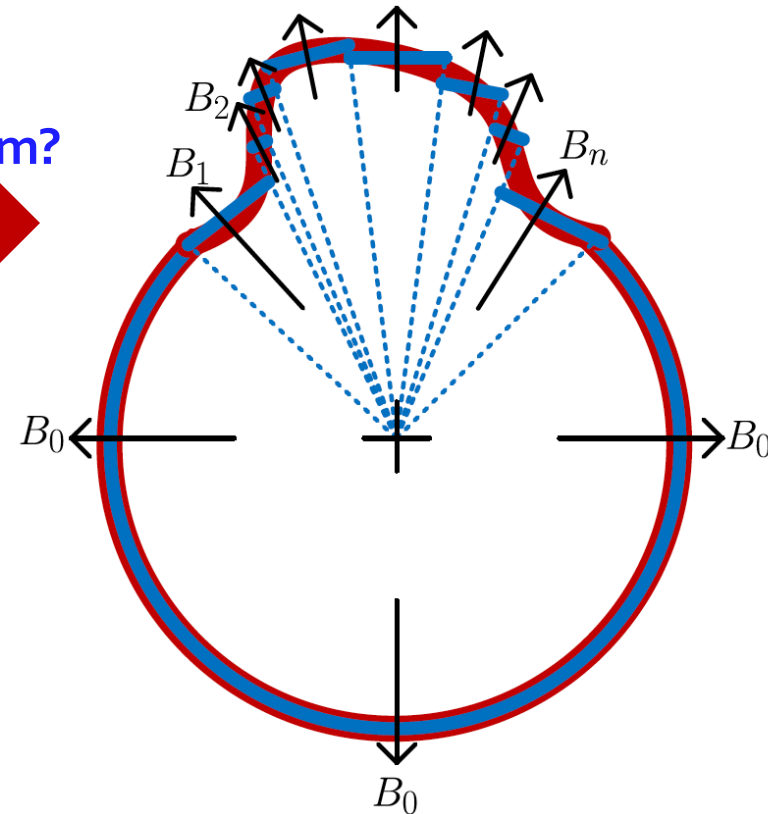
Shielding: ★ ★ ★ ★ ★

KB magnet-coil history:

Radial PM system



Why a radial PM system?



$$\vec{\nabla} \cdot \vec{B} = \frac{1}{r} \frac{\partial(rB_r)}{\partial r} + \frac{1}{r} \frac{\partial B_\varphi}{\partial \varphi} + \frac{\partial B_z}{\partial z} = 0$$

$$\frac{B_r}{r} + \frac{\partial B_r}{\partial r} = -\frac{\partial B_z}{\partial z}$$

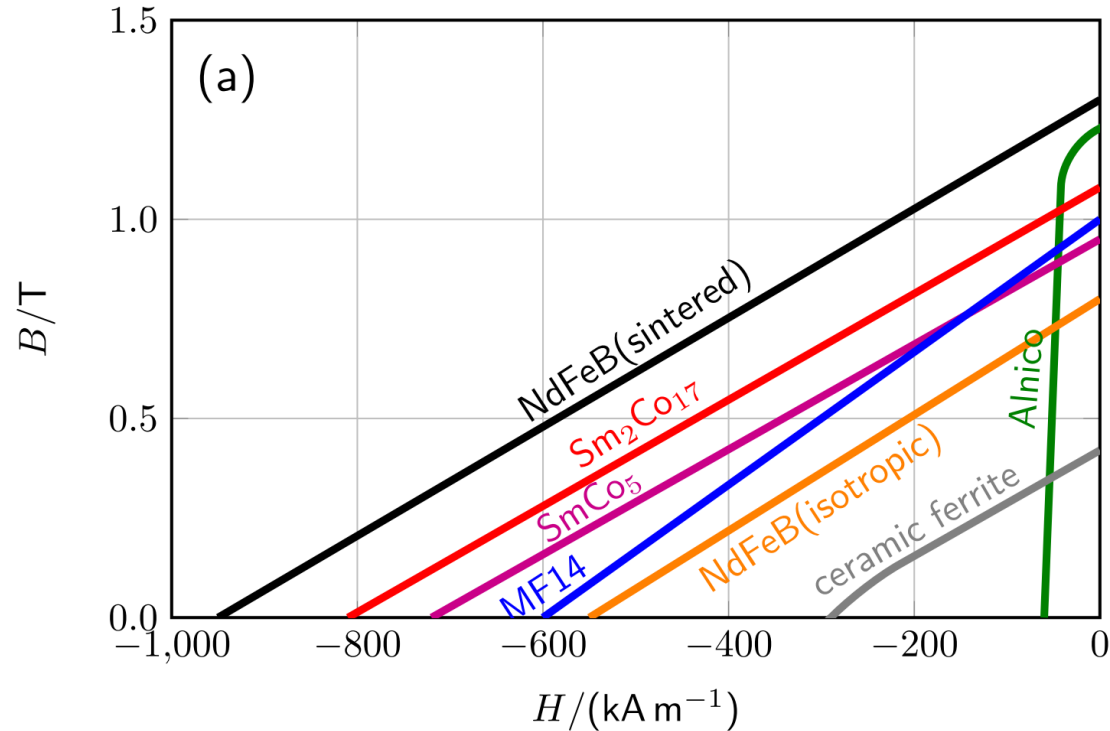
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KB magnet design:

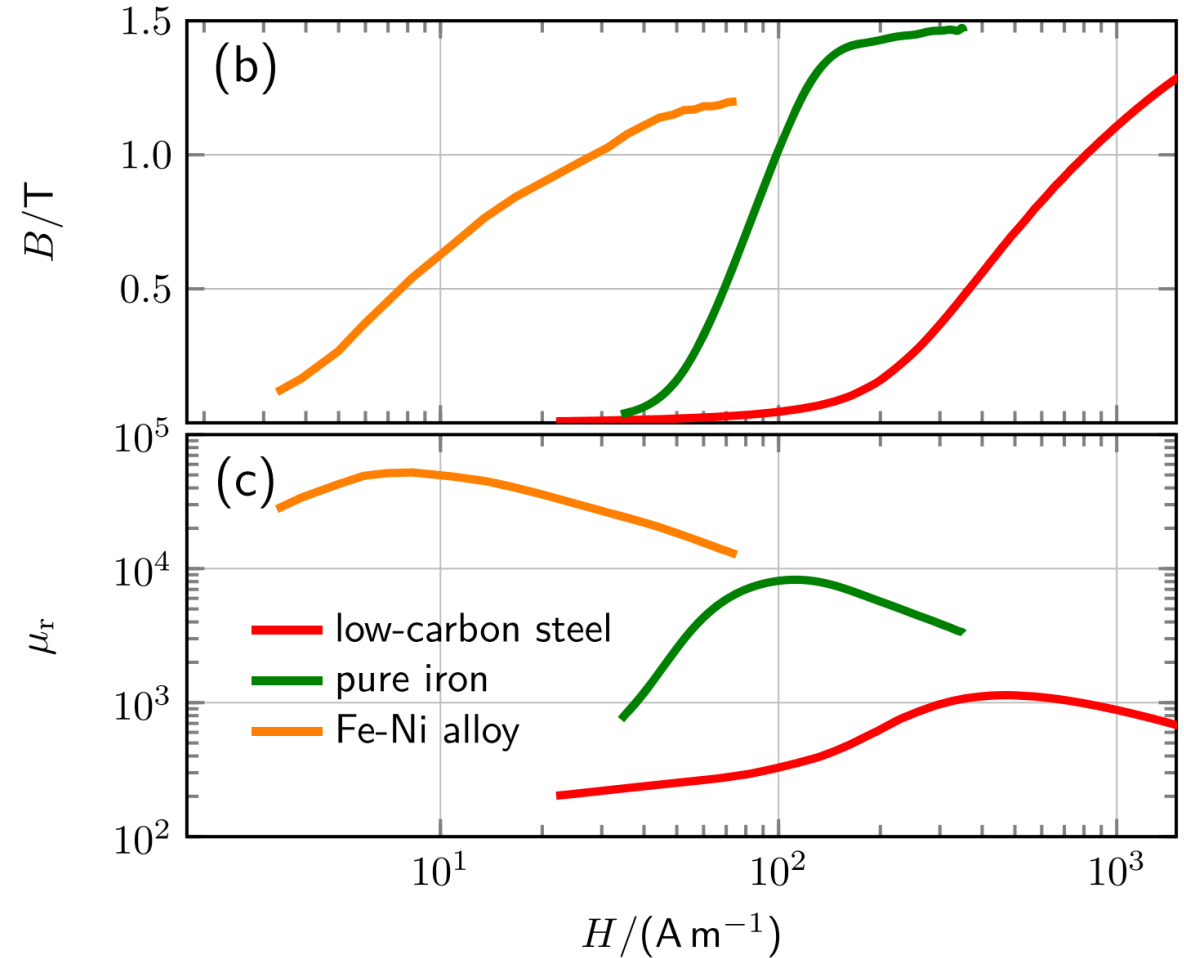
Magnetic materials



NdFeB: high H_c , high T_c (-1000ppm/K)

Sm₂Co₁₇: T_c (-300ppm/K), lower H_c

(Gd,Sm)₂Co₁₇: low T_c , H_c is 20% less than Sm₂Co₁₇

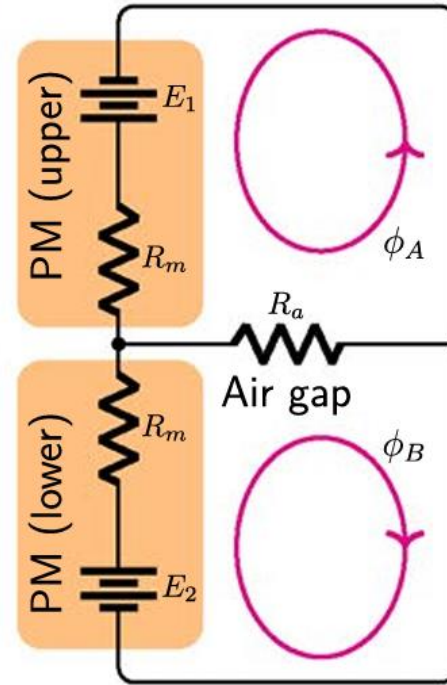
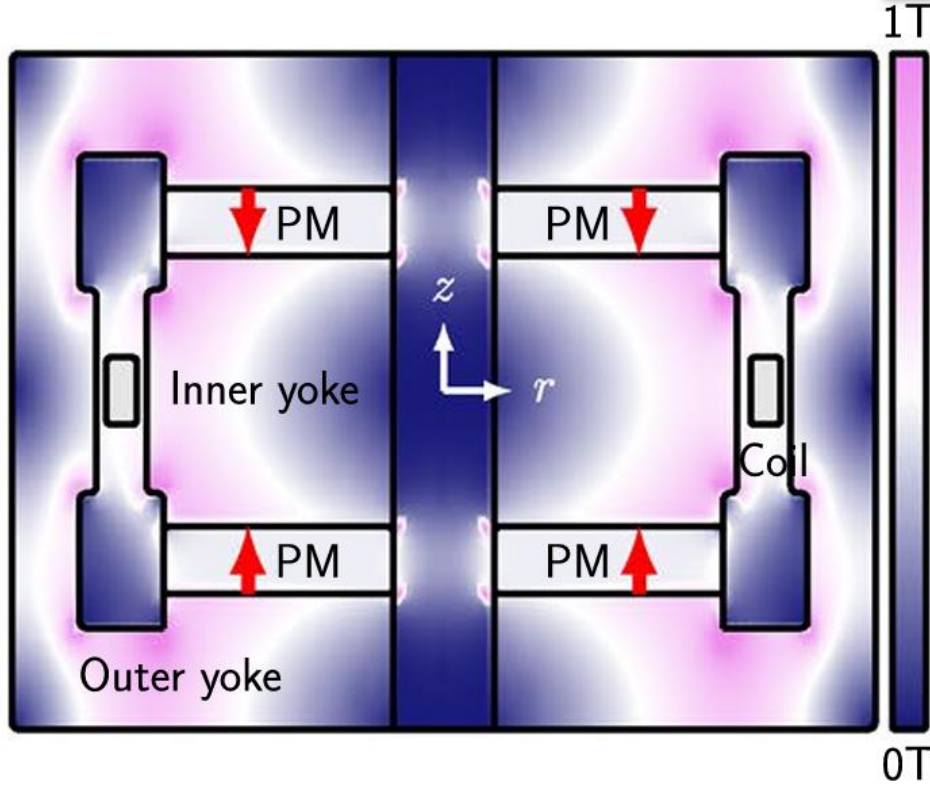


Pure iron seems to be a good choice.

KB magnet design:

Field strength at coil pos

↑↓ BIPM



$$R_m = \frac{\delta_m}{\mu_m \mu_0 S_m}, \quad R_y = \frac{\delta_y}{\mu_y \mu_0 S_y}, \quad R_a = \frac{\delta_a}{\mu_0 S_a},$$

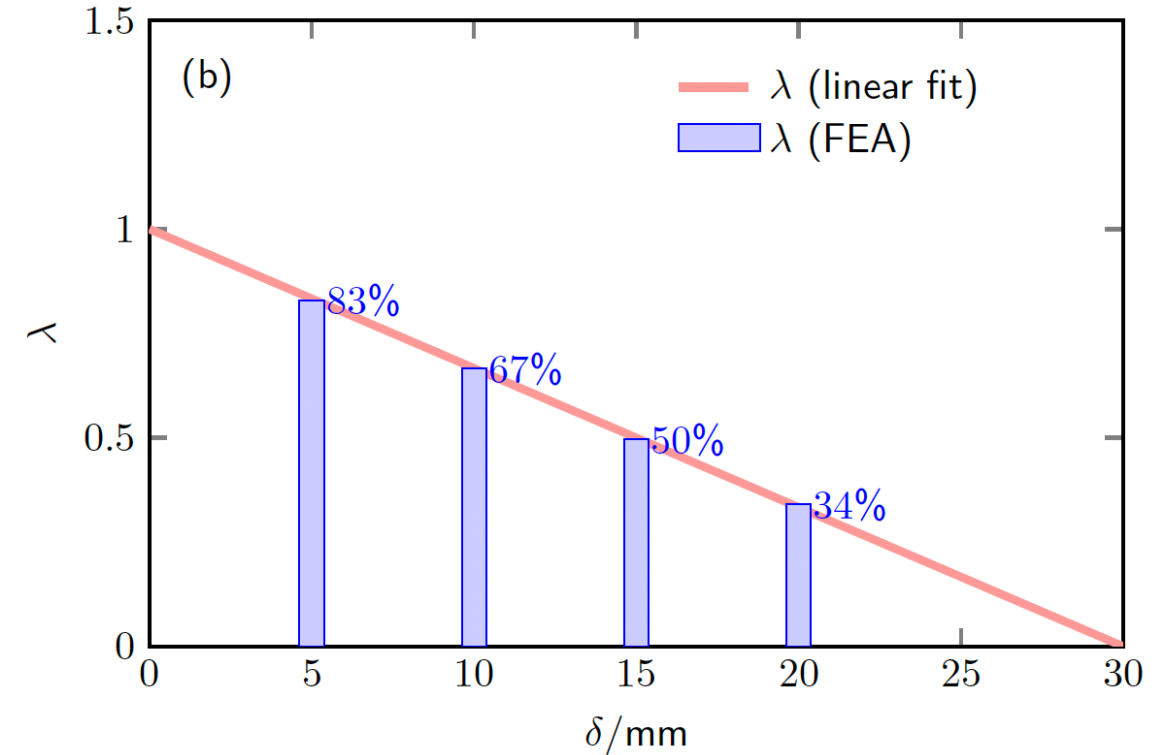
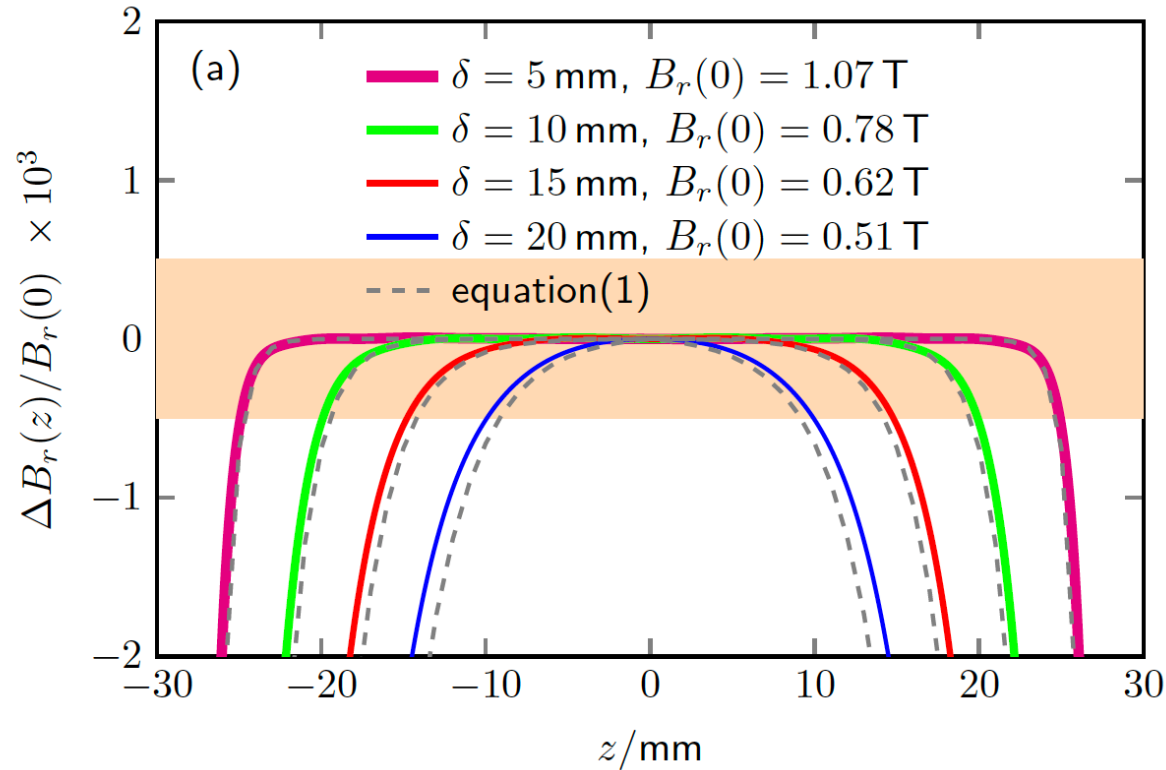
$$(R_m + R_y + R_a)\phi = -H_C \delta_m.$$

$$\phi = S_m B_m = B_a S_a$$

$$B_a = \frac{-H_C}{\frac{S_a}{S_m} \frac{1}{\mu_m \mu_0} + \frac{\delta_a}{\delta_m} \frac{1}{\mu_0}} \approx \frac{-\mu_0 H_C}{\frac{S_a}{S_m} + \frac{\delta_a}{\delta_m}} \approx \frac{1 \text{ T}}{\frac{S_a}{S_m} + \frac{\delta_a}{\delta_m}}.$$

KB magnet design:

Improve field uniformity



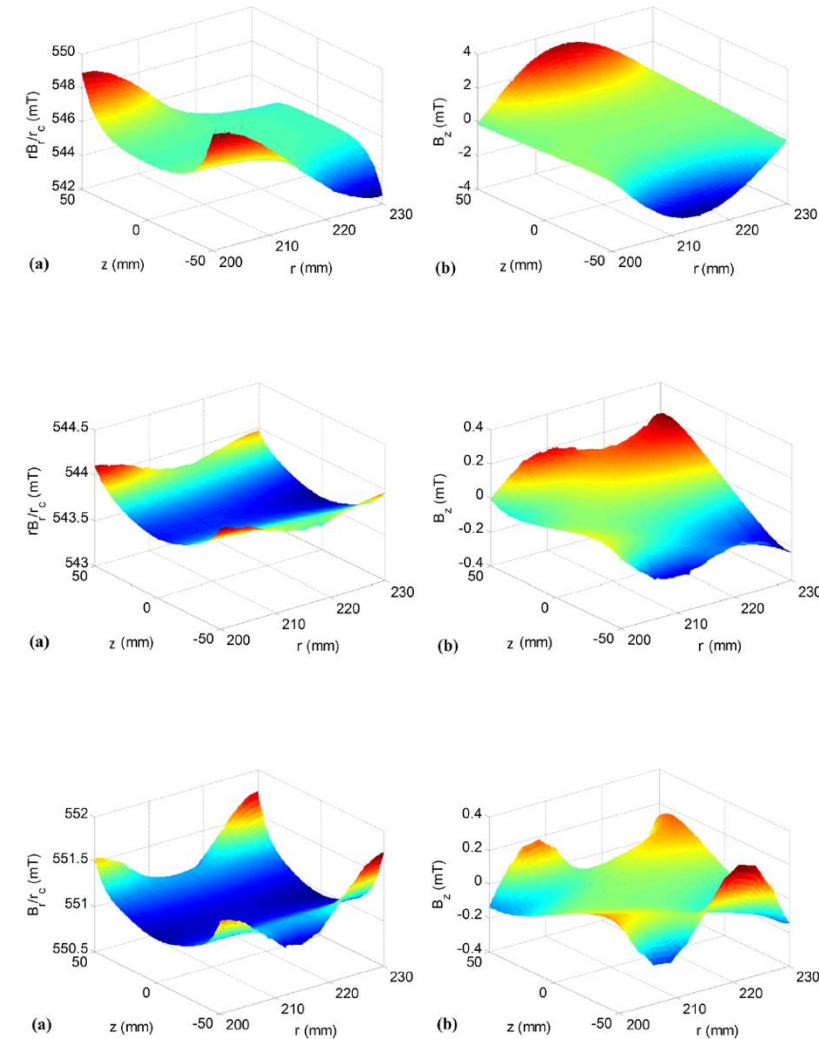
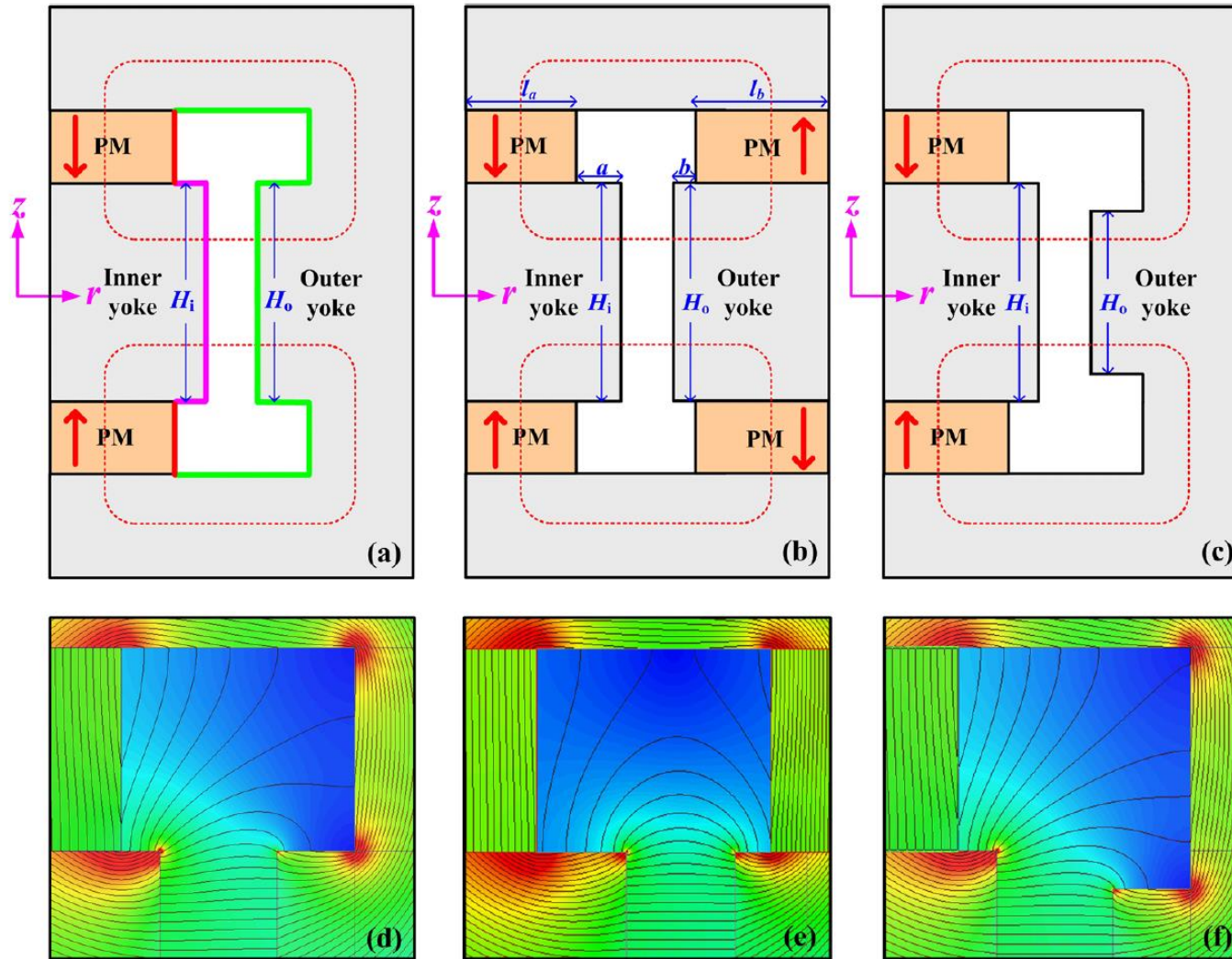
$$\frac{B_r(z)}{B_r(0)} - 1 = -\exp\left[-\left(1 + \frac{\pi(h_a - 2|z|)}{\delta_a}\right)\right] + \exp\left[-\left(1 + \frac{\pi h_a}{\delta_a}\right)\right],$$

KB magnet design:

Improve field uniformity

↑↓ BIPM

A

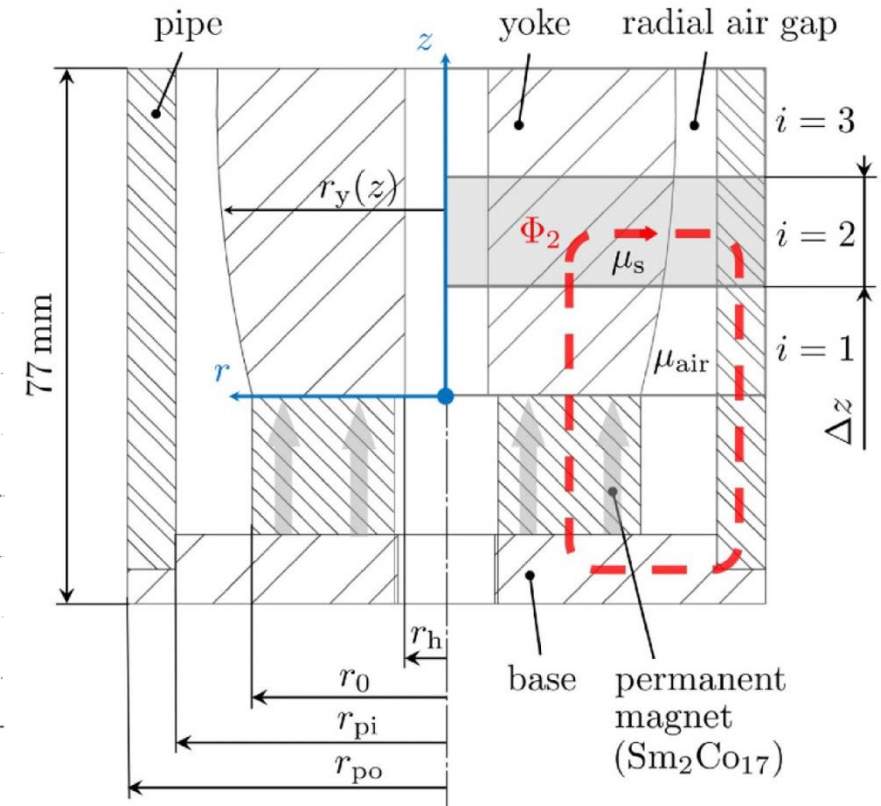
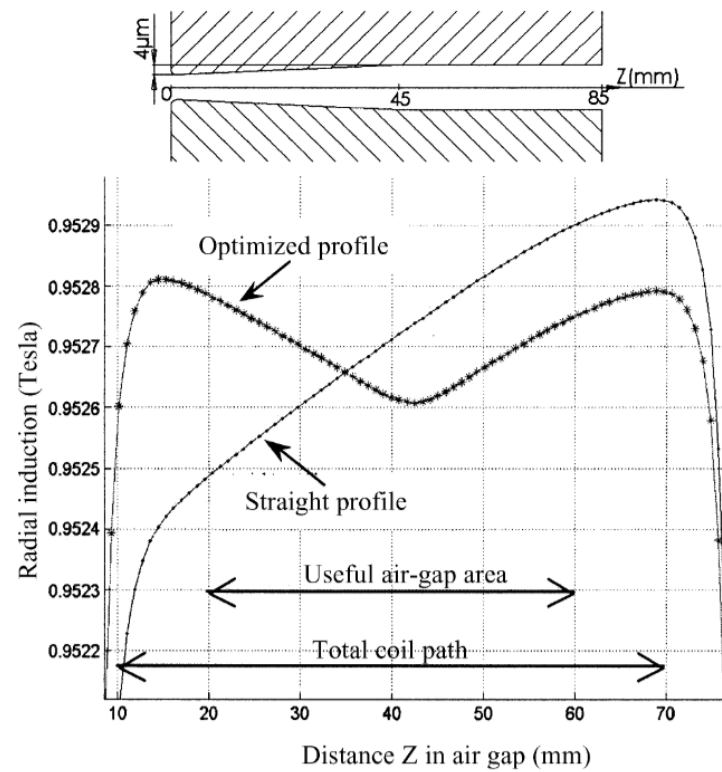
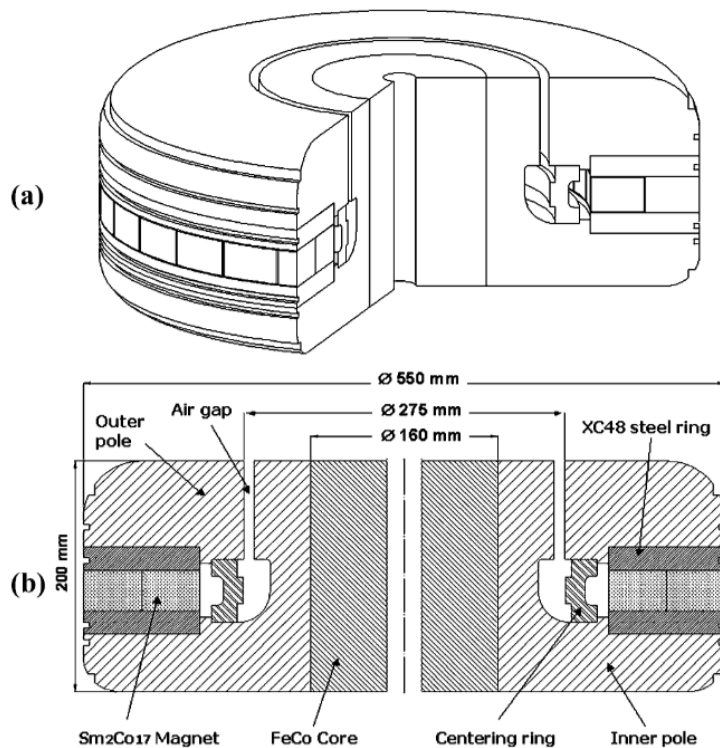


KB magnet design:

Improve field uniformity

↑↓ BIPM

B

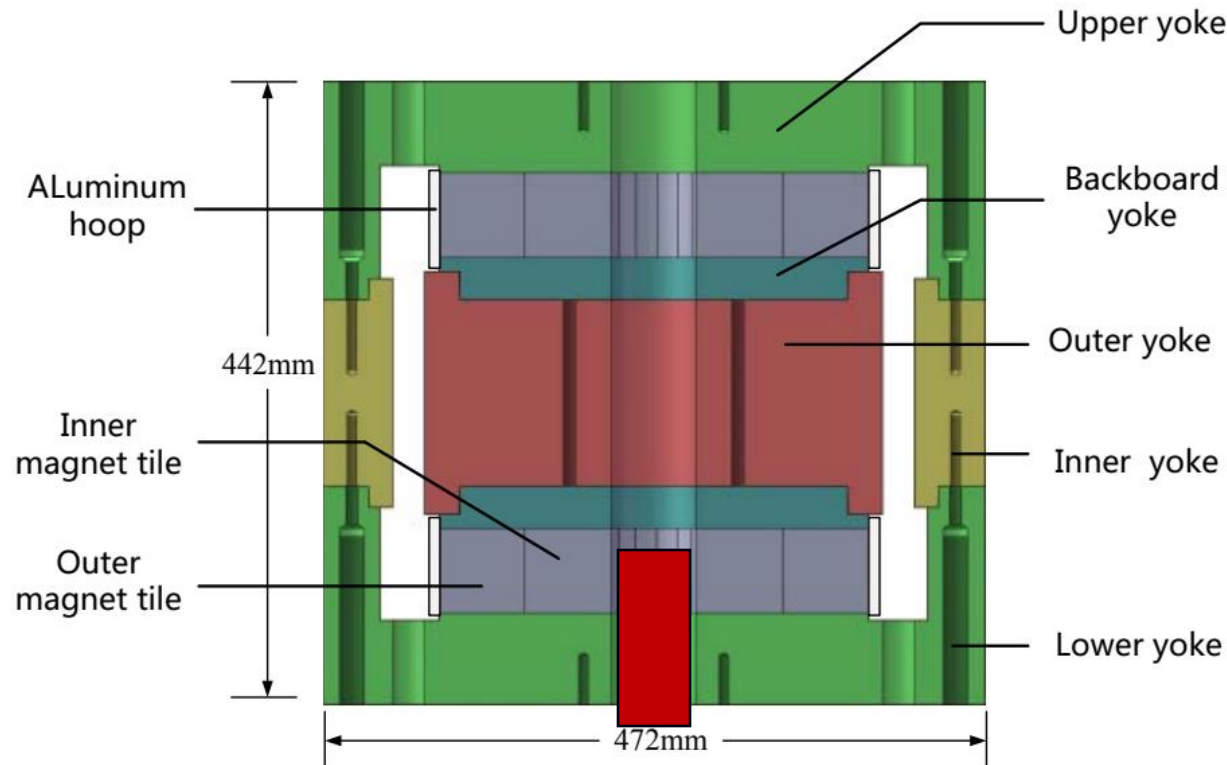


KB magnet design:

Improve field uniformity

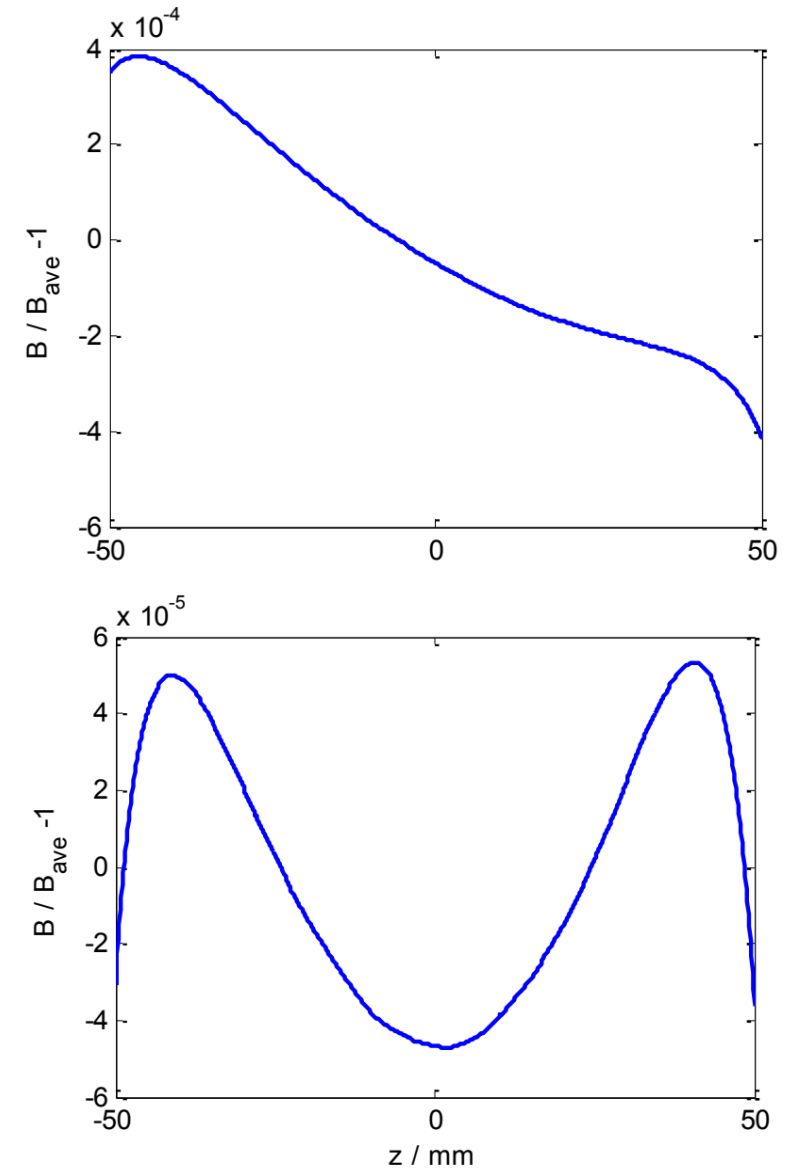
↑↓ BIPM

C



HiperCo50 (saturation magnetic flux density is about 2.4T)

You Q et al CPEM2018

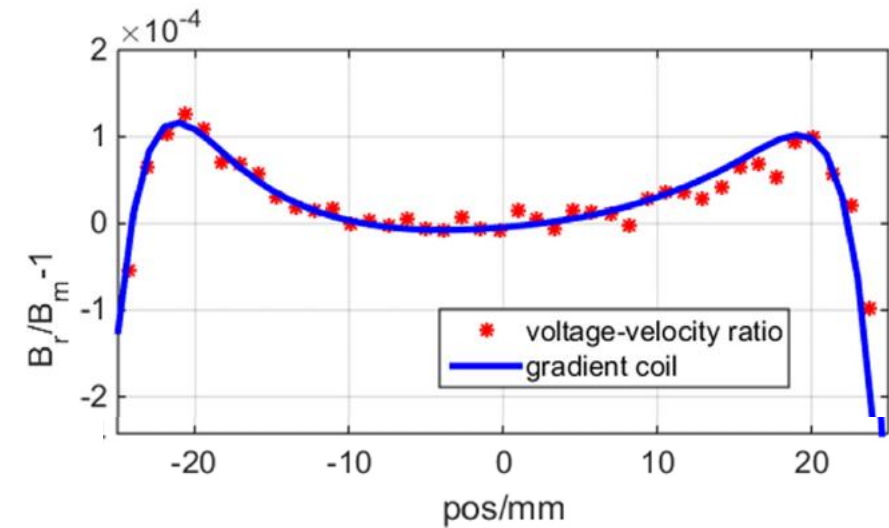
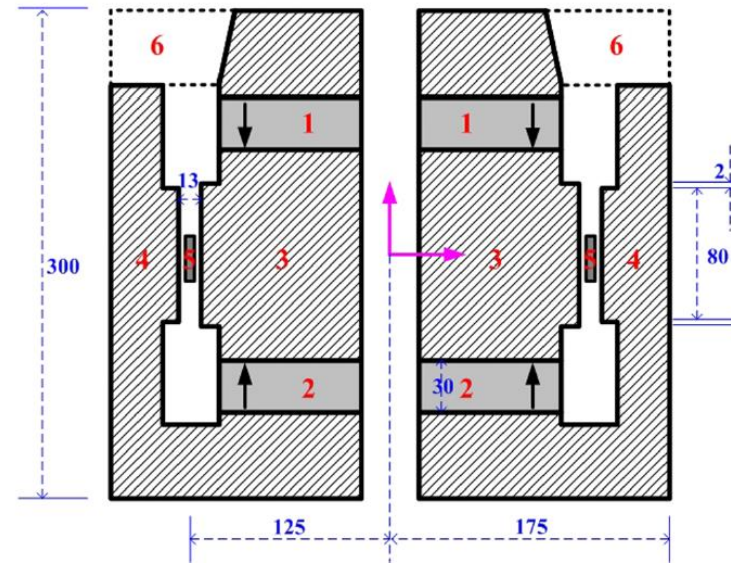
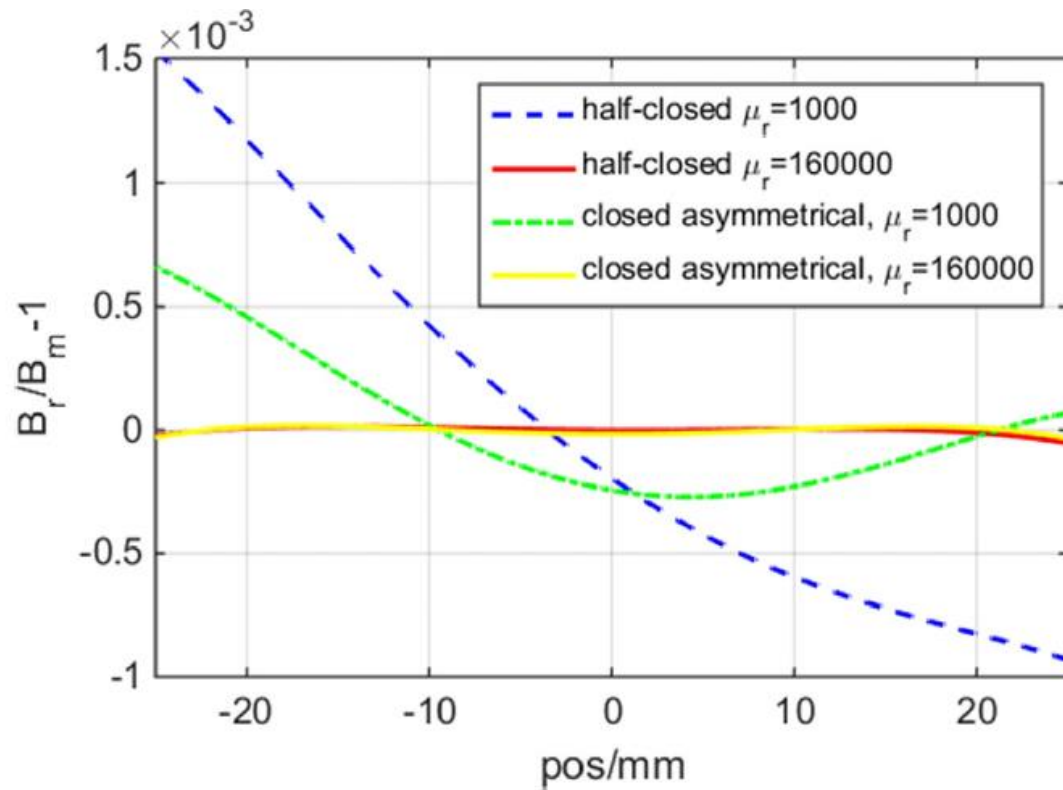


KB magnet design:

Improve field uniformity



D

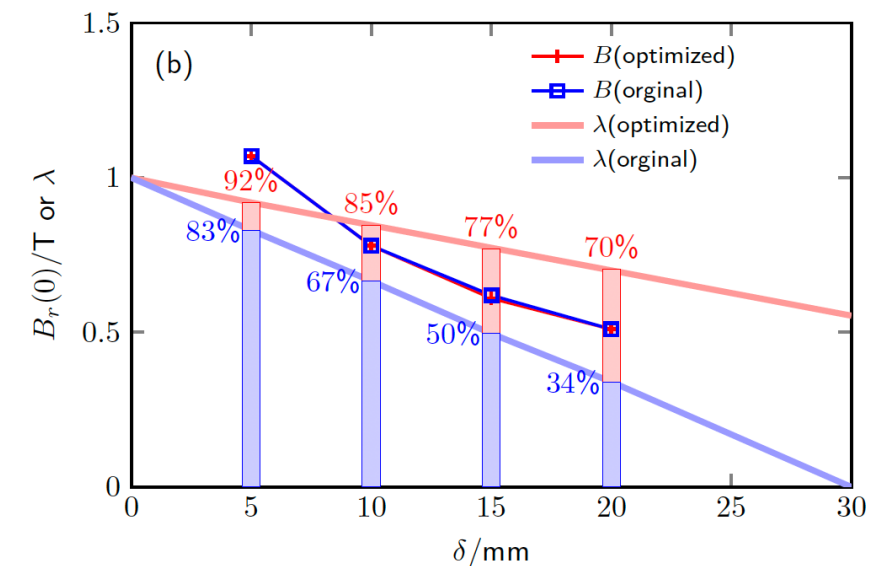
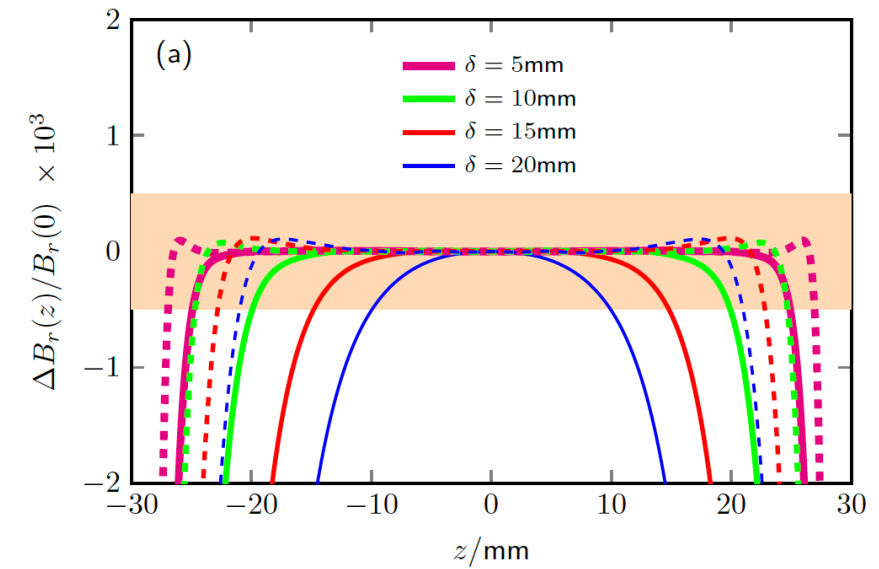
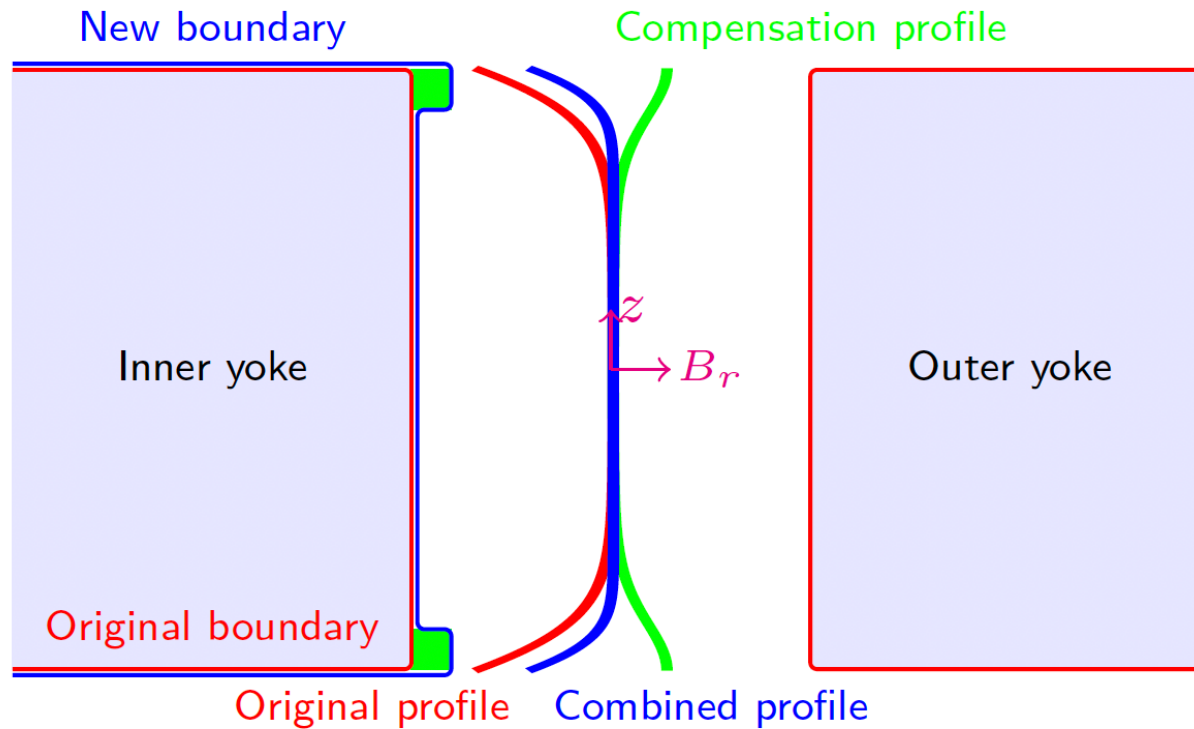


KB magnet design:

Improve field uniformity



E

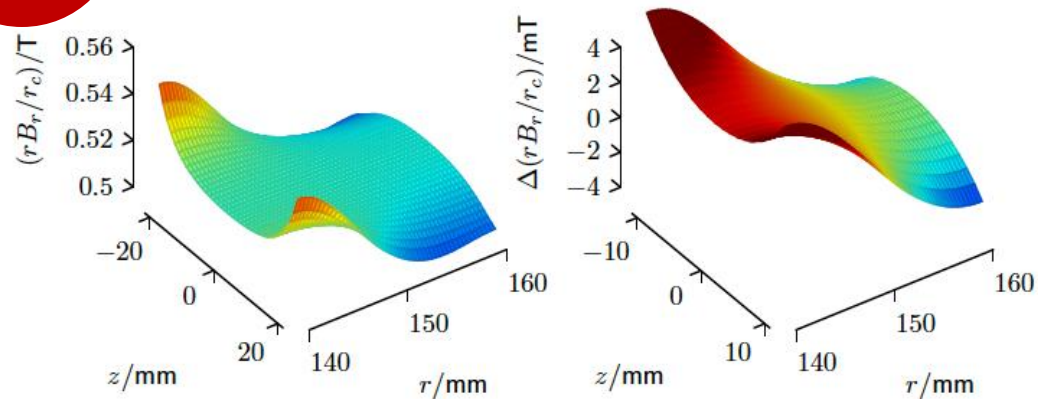


KB magnet design:

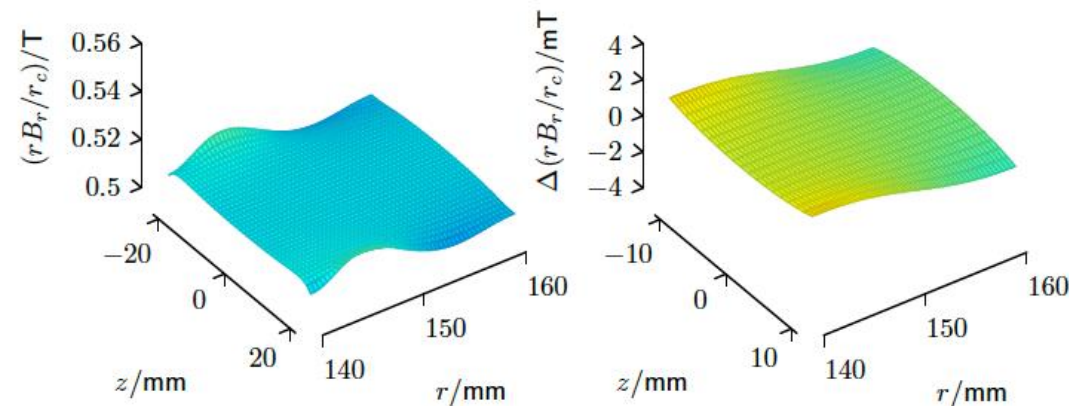
Improve field uniformity



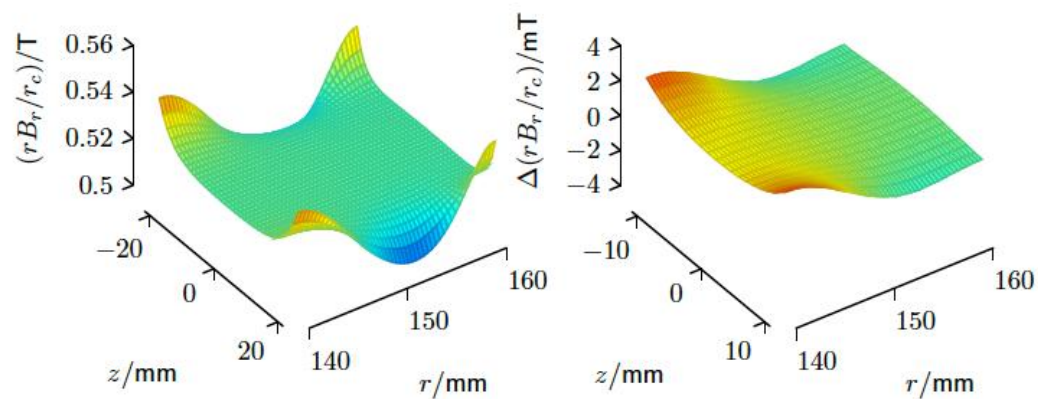
Inner yoke compensation



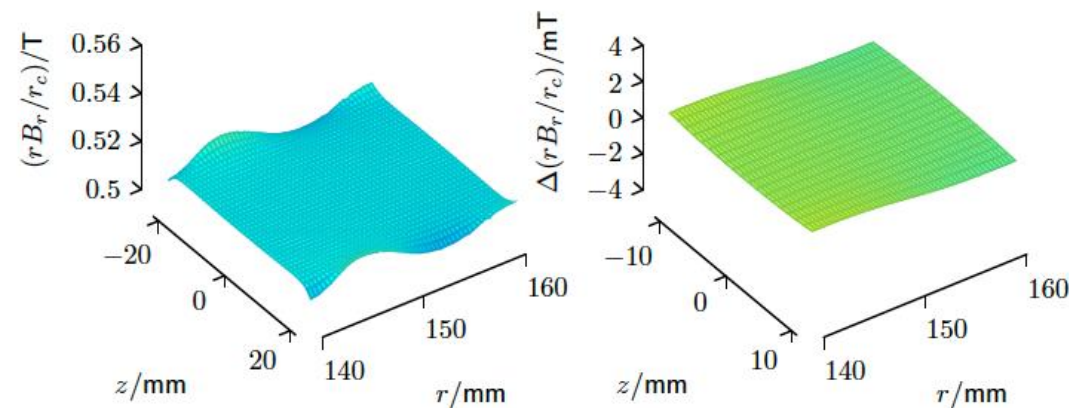
(a)



(c)



(b)



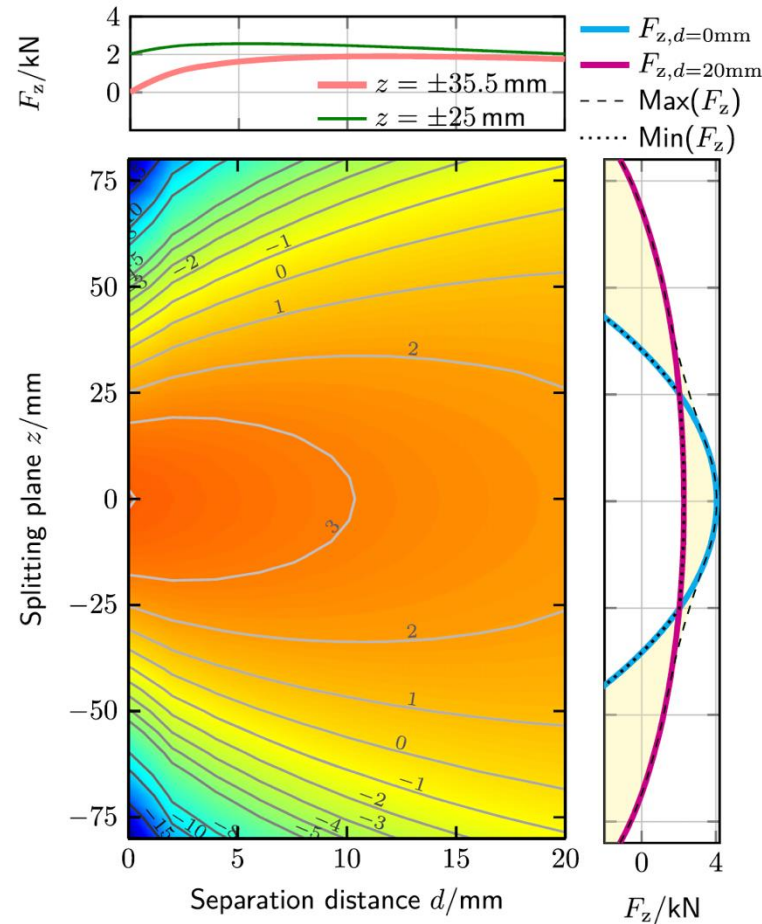
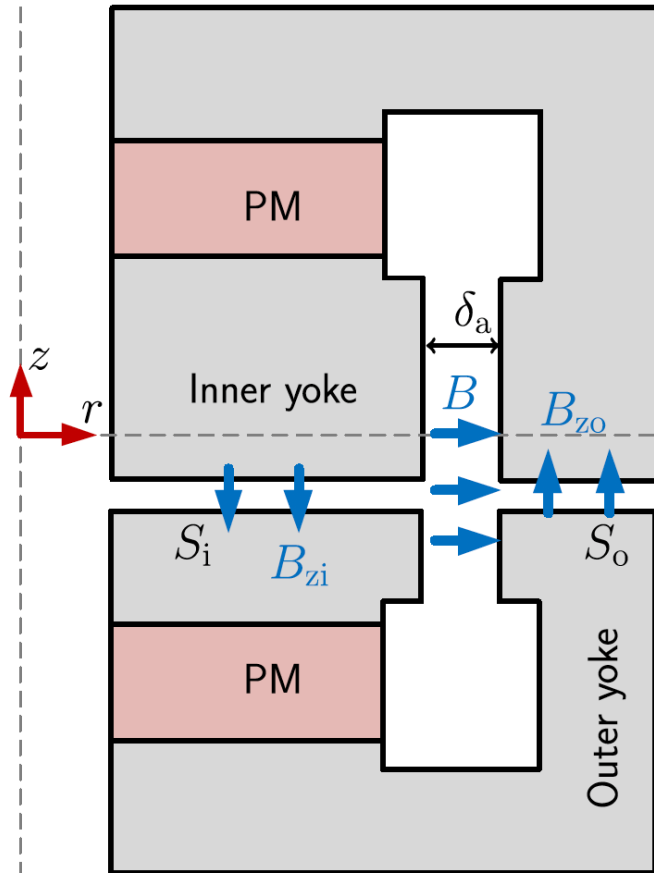
(d)

Lowering outer yoke height

Both methods applied

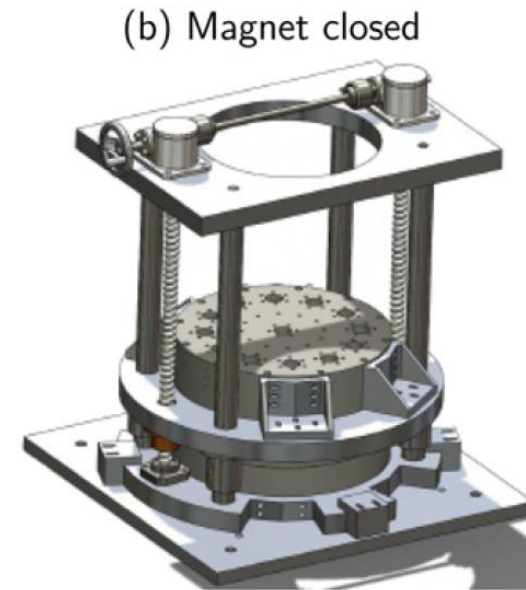
KB magnet design:

Splitting force



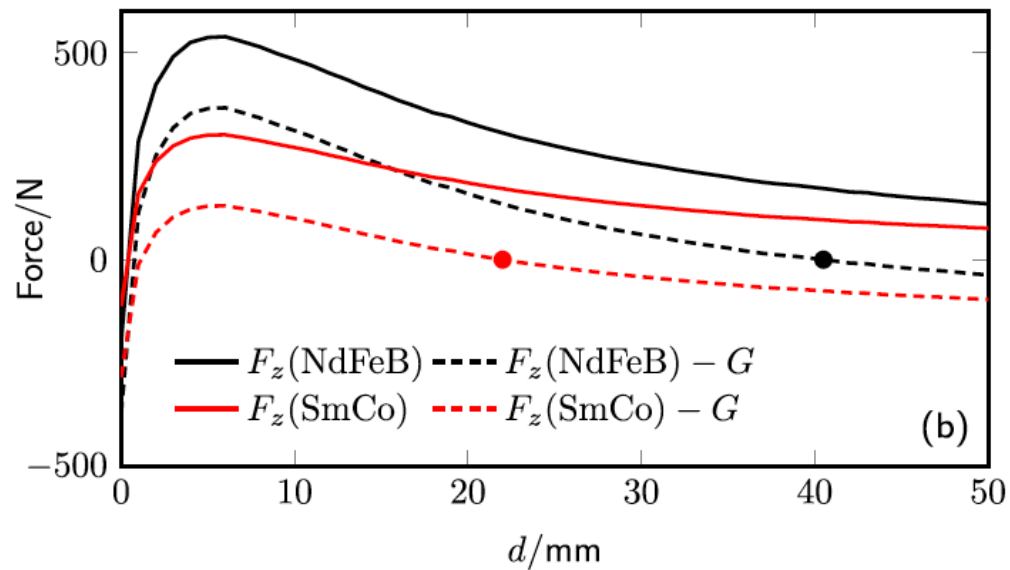
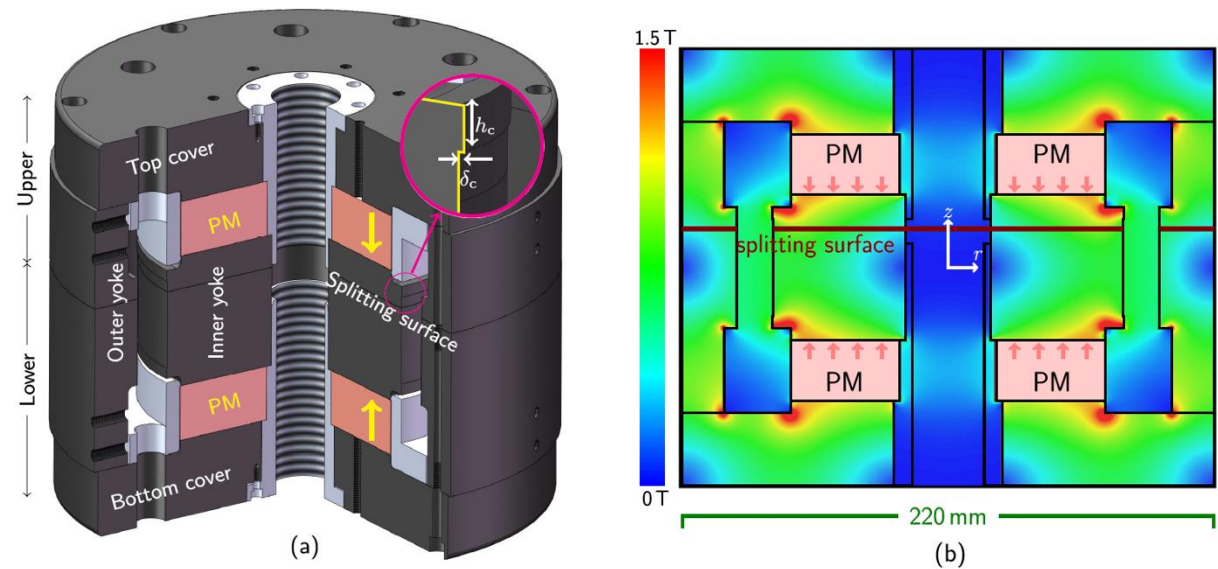
$$F_z(z) = \frac{\pi r_a B^2}{\mu_0} \left(\delta_a - \frac{2\pi r_a (S_i + S_o)}{S_i S_o} z^2 \right)$$

Attractive OR
Repulsive?



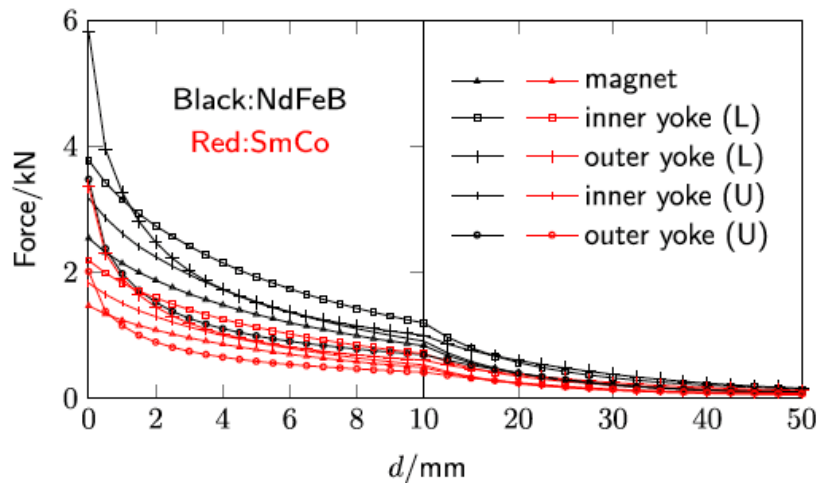
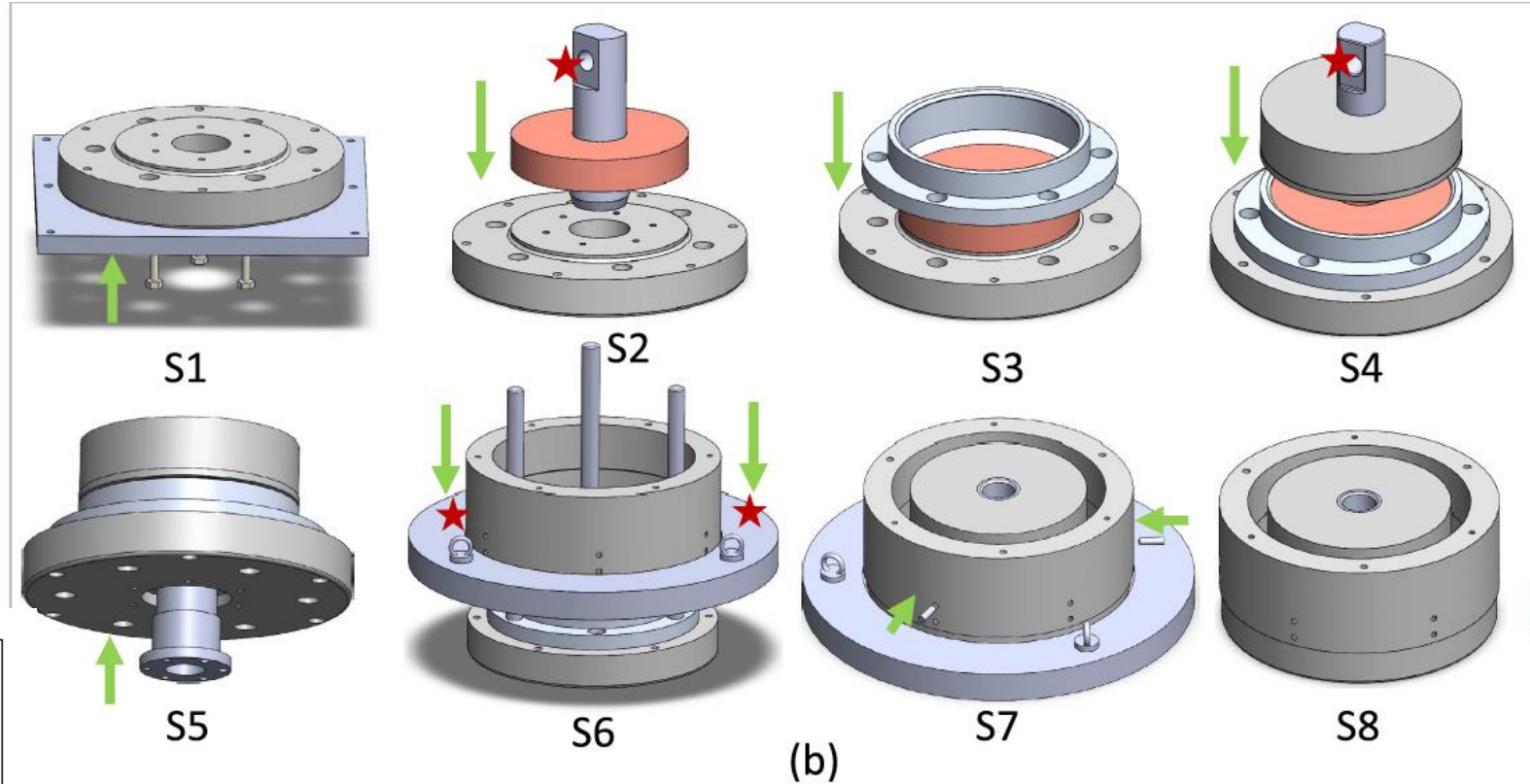
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Splitting force



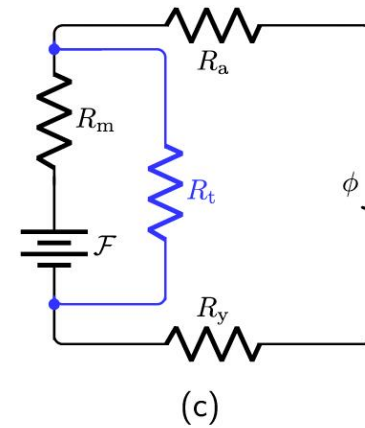
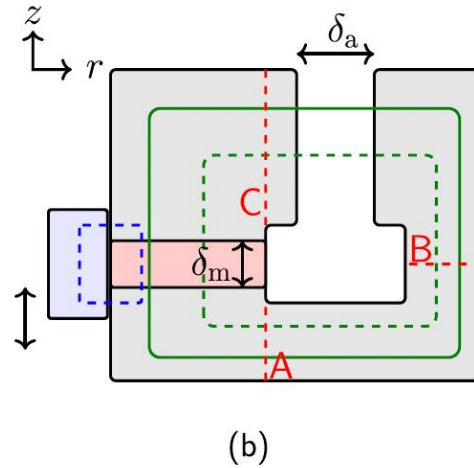
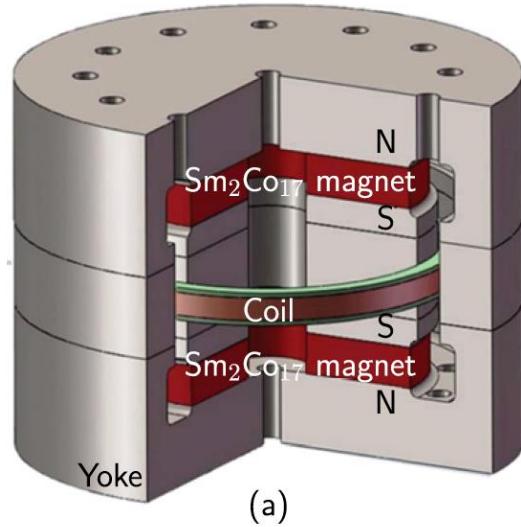
KB magnet design:

Installation force



KB magnet design:

Thermal considerations

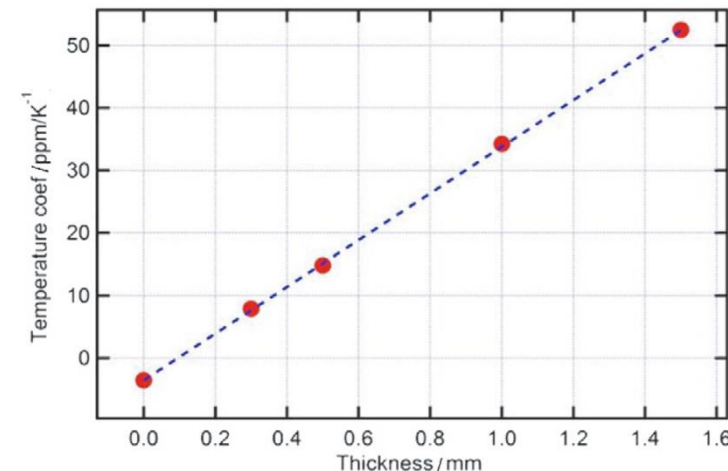
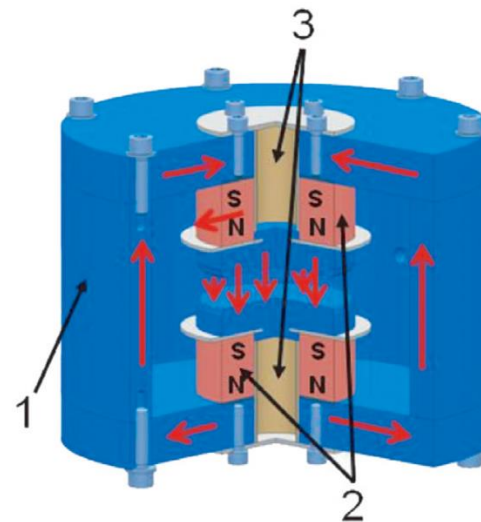
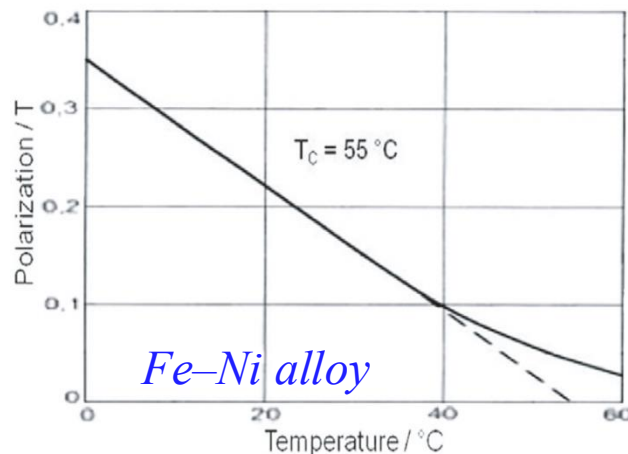


$$(\phi + \phi')R_m + \phi(R_a + R_y) = \mathcal{F}$$

$$(\phi + \phi')R_m + \phi'R_t = \mathcal{F}.$$

$$\frac{1}{\mathcal{F}} \frac{\partial \mathcal{F}}{\partial T} = -\frac{1}{R_m + R_a} \frac{R_m R_a}{R_t^2} \frac{\partial R_t}{\partial T},$$

$$\left[\frac{1}{\mathcal{F}} \frac{\partial \mathcal{F}}{\partial T} \right] = -\frac{R_m R_a}{(R_m + R_a) R_t} \left[\frac{1}{R_t} \frac{\partial R_t}{\partial T} \right]$$



Outlines



1. Magnetic basics in KB experiments
2. A Brief history KB magnet-coil designs
3. How to design a KB magnet system
4. **Evaluation performance of a KB magnet**
5. Systematic effects and how to reduce
6. Summary & outlook

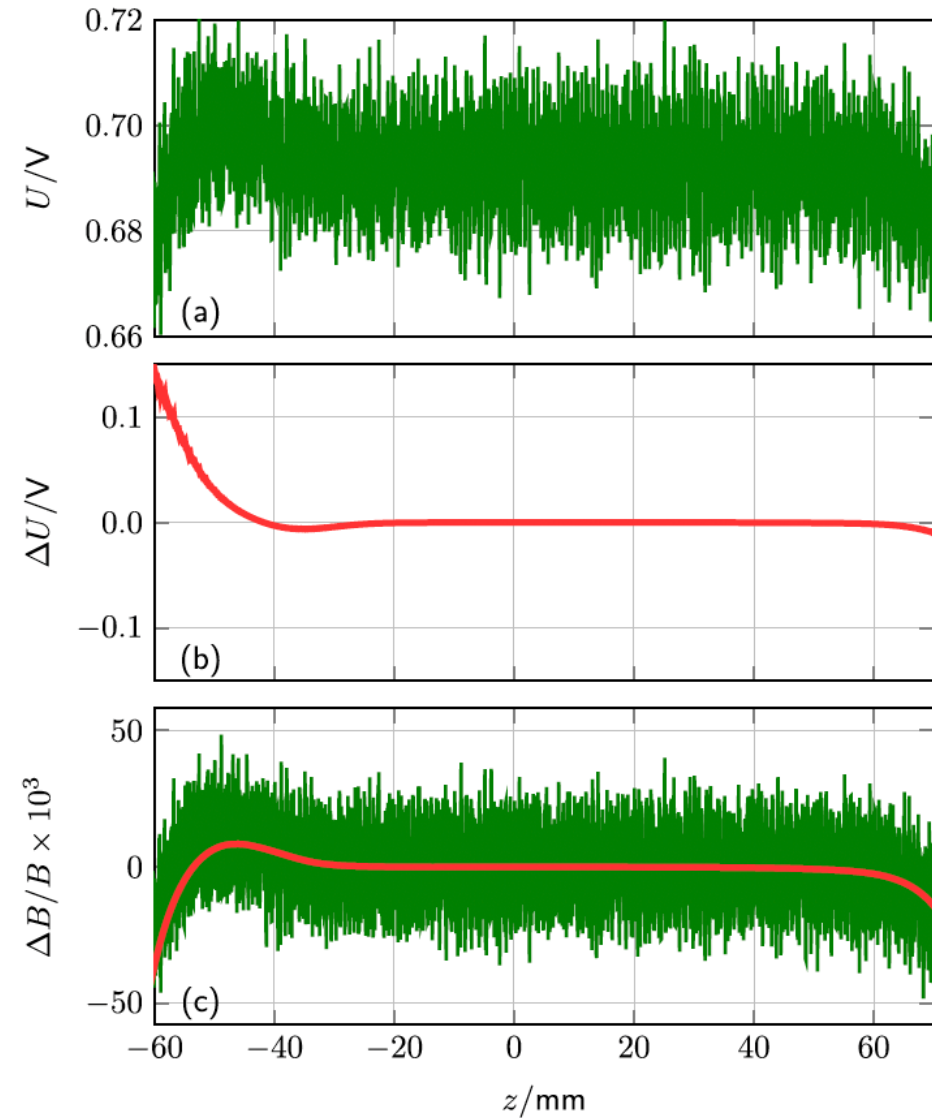
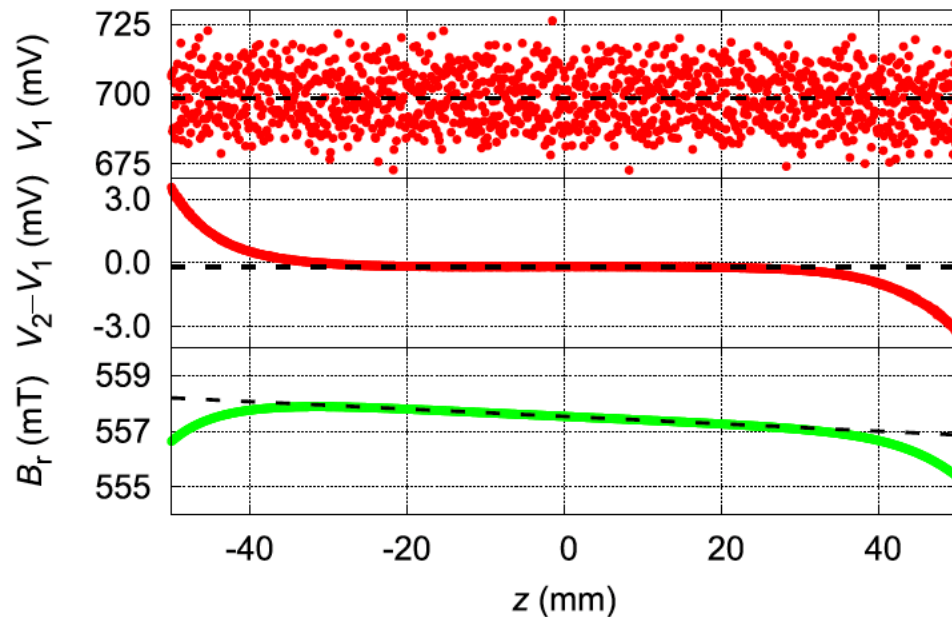
Performance evaluation:

Magnetic profile



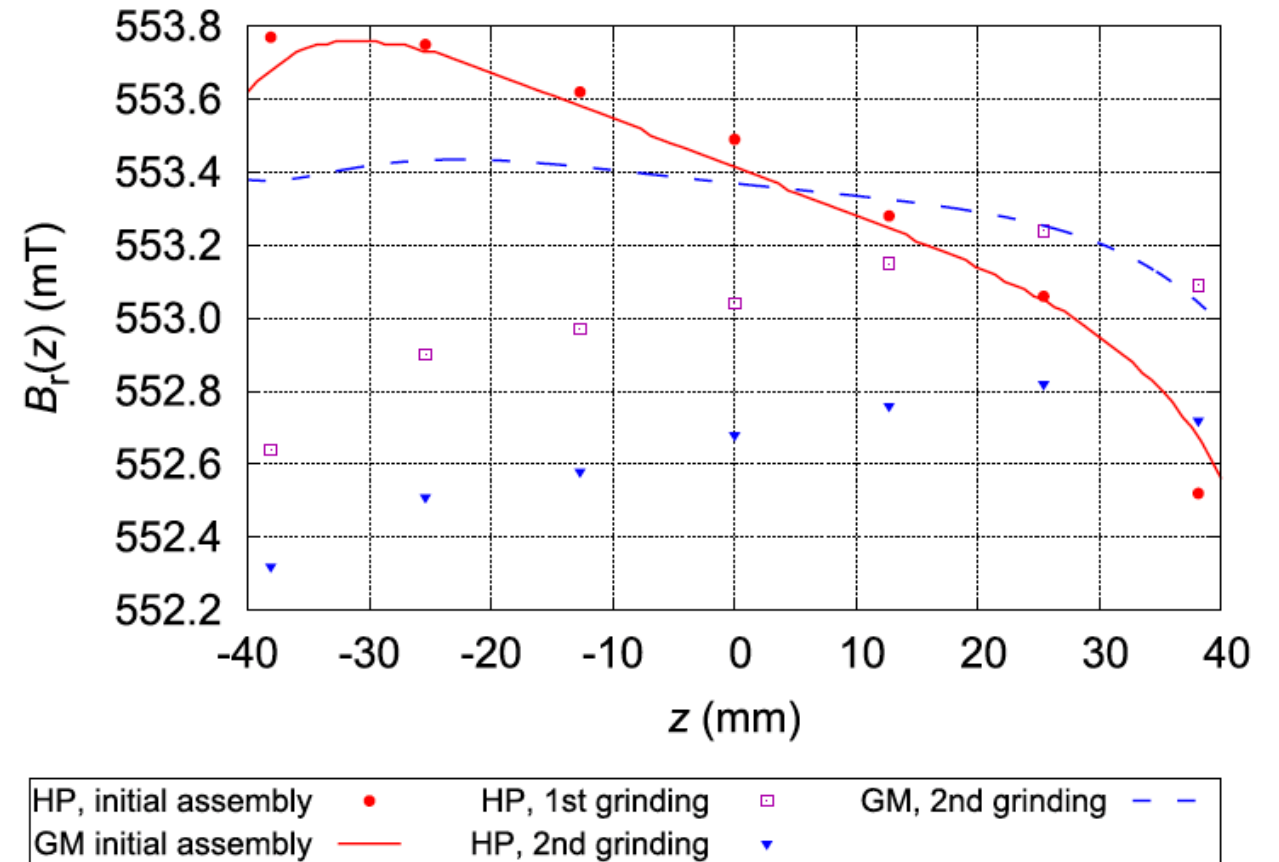
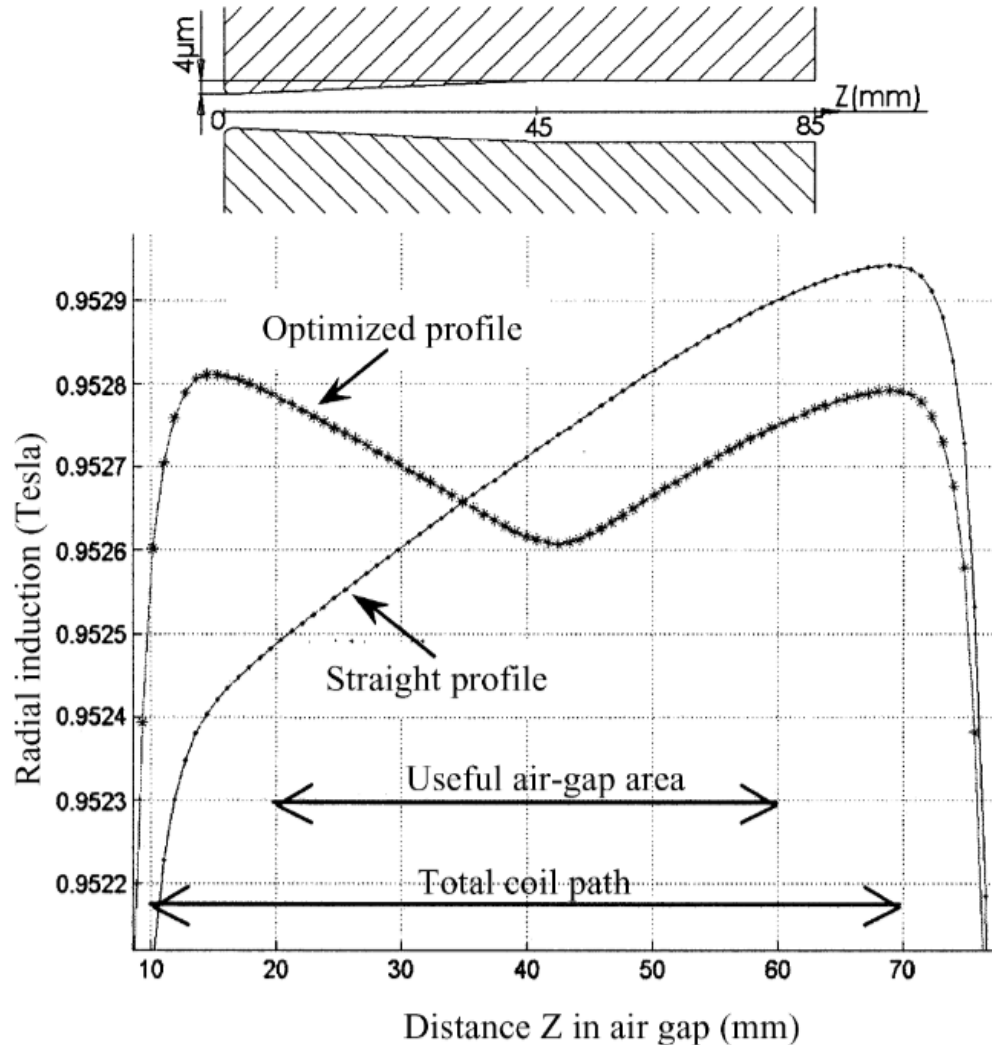
$$\frac{V_1(z) - V_2(z)}{V_1(z)} = \frac{B_r(z) - B_r(z - \Delta z)}{B_r(z)} \approx \frac{\Delta z \frac{dB_r(z)}{dz}}{B_r(z)}.$$

$$B_r(z) = \frac{\bar{B}_r}{\bar{V}_1 \Delta z} \int_b^z (V_1(z') - V_2(z')) dz' + O$$



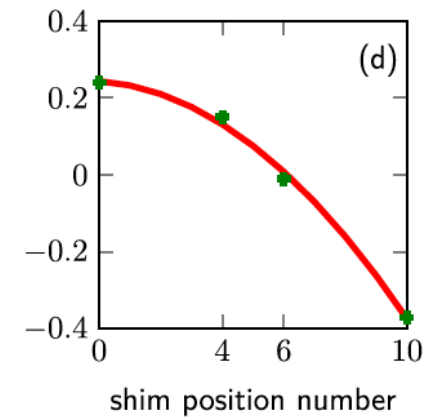
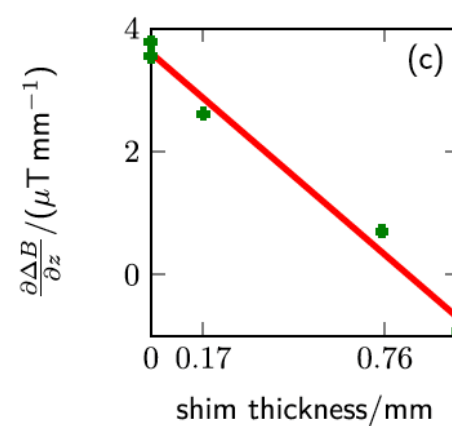
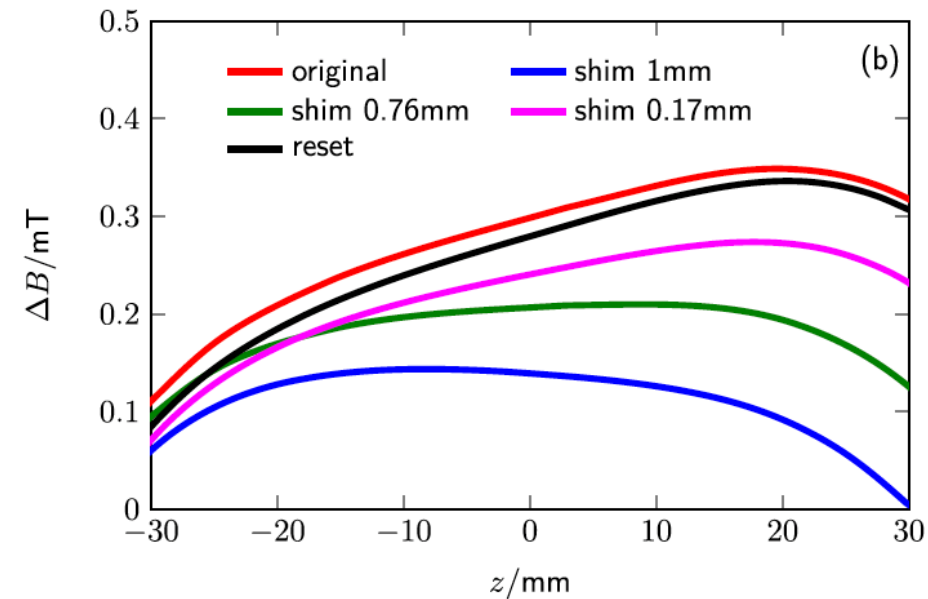
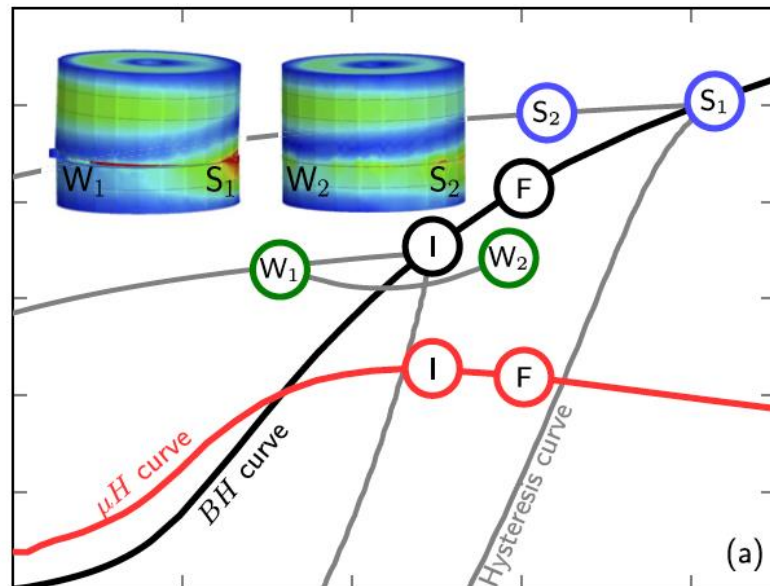
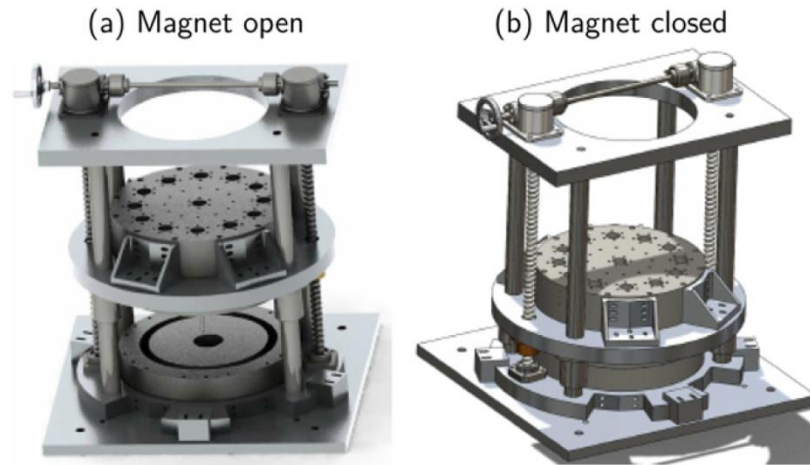
Performance evaluation:

Flattening profile



Performance evaluation:

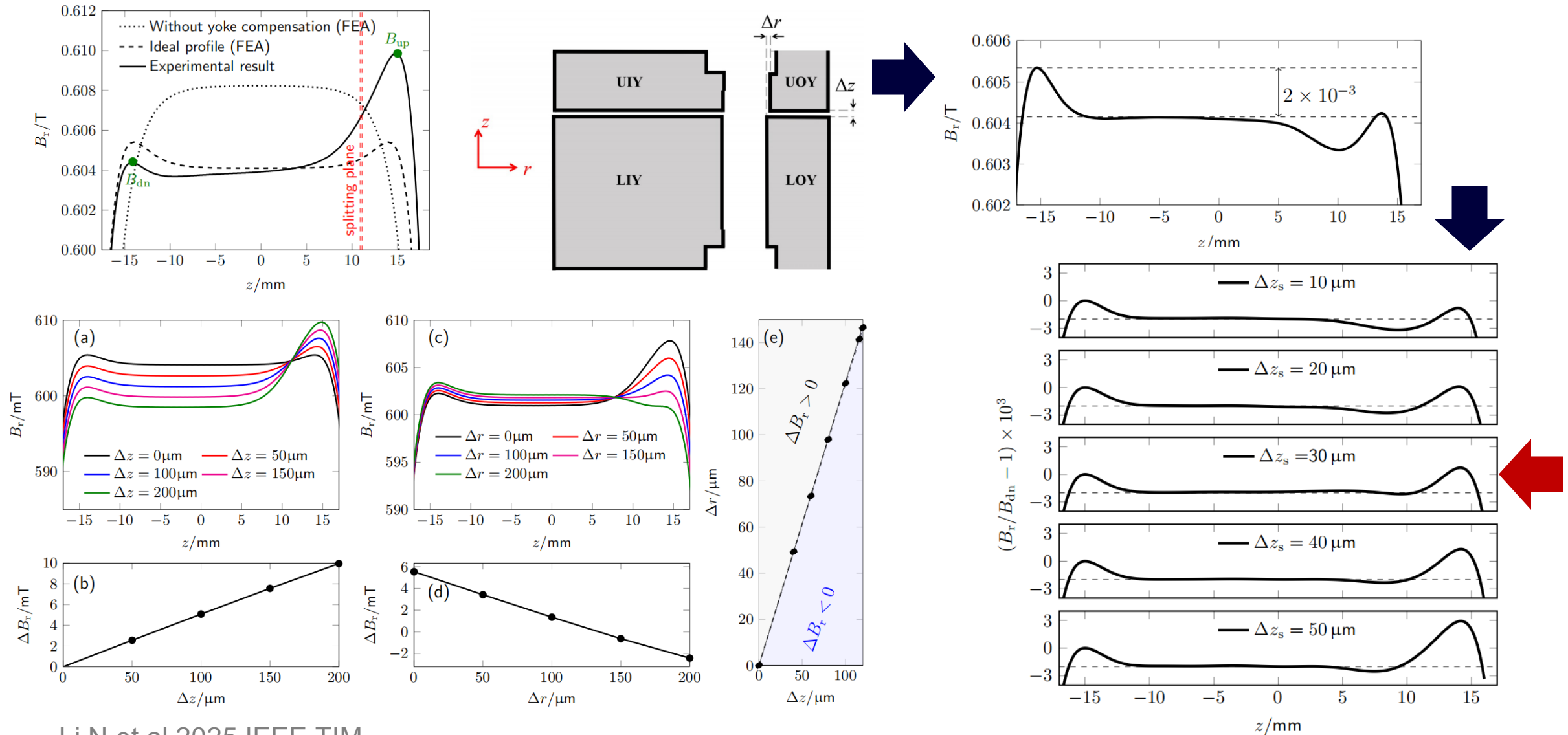
Flattening profile



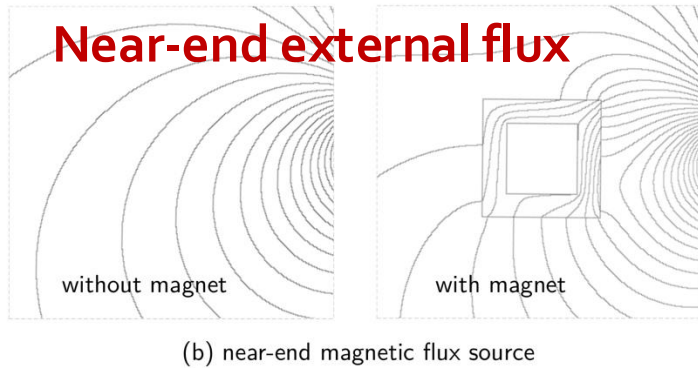
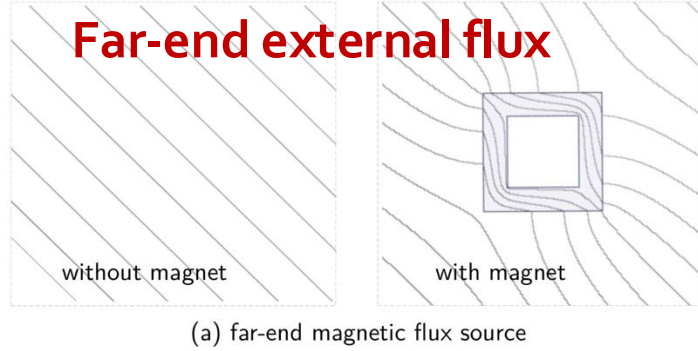
Performance evaluation:

Flattening profile

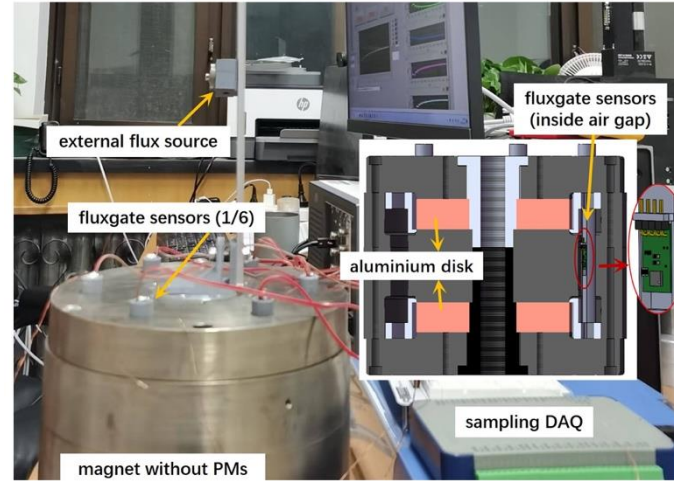
↑↓ BIPM



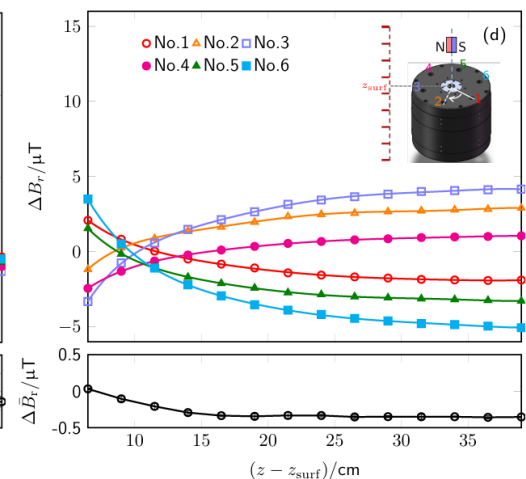
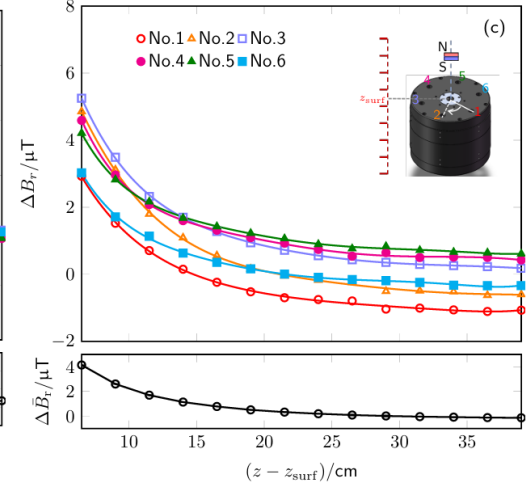
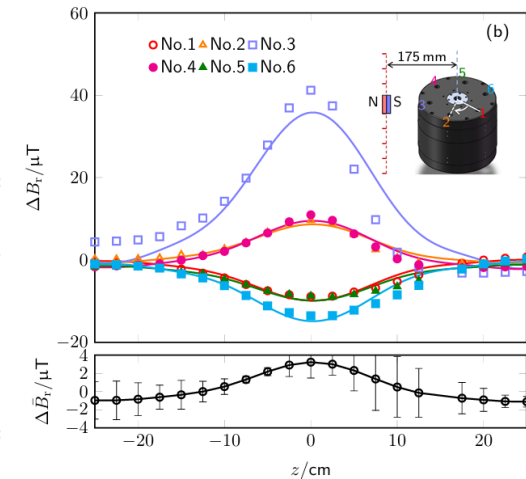
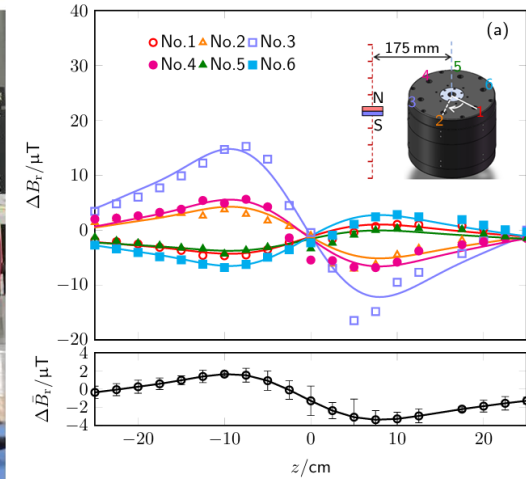
Performance evaluation: *Shielding performance*



$$\varepsilon = \frac{\Delta m}{m} = \frac{B(z_w) + \Delta \bar{B}_w(z_w)}{B(z_w) + \Delta \bar{B}_v(z_w)} \approx \frac{\Delta \bar{B}_w(z_w) - \Delta \bar{B}_v(z_w)}{B(z_w)}$$

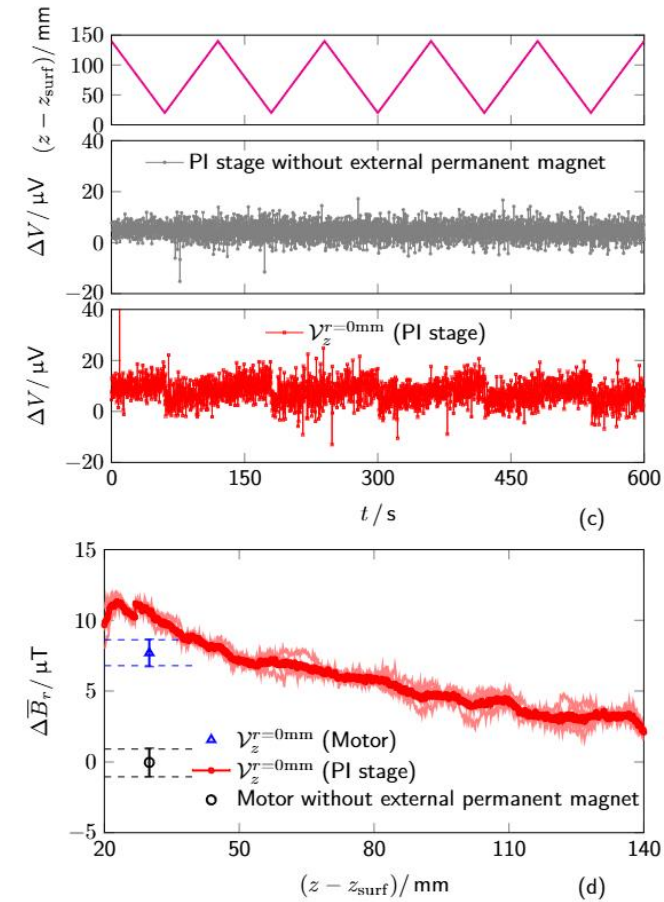
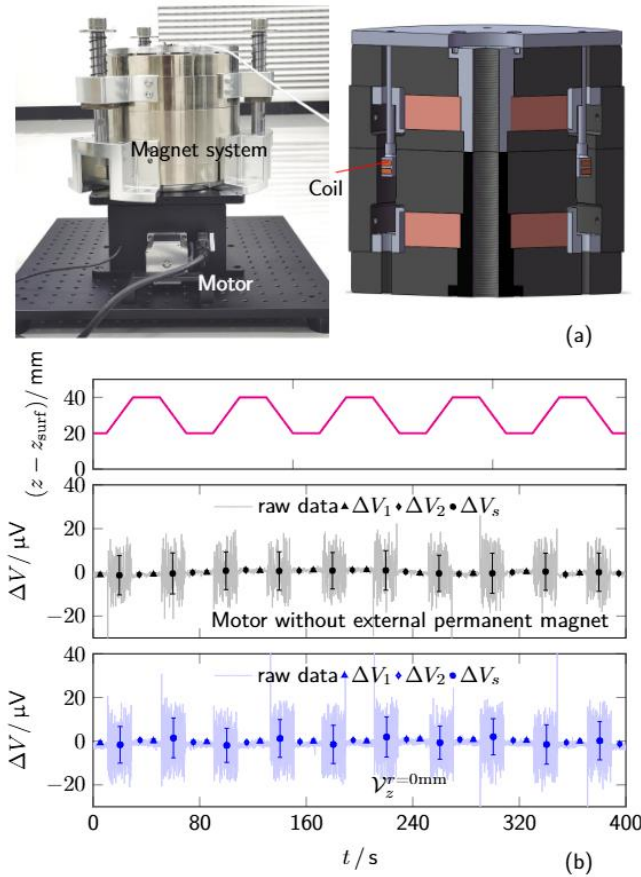
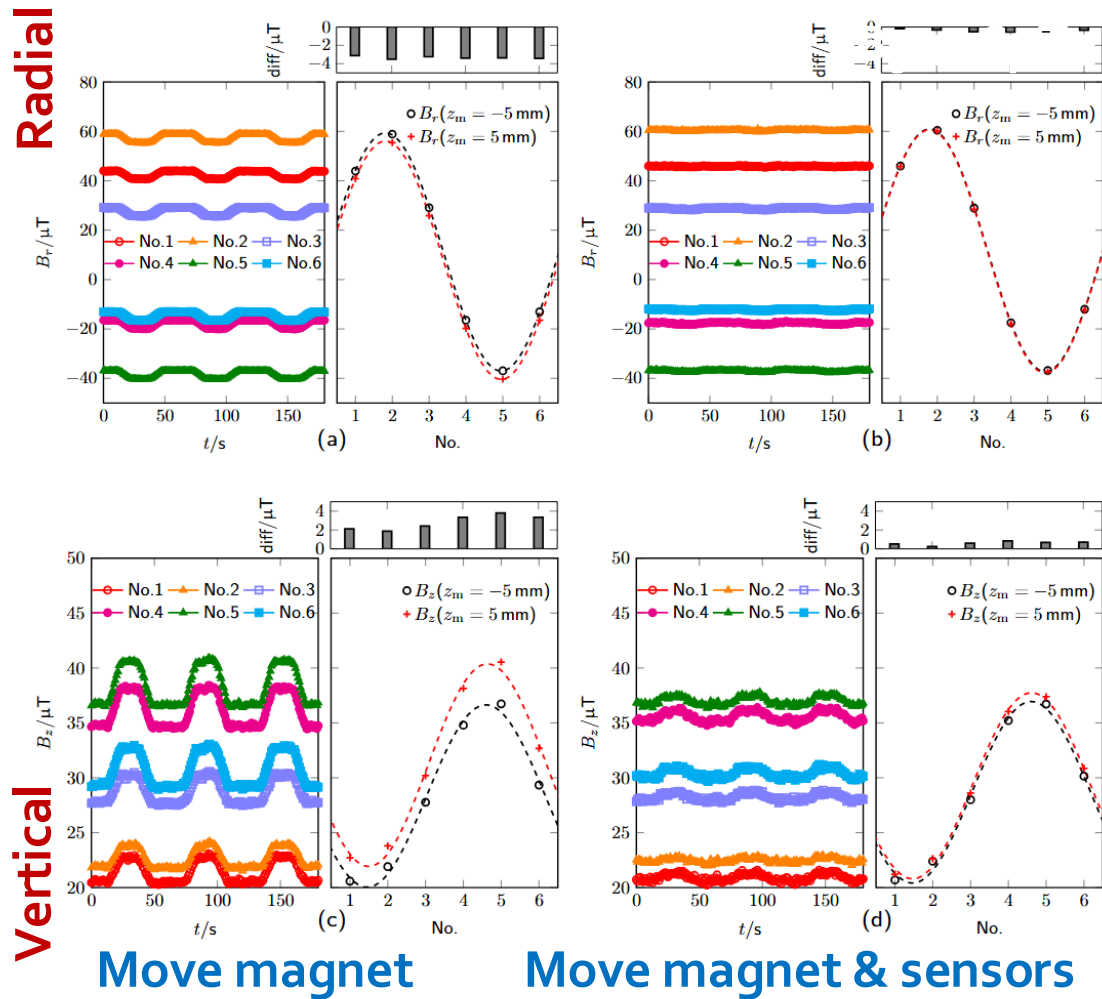


Type	$\gamma_{\max}$, FEA, $\mu_r = 1000$	$\gamma_{\max}$, Exp. without PMs	$\gamma_{\max}$, KB working status
$\mathcal{V}_{z=175\text{ mm}}^{r=175\text{ mm}}$	7.4×10^{-5}	5.9×10^{-5}	6.0×10^{-6}
$\mathcal{H}_{z=0\text{ mm}}^{r=175\text{ mm}}$	7.7×10^{-5}	1.4×10^{-4}	1.4×10^{-5}
$\mathcal{V}_{z=155\text{ mm}}^{r=0\text{ mm}}$	3.9×10^{-5}	2.0×10^{-5}	2.0×10^{-6}
$\mathcal{H}_{z=155\text{ mm}}^{r=0\text{ mm}}$	2.4×10^{-5}	1.3×10^{-5}	1.4×10^{-6}



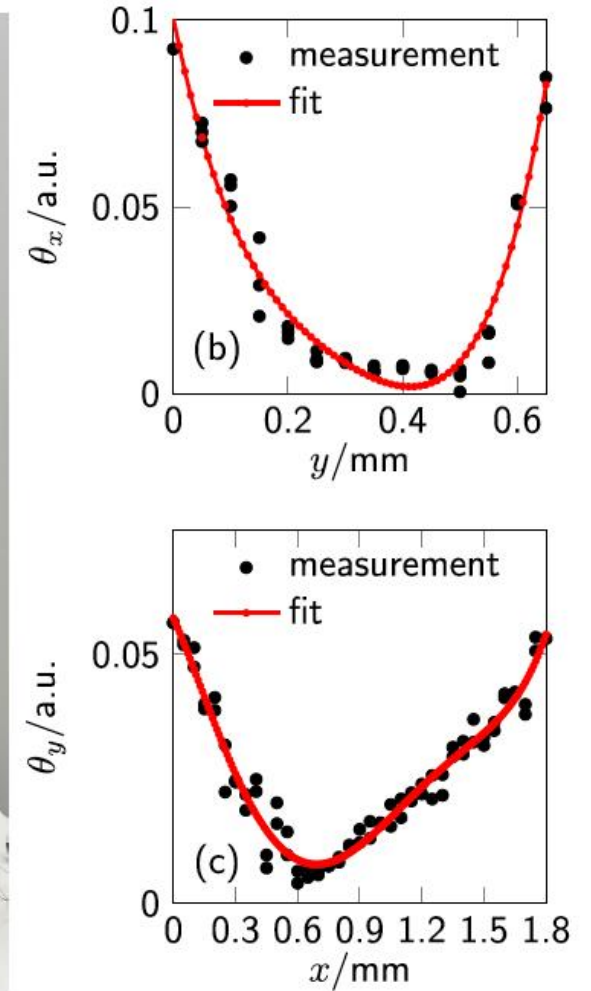
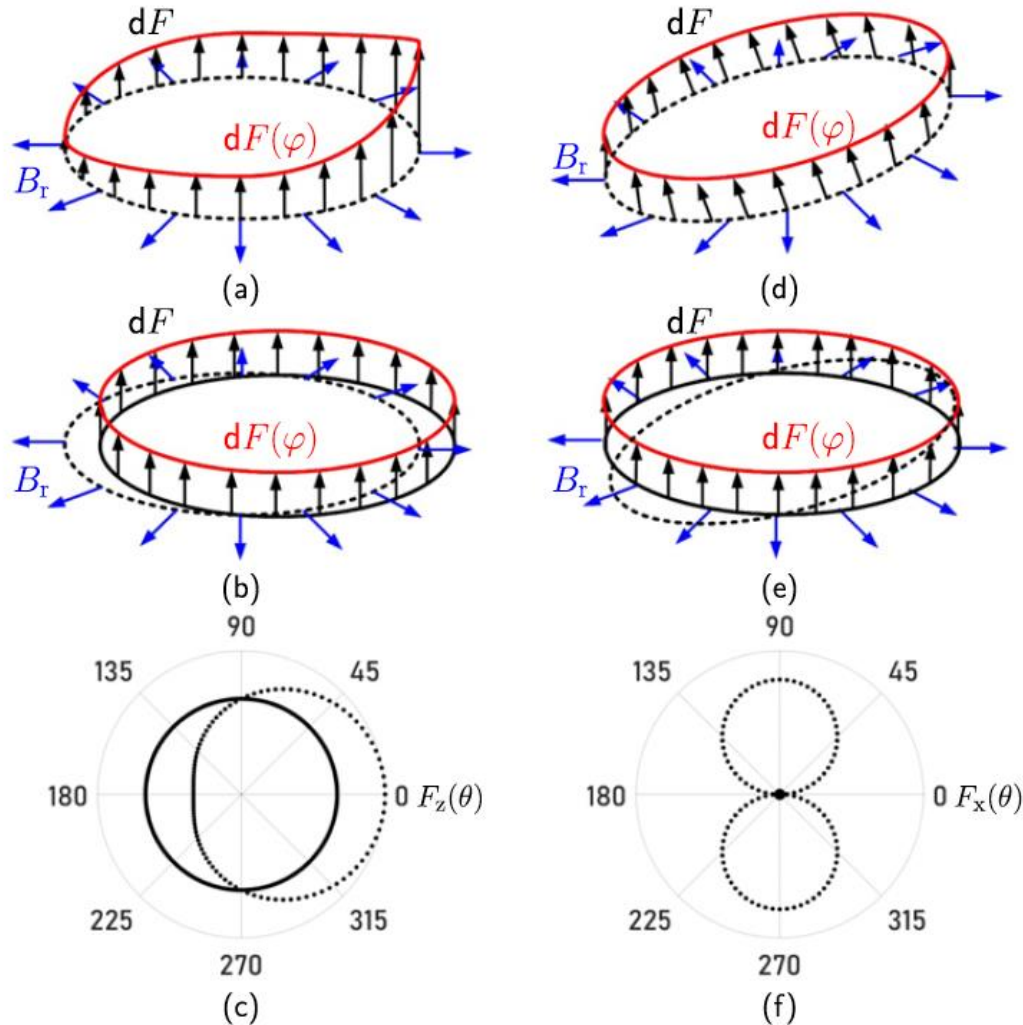
Performance evaluation:

Shielding performance



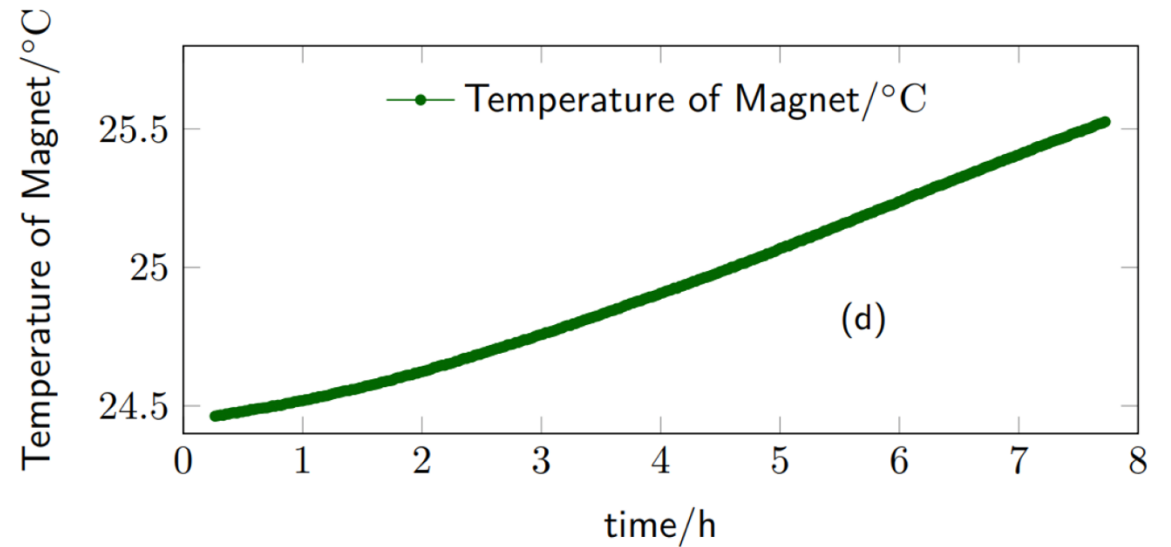
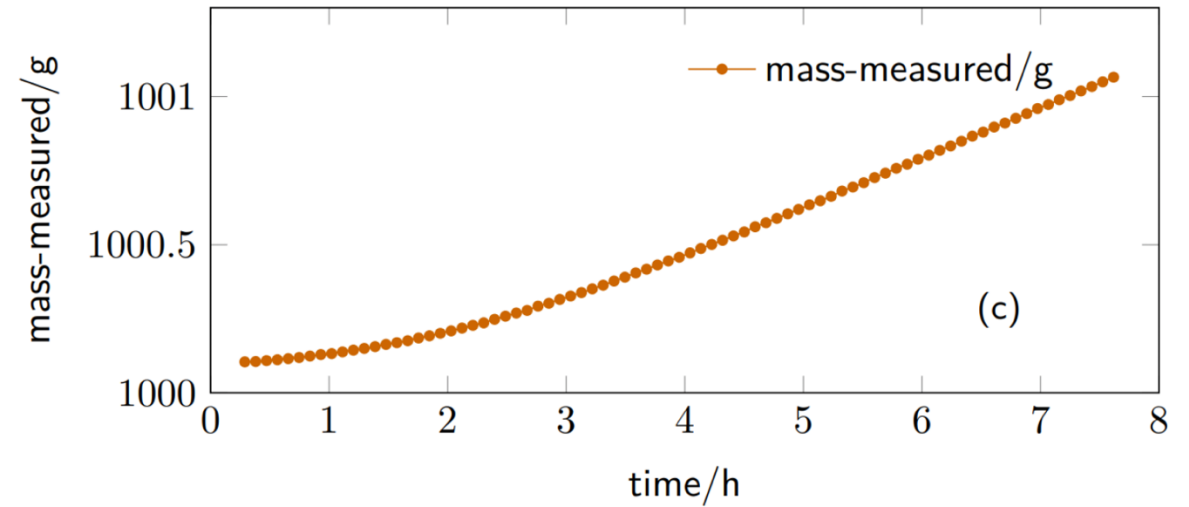
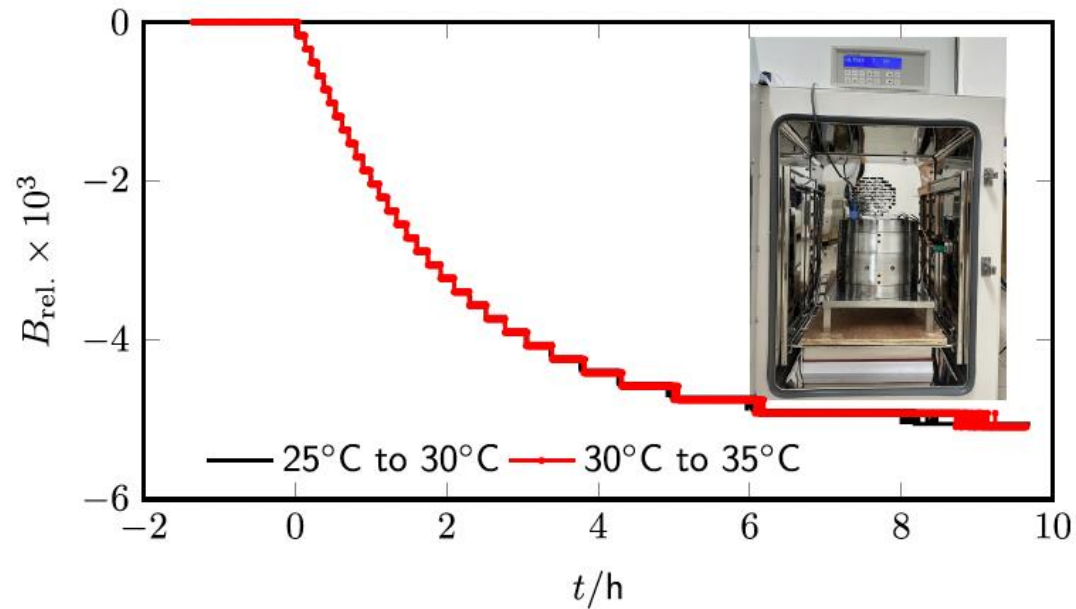
Performance evaluation:

Meet alignment range



Performance evaluation:

T_c measurement



Outlines



1. Magnetic basics in KB experiments
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KB magnetic effects:

Coil self inductance

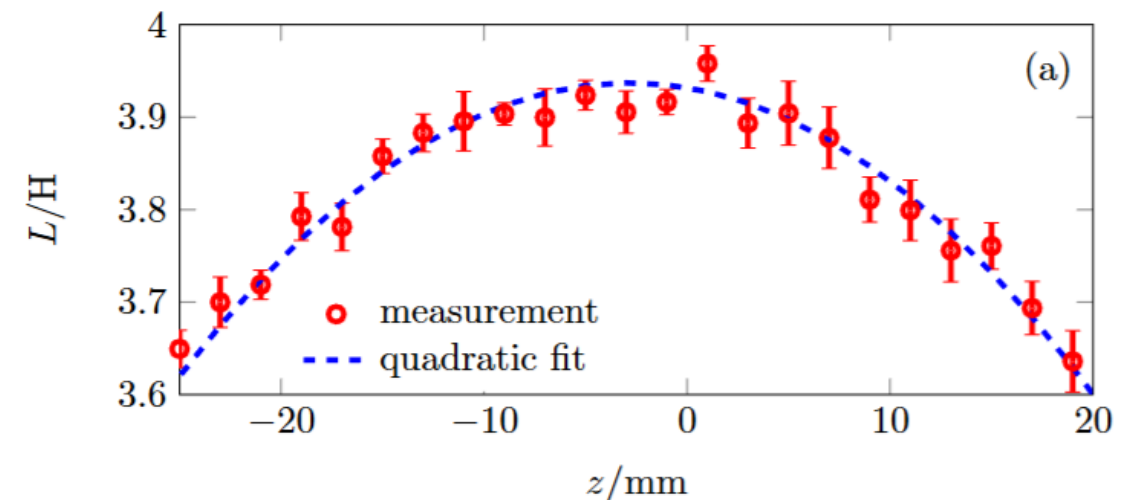
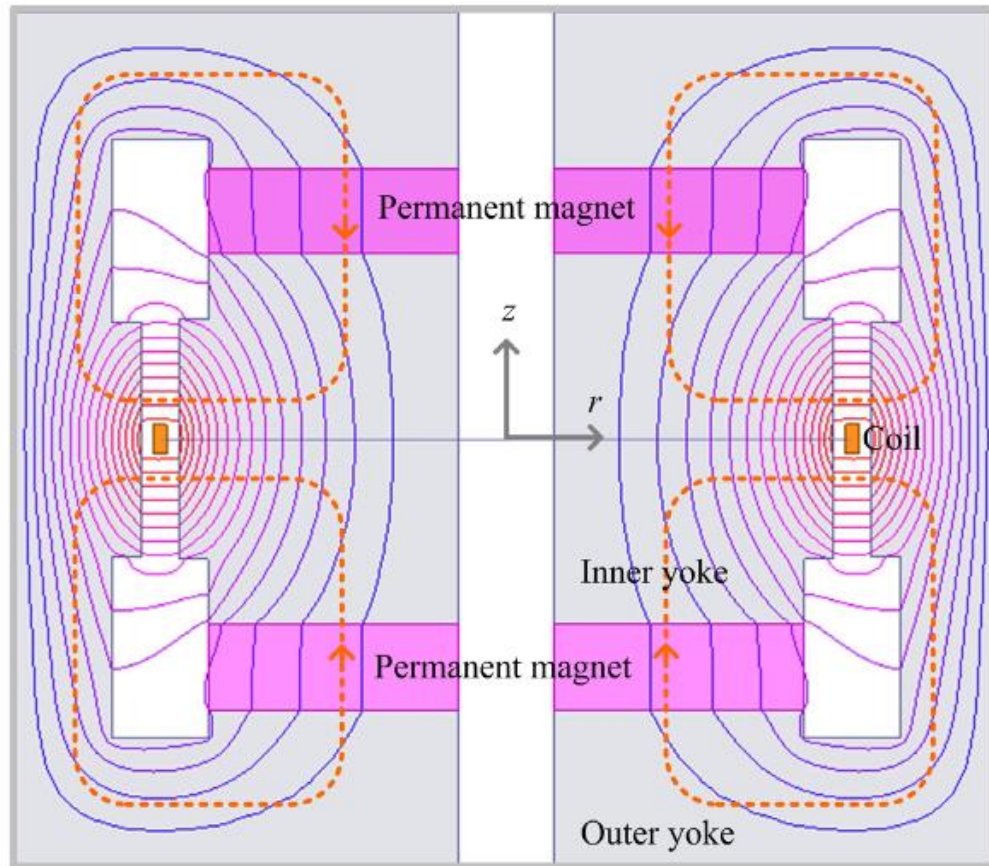


$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$

$$E = \frac{L}{2}I_1^2 + \frac{L}{2}I_2^2 + MI_1I_2$$

$$\Delta F = \frac{\partial E}{\partial z} = \frac{I^2}{2} \frac{\partial L}{\partial z}$$

$$\Delta(Bl)_w = \frac{\Delta F}{I} = \frac{I}{2} \frac{\partial L}{\partial z}$$



KB magnetic effects:

Coil self inductance

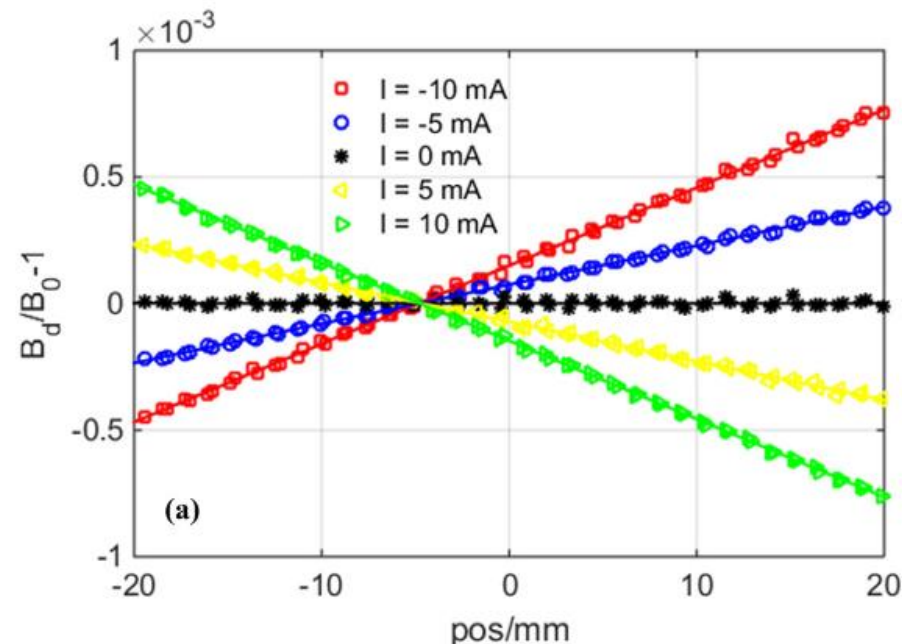
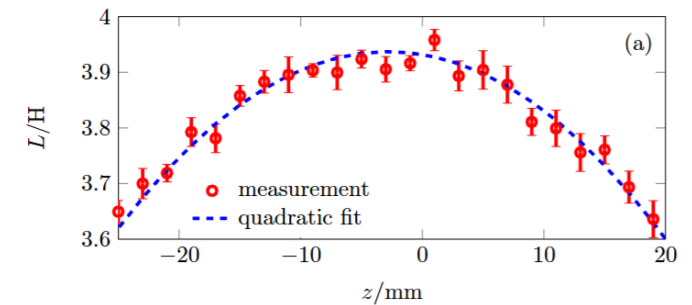
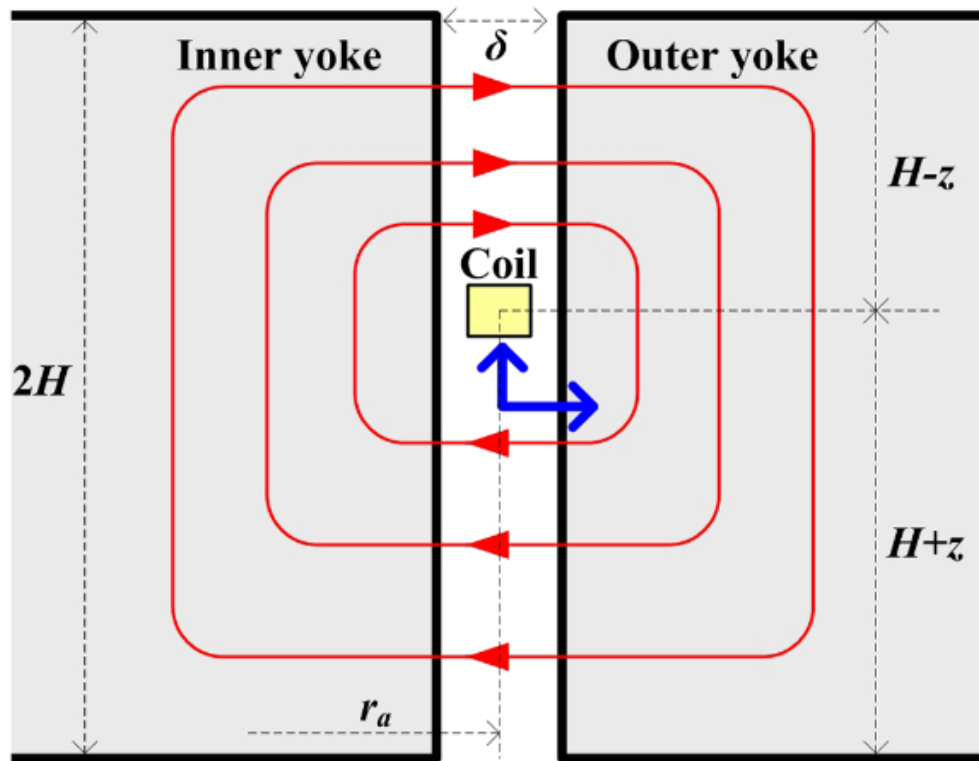


$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$

$$NI = \left(\frac{\delta}{\mu_0 2\pi r_a (H - z)} + \frac{\delta}{\mu_0 2\pi r_a (H + z)} \right) \phi$$

$$L = N\phi/I$$

$$L = \frac{\pi \mu_0 N^2 r_a (H^2 - z^2)}{H\delta}$$



KB magnetic effects:

Coil self inductance



$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$

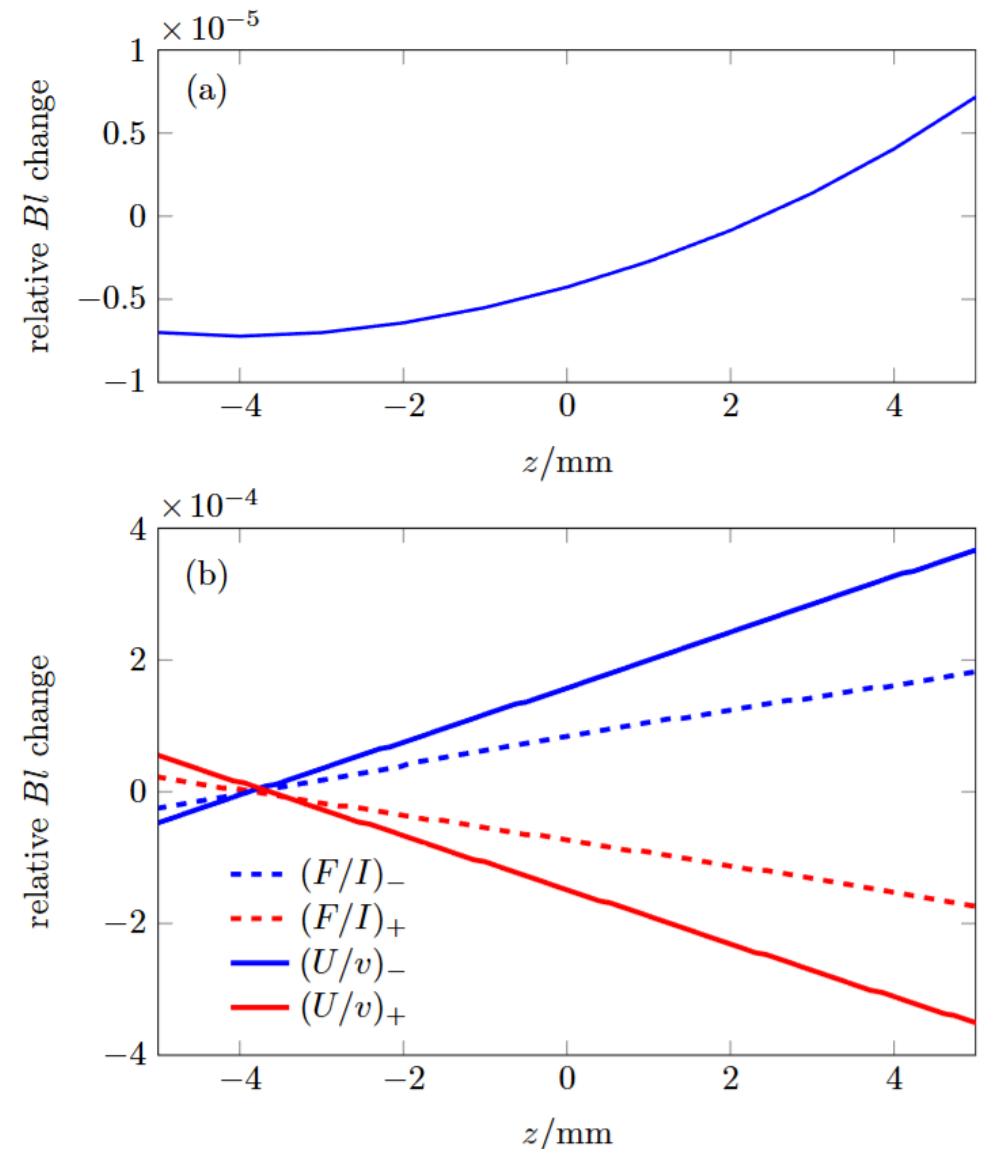
$$\begin{bmatrix} \Delta U_1 \\ \Delta U_2 \end{bmatrix} = \begin{bmatrix} U_1 - Blv \\ U_2 - Blv \end{bmatrix}$$

$$= \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} \partial I_1 / \partial t \\ \partial I_2 / \partial t \end{bmatrix} + \begin{bmatrix} \partial L_1 / \partial t & \partial M / \partial t \\ \partial M / \partial t & \partial L_2 / \partial t \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta U = \Delta U_2 = I \frac{\partial M}{\partial t} = I \frac{\partial M}{\partial z} \frac{\partial z}{\partial t} = Iv \frac{\partial L}{\partial z}$$

$$\Delta(Bl)_v = \frac{\Delta U}{v} = I \frac{\partial L}{\partial z}$$

$$\Delta(Bl)_v = 2\Delta(Bl)_w$$



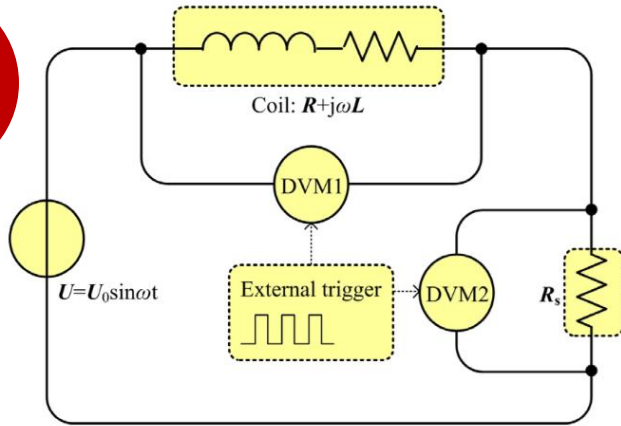
KB magnetic effects:

Coil self inductance



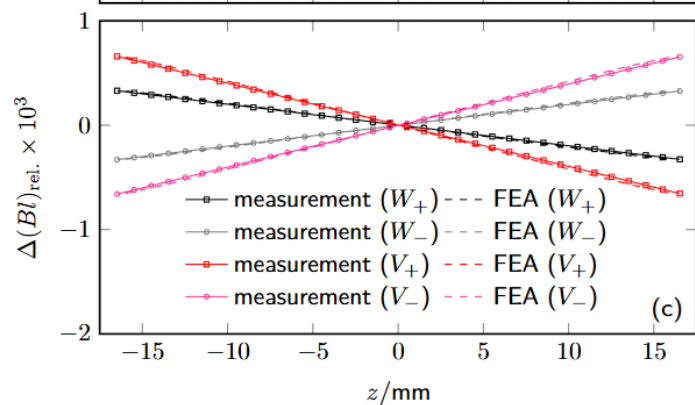
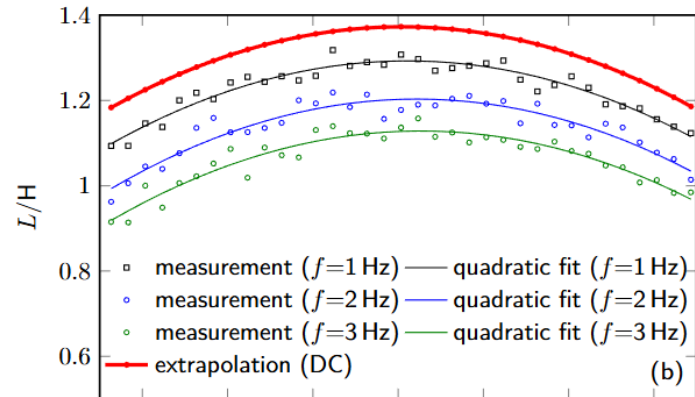
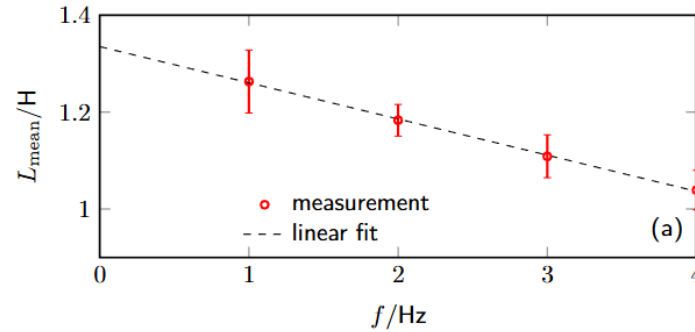
A

$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$

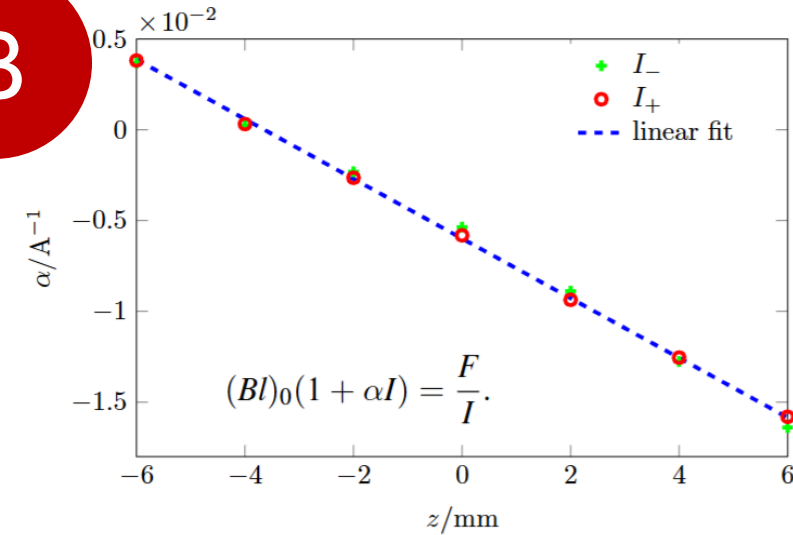


$$\frac{R_s^2}{R_s^2 + (\omega L)^2} = \left(\frac{V_R}{V_L}\right)^2,$$

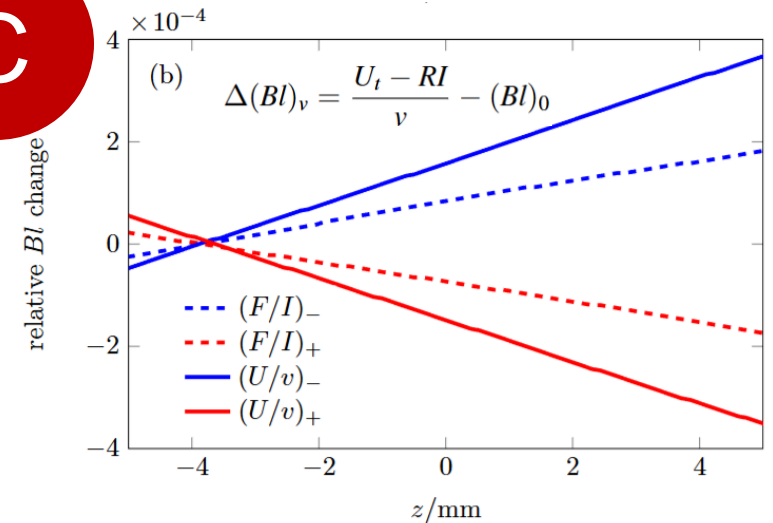
$$\frac{\omega L}{R_L} = \tan(\phi_L - \phi_R),$$



B



C



KB magnetic effects:

Coil self inductance



$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$

$$\begin{cases} B l I_+ [1 + \alpha(z_+) I_+] = mg - m_c g - m_e g \\ B l I_- [1 + \alpha(z_-) I_-] = -m_c g - m_e g \end{cases}$$

$$(Bl)_w = \frac{mg}{I_+ - I_-} = Bl \left[1 + \frac{\alpha(z_+) I_+^2 - \alpha(z_-) I_-^2}{I_+ - I_-} \right]$$

$$\varsigma \approx \frac{k(z_+ - z_-)(I_+ - I_-)}{4} + k\left(\frac{z_+ + z_-}{2} - z_0\right)(I_+ + I_-).$$

Linear current effect, ~10ppb/μm for the BIPM system

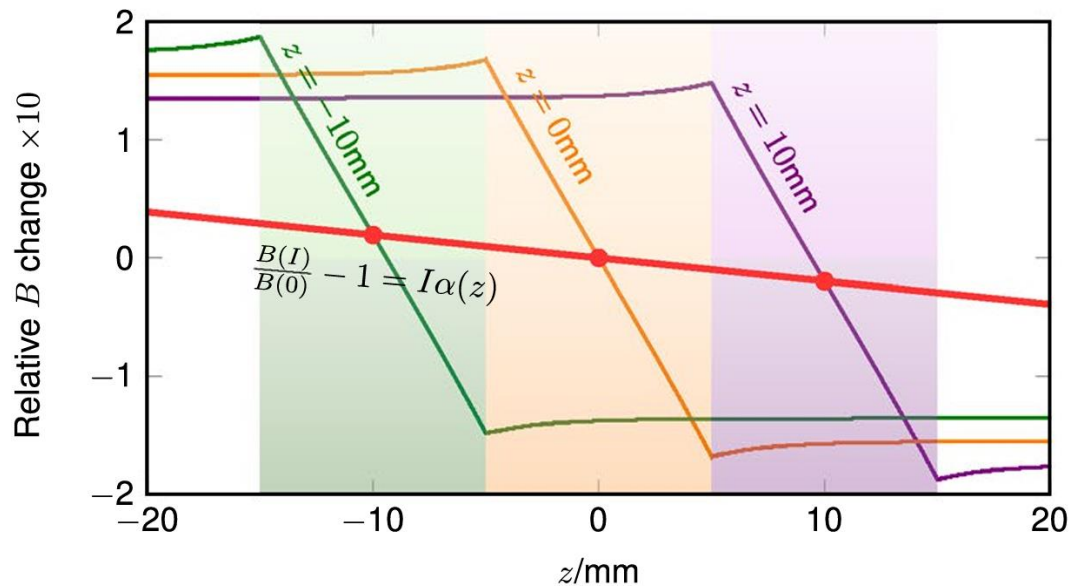
KB magnetic effects:

Diamagnetic effect



$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$

$$F_d = \frac{\int \chi dV}{\mu_0} \frac{1}{2} \frac{\partial}{\partial z} (B + b)^2,$$



Positive current, negative field gradient
Negative current, positive field gradient!



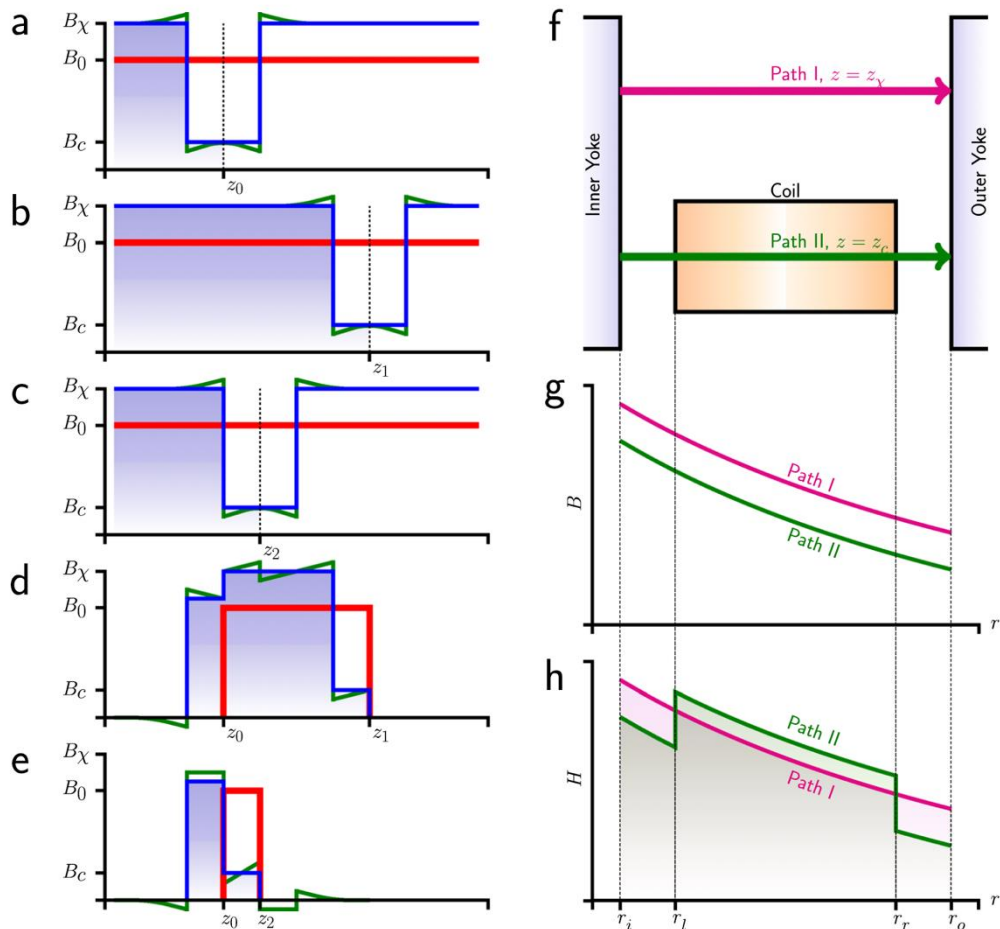
The effect is up to 1ppm!

KB magnetic effects:

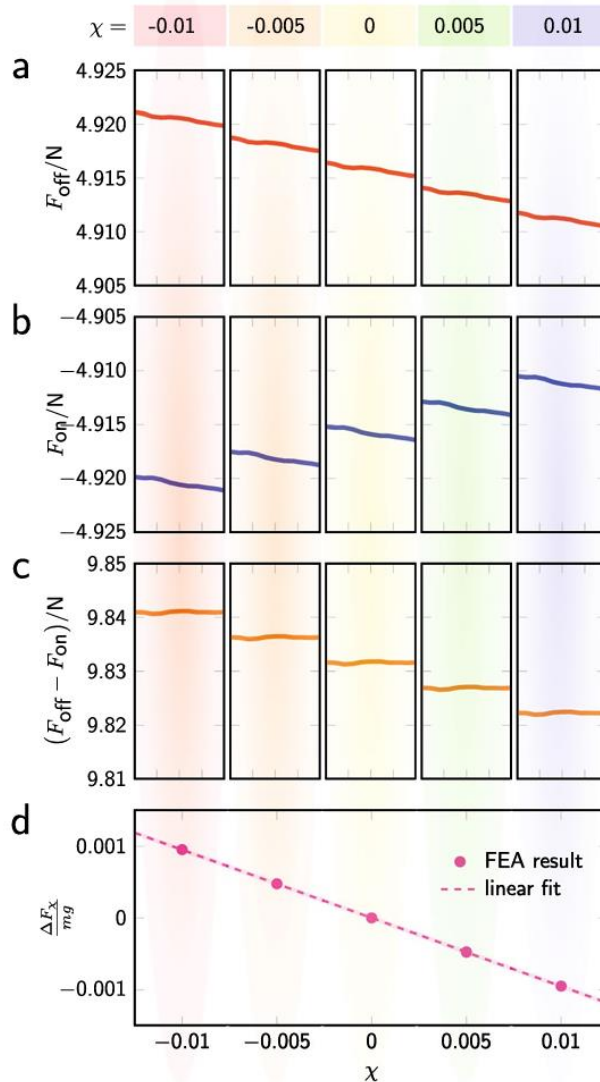
Diamagnetic effect



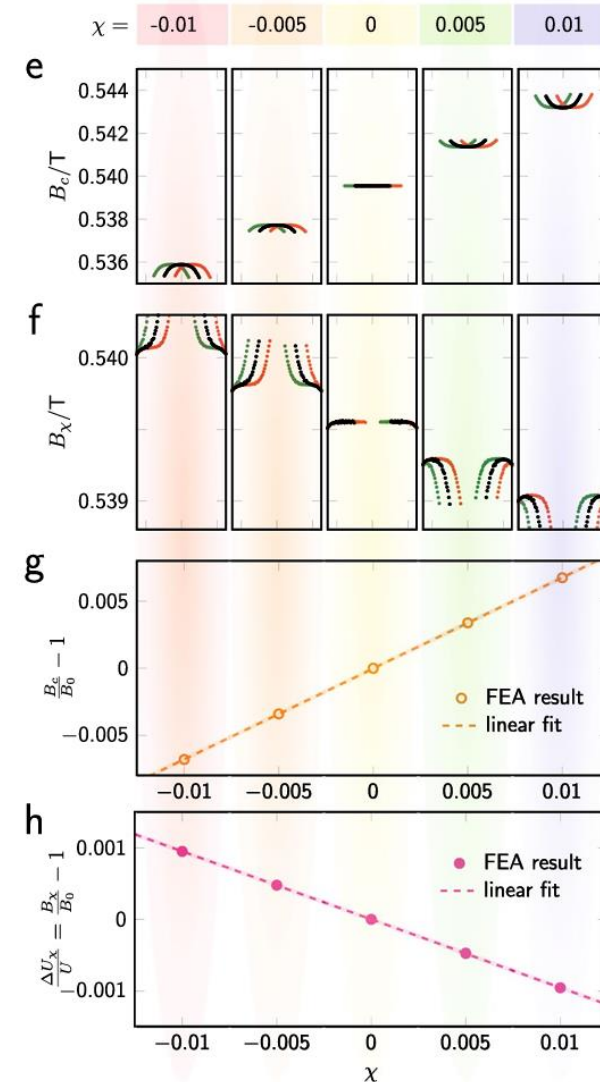
$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$



Weighing Measurement



Velocity Measurement



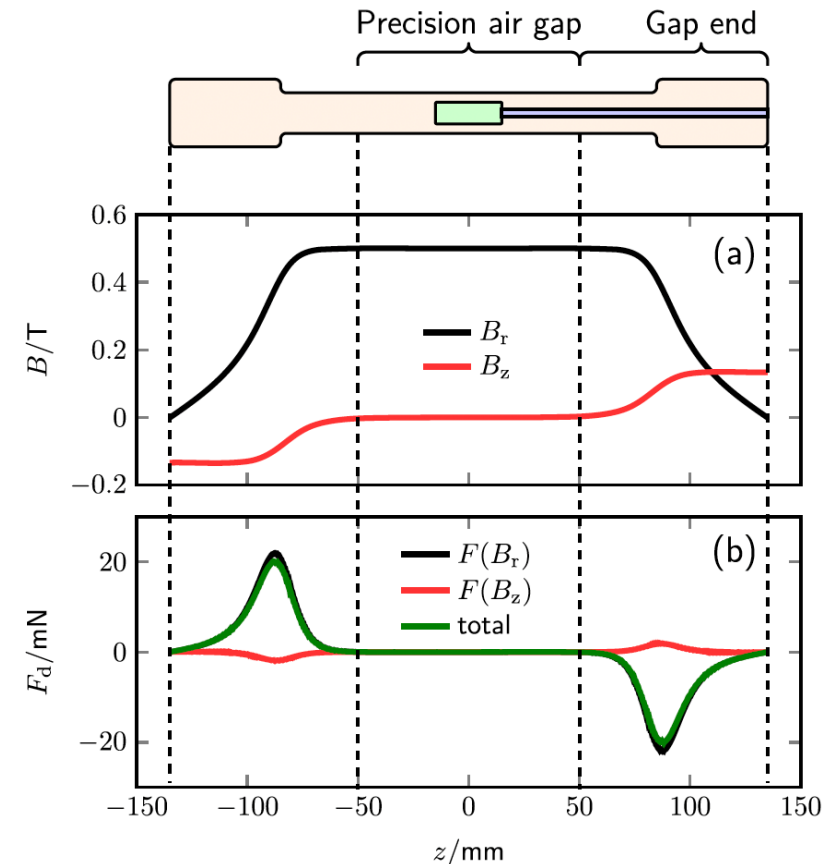
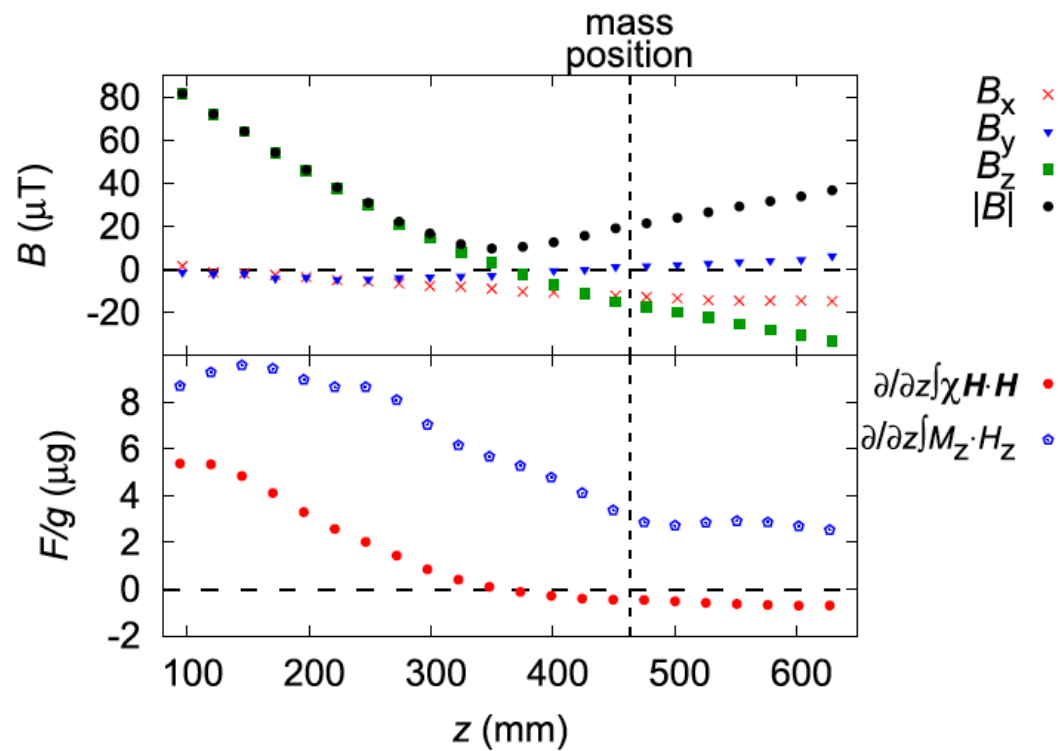
KB magnetic effects:

magnetization effect



$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$

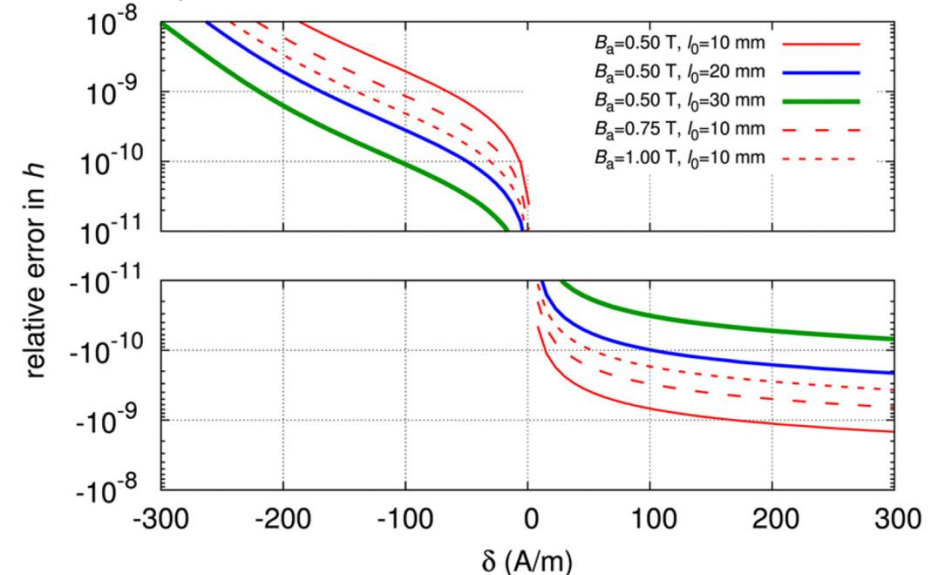
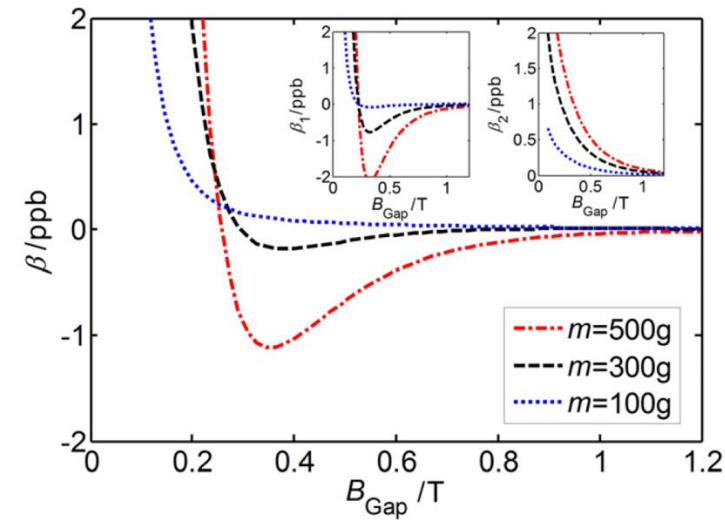
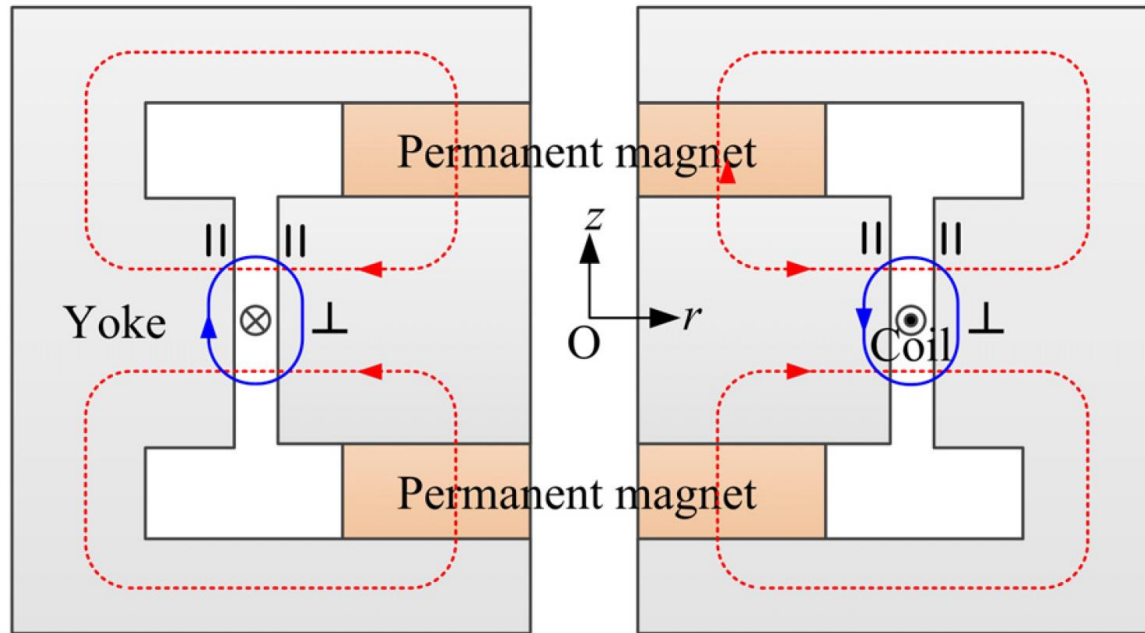
$$F_z = -\frac{\mu_0}{2} \frac{\partial}{\partial z} \int \chi \mathbf{H} \cdot \mathbf{H} dV - \mu_0 \frac{\partial}{\partial z} \int \mathbf{M} \cdot \mathbf{H} dV.$$



KB magnetic effects: *Nonlinear current effects*



$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$



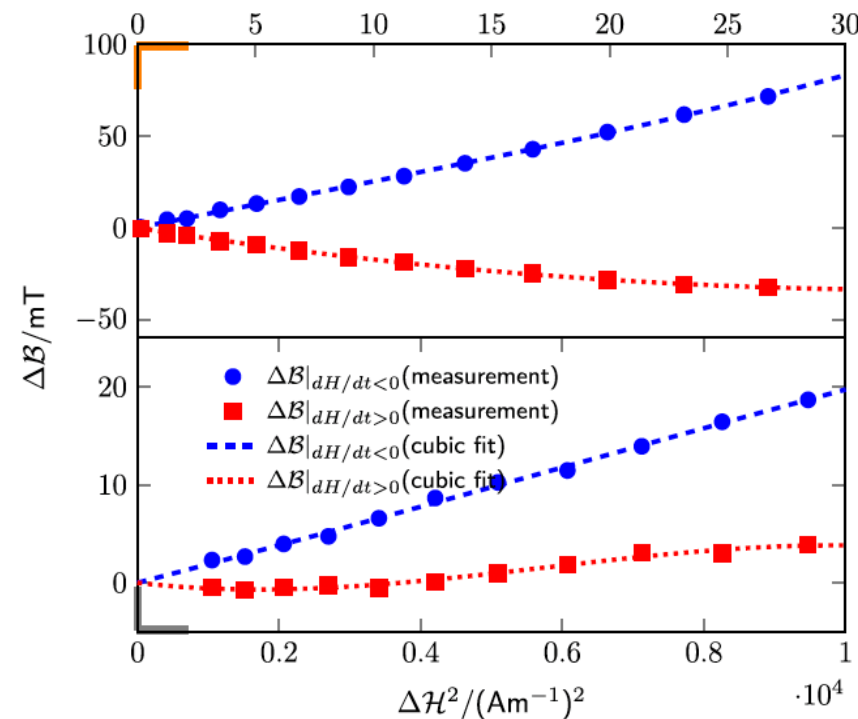
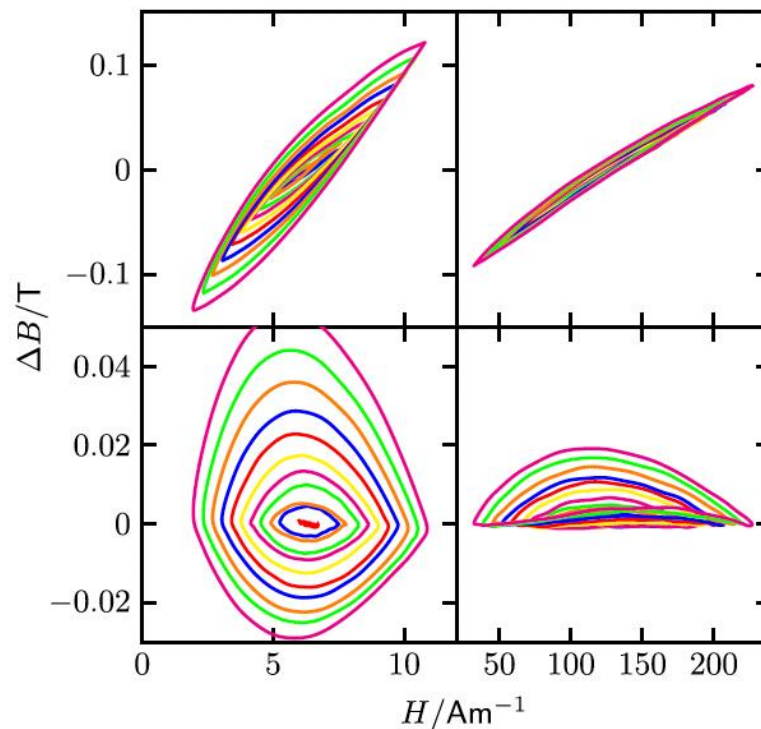
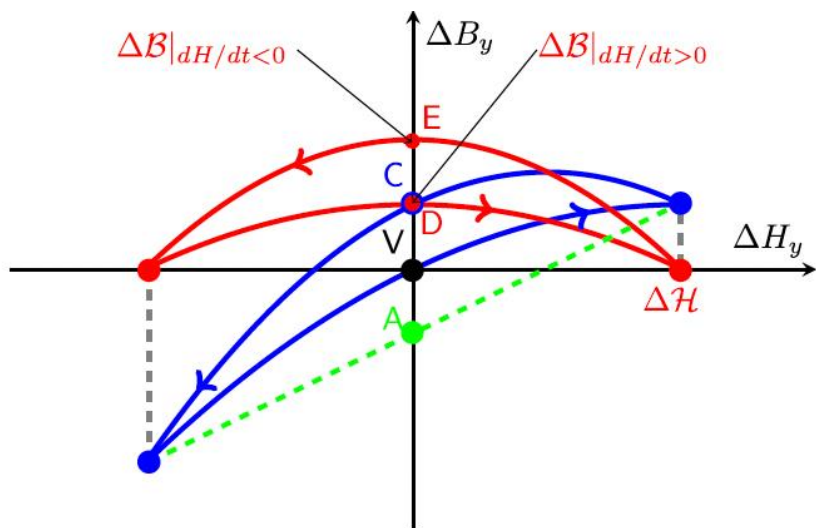
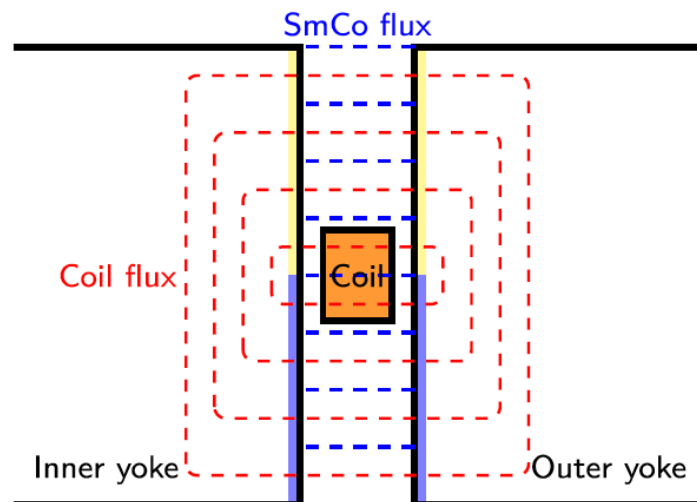
Yoke nonlinearity, considering the main BH curve. The effect is negligible.

KB magnetic effects:

Nonlinear current effects



$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$



BIPM system

Yoke no heat treatment: (-16.8 ± 2.8) ppb

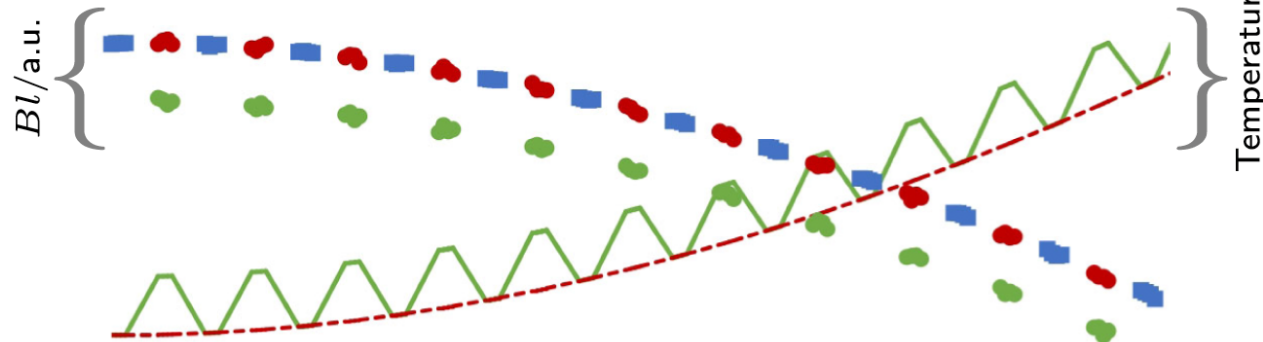
Yoke with heat treatment: (-21 ± 4.8) ppb

KB magnetic effects: *Thermal-magnetic effect*

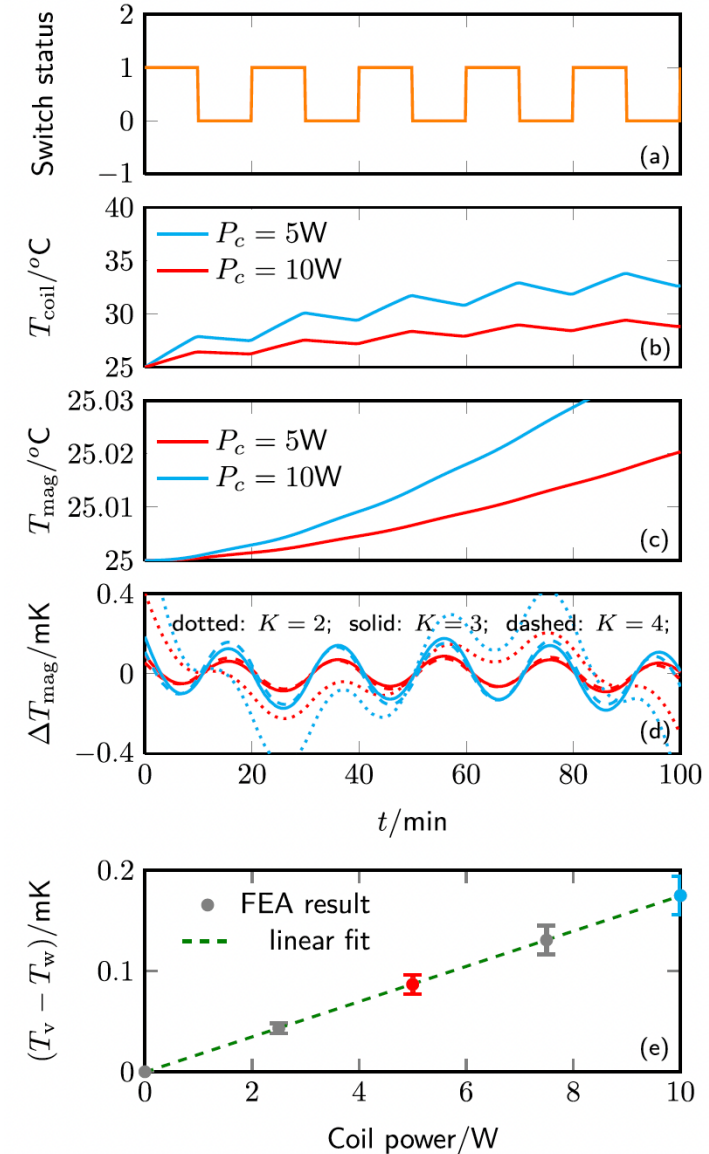
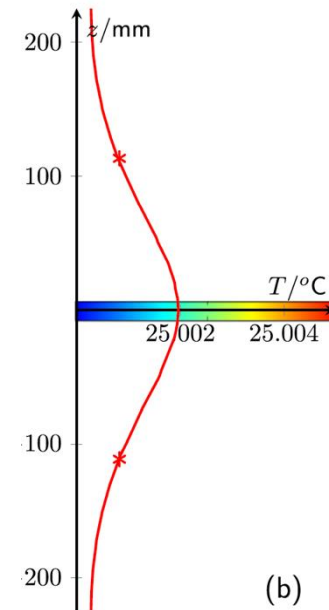
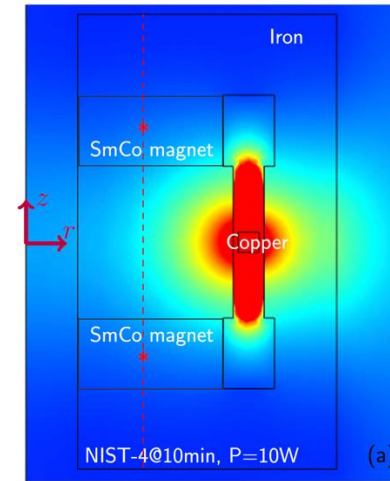
↑↓ BIPM

$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$

- $(Bl)_v$
- $(Bl)_w$ (T_m is coherent with the measurement phase)
- $(Bl)_w$ (T_m is not coherent with the measurement phase)
- T_m (coherent with the measurement phase)
- - - T_m (not coherent with the measurement phase)



Effect is found proportional to p^{-8}

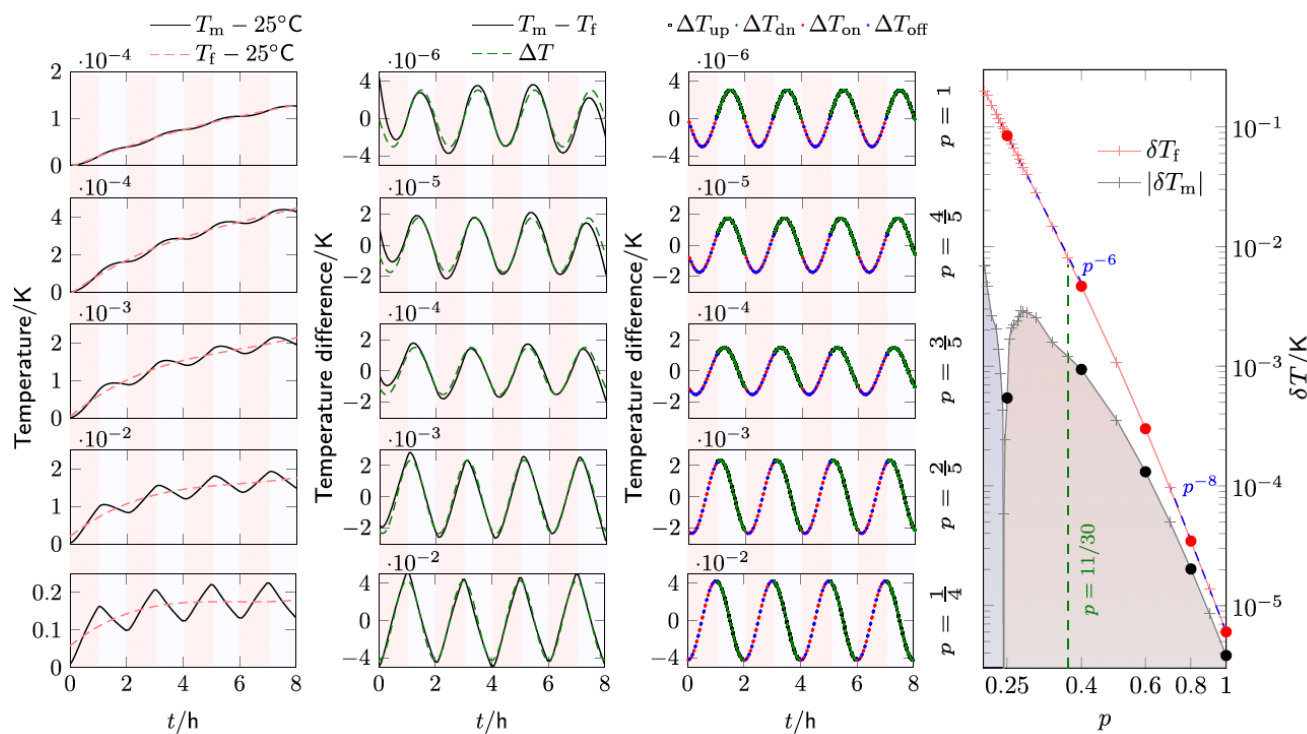


KB magnetic effects:

Thermal-magnetic effect



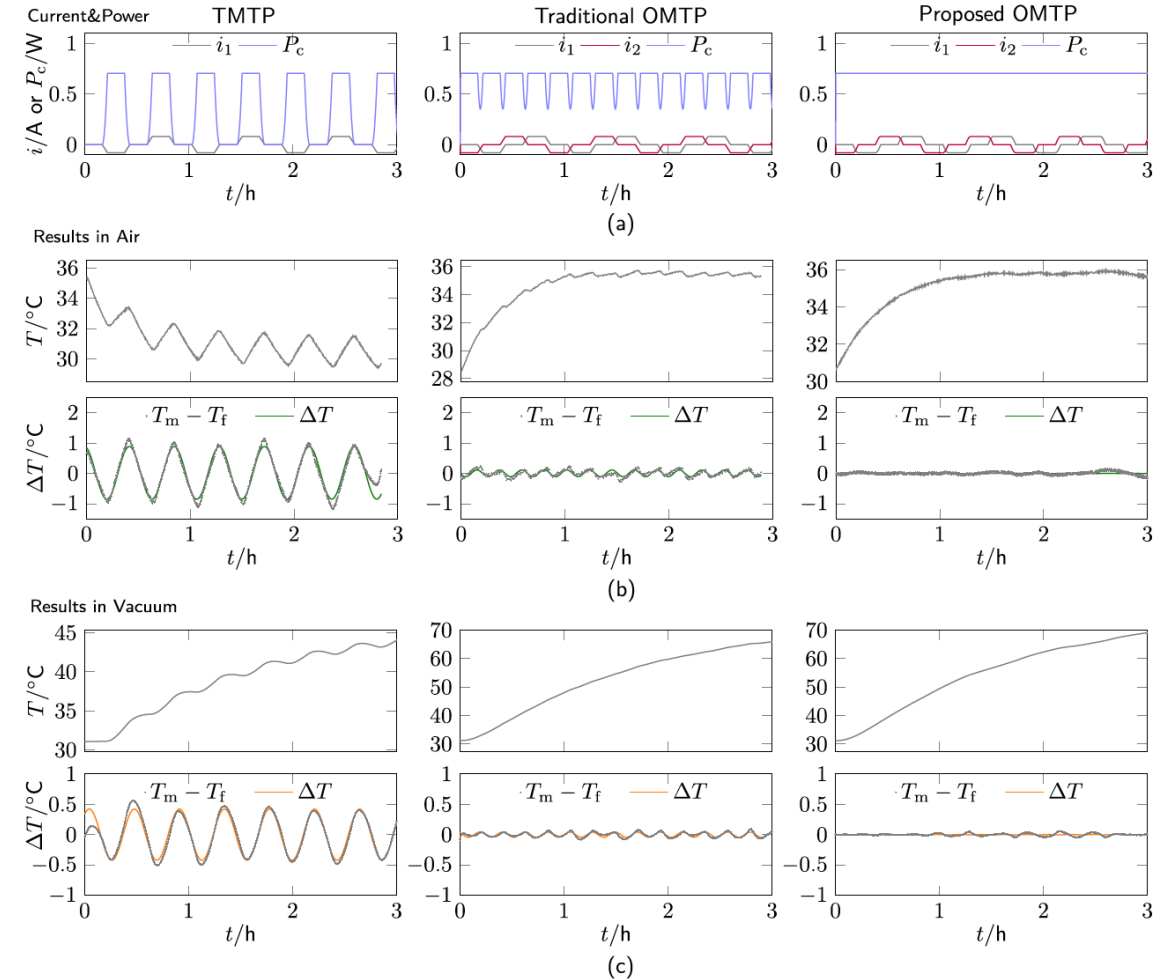
$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$



Experimental verification ~

One mode scheme can reduce the effect to 10%, and a further sin/cos current ramping to ~1%.

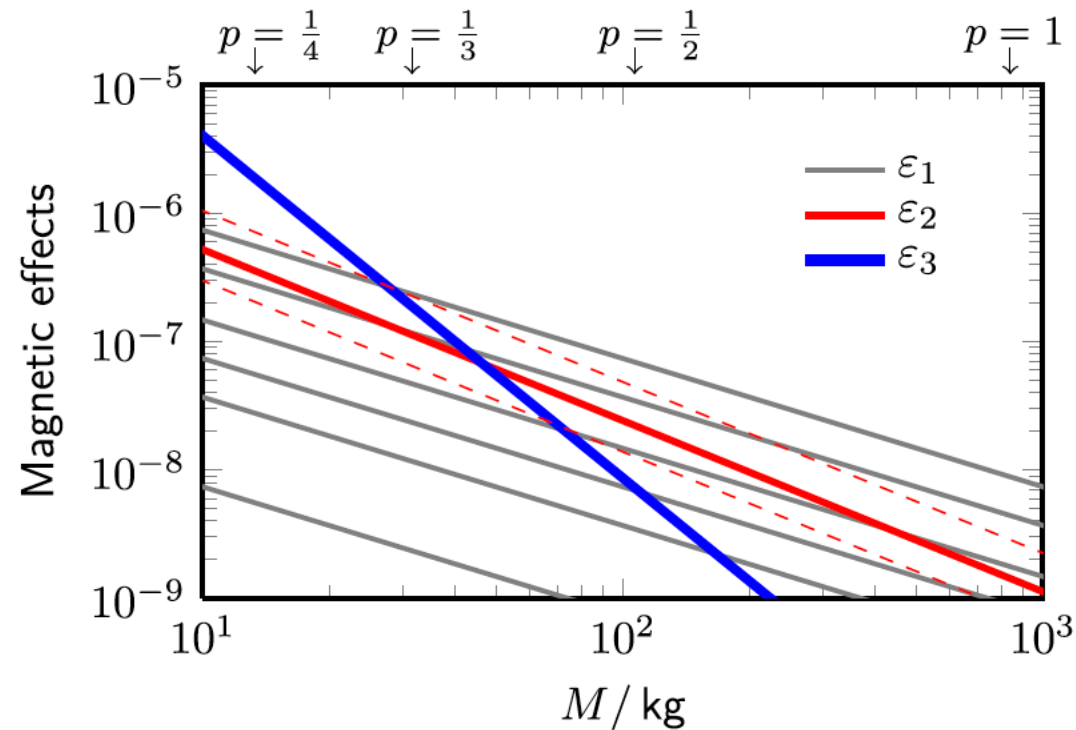
Liu W et al 2025 Metrologia 62 035009



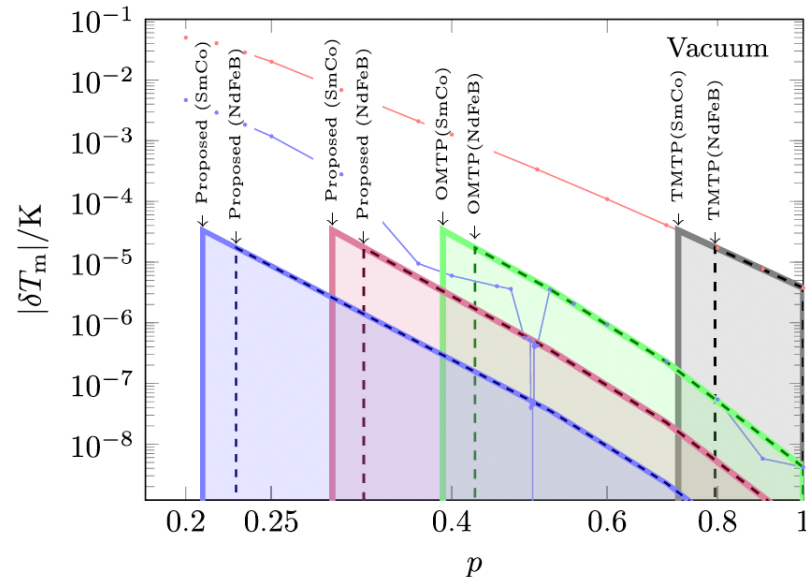
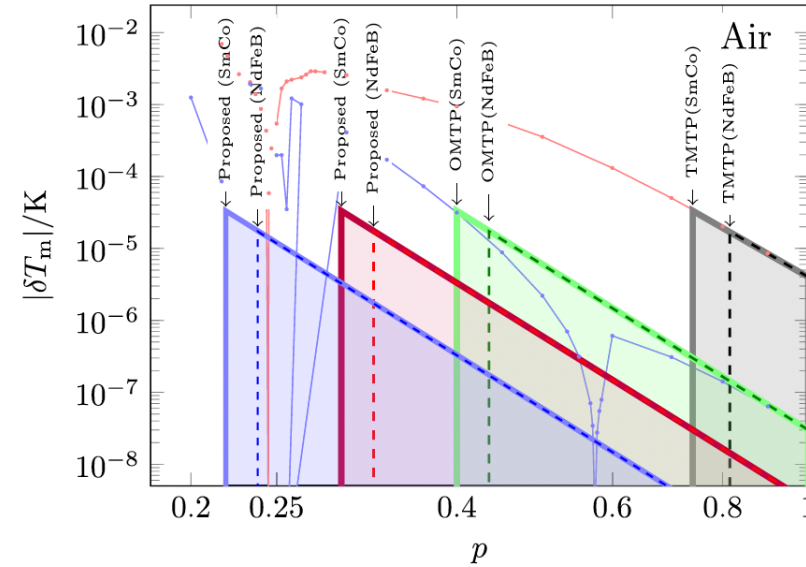
KB magnetic effects: *Thermal-magnetic effect*



$$(Bl)_w = (Bl)_v(1 + \alpha I + \beta I^2)$$



Set a size boundary for KB magnets!



Outlines



1. Magnetic basics in KB experiments
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6. **Summary & outlook**

6 Summary & outlook



6.1 Tabletop considerations

6.2 Open-hardware magnet system

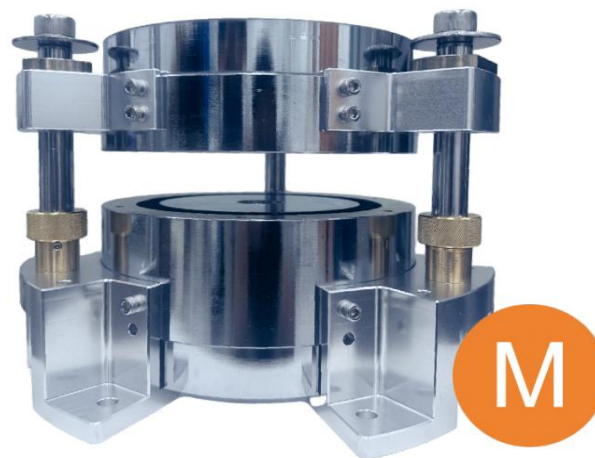
6.3 Future trends

Thanks!

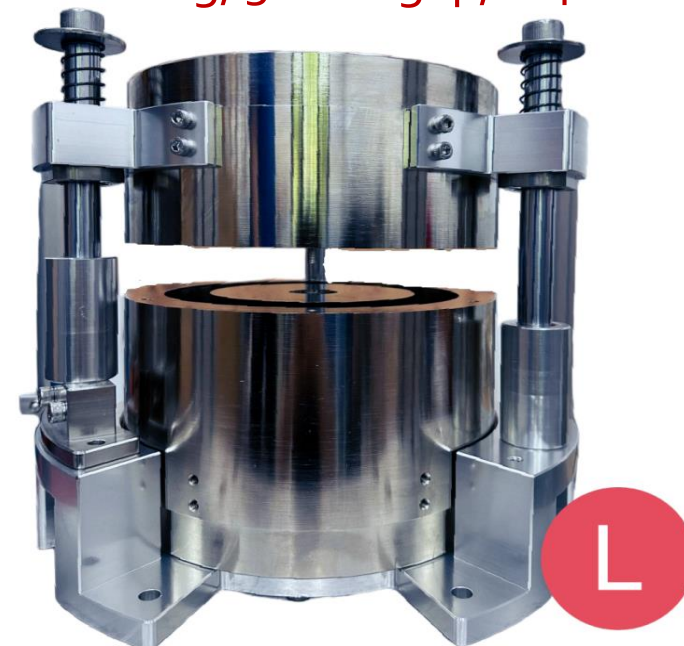
20kg, 10mm gap, 0.6T



40kg, 15mm gap, 0.6T



60kg, 30mm gap, 0.4T



Thank you.

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