International Association of Geodesy

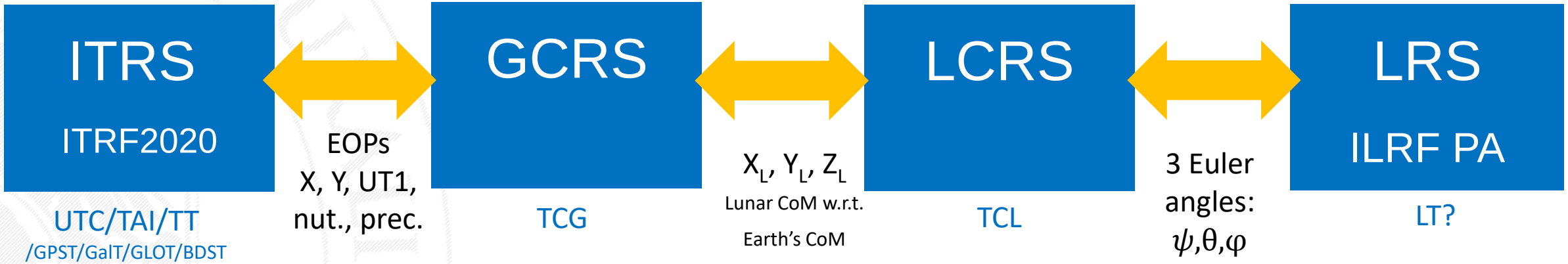
Wrocław University of Environmental and Life Sciences

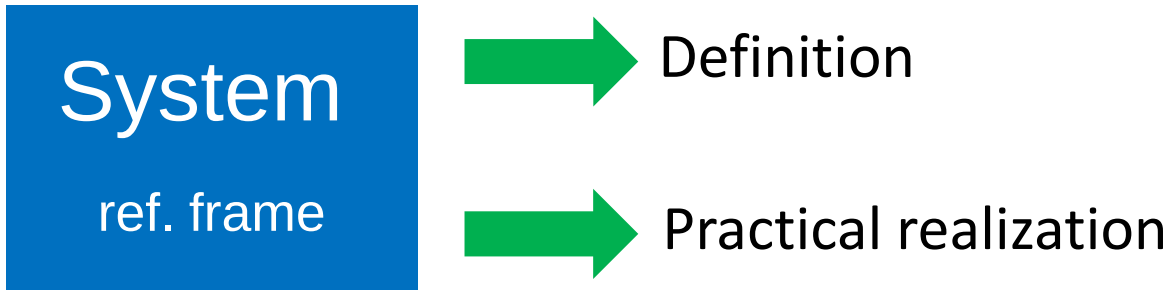
# Lunar Reference Frames – IAG

Krzysztof Sośnica, Agnes Fienga, IAG JWG 1.1.3

# Proposals

TDB - ephemerides



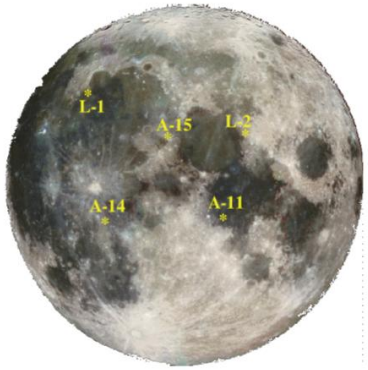




LCRS=Lunar Celestial Reference System (quasi-inertial)
LRS=Lunar Reference System (body-fixed)
ILRF=International Lunar Reference Frame
PA= Principal Axis

# Proposals

**The Lunar Reference System (LRS) is defined in such a way that:**

- its origin coincides with the center of mass of the Moon,
- its orientation is defined by the diagonal matrix of the lunar tensor of inertia, thus, co-rotates with the Moon,
- its scale follows the GR framework as defined by the IAU 2024 resolutions.




**The International Lunar Reference Frame (ILRF) is realized in such a way that:**

- its origin is based on the combination of INPOP21a, DE430, and EPM2021, using Variance Component Estimation (VCE) and referred to the mean center of mass of the Earth.
- its orientation is provided by three Euler angles from the VCE combined solution based on INPOP21a, DE430, and EPM2021.
- its scale is realized contemporarily with the origin and orientation.
- the weights for the origin and orientation derived from VCE are the same for all components.

# Proposals

```
Time scale            : TDB
Object                : Moon
Relative to           : Observer
Planetary theory      : Combination of EPM2021, INPOP21, and DE430
Coordinates           : Equatorial
Equator and equinox   : Mean J2000
Observer              : Geocentric
Light time correction : no
Relativistic effects  : no

          Date                                Position                                  Velocity
YYYY MM DD HH MM SS.sss        X(m)             Y(m)             Z(m)          Vx(m/s)          Vy(m/s)          Vz(m/s)
1970 01 01 00 00 00.000 -384373066.230  -63032035.128  -44389491.055   254.95073774  -851.93857476  -452.80022984
1970 01 01 18 00 00.000 -362447700.446 -117079864.046  -72959754.781   421.04387129  -812.06166723  -426.77493420
1970 01 02 12 00 00.000 -329950245.377 -167733789.338  -99410379.440   580.43055965  -747.13866643  -387.37500649
1970 01 03 06 00 00.000 -287480314.220 -213368562.660 -122878045.730   727.93017141  -657.22268029  -334.77313597
1970 01 04 00 00 00.000 -235989005.358 -252389120.504 -142525562.681   857.89682128  -543.23719303  -269.62674217
1970 01 04 18 00 00.000 -176803222.687 -283295881.582 -157577921.035   964.42508966  -407.25121194  -193.22084080
1970 01 05 12 00 00.000 -111631317.464 -304767276.383 -167367130.815  1041.72470952  -252.72414697  -107.58979150
1970 01 06 06 00 00.000  -42538788.822 -315754973.742 -171383074.829  1084.66611505   -84.63731266   -15.57295728
1970 01 07 00 00 00.000   28115565.535 -315581419.091 -169324494.299  1089.44030400    90.58085863    79.24435808
1970 01 07 18 00 00.000   97786738.973 -304023900.051 -161141470.495  1054.20745130   265.40230164   172.73707175
1970 01 08 12 00 00.000  163887672.813 -281367117.968 -147059833.190   979.56267651   431.81542969   260.58658092
1970 01 09 06 00 00.000  223953269.319 -248409567.945 -127580032.176   868.66236808   582.11043566   338.70899266
1970 01 10 00 00 00.000  275799423.226 -206418150.853 -103448151.324   726.94325833   709.65948800   403.66921956
1970 01 10 18 00 00.000  317651973.787 -157037461.315  -75603249.317   561.49574639   809.50836627   452.98349606
1970 01 11 12 00 00.000  348228182.174 -102170287.789  -45110451.391   380.25835488   878.66222401   485.25155832
1970 01 12 06 00 00.000  366765613.922  -43850109.051  -13091170.474   191.23096403   916.05360688   500.11757800
1970 01 13 00 00 00.000  373004940.739   15875992.913   19339765.760     1.85778600   922.27108532   498.10640303
1970 01 13 18 00 00.000  367140263.944   75042264.563   51124113.558  -181.35441290   899.16698560   480.40096642
1970 01 14 12 00 00.000  349752138.519  131832893.186   81295507.073  -352.99549697   849.45271142   448.61914356
1970 01 15 06 00 00.000  321735990.803  184626024.794  109001330.585  -508.74641057   776.35254640   404.62676265
1970 01 16 00 00 00.000  284234355.978  232018147.654  133514081.685  -645.27788425   683.34589113   350.40103572
1970 01 16 18 00 00.000  238577157.616  272834686.624  154235499.576  -760.11174903   573.99761239   287.94261574
1970 01 17 12 00 00.000  186231067.126  306132030.969  170696307.738  -851.48433035   451.85979341   219.22615054
1970 01 18 06 00 00.000  128757033.073  331194913.755  182553661.120  -918.23311781   320.42279777   146.17681118
1970 01 19 00 00 00.000   67774173.911  347531645.531  189587598.322  -959.71402144   183.09481696    70.66133598
1970 01 19 18 00 00.000    4928070.219  354868486.275  191697136.514  -975.74753090    43.19340740    -5.51527038
```


VCE for the orbit combination:

Zajdel, R., Mansur, G., Sakic, P. *et al.* Advancing multi-GNSS orbit combination in the variance component estimation framework. *J Geod* **99**, 90 (2025). https://doi.org/10.1007/s00190-025-02005-w

ILRF is expressed as a time series of positions, velocities, orientation angles and their first derivatives (in ASCII format sampled every 0.75 days), as well as in the form of Chebyshev polynomials (continuous function).
Implementation period: 1970-2052
→ Easily accessible to users (calceph)

Additionally, the positions of the retroreflectors, the kinematic corrections, the Love number corresponding to the realization, and the transformation parameters to PA and ME are provided, see:

Definition and Realization of the International Lunar Reference Frame: https://doi.org/10.48550/arXiv.2510.15484

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**Definition and Realization of the International Lunar Reference Frame**

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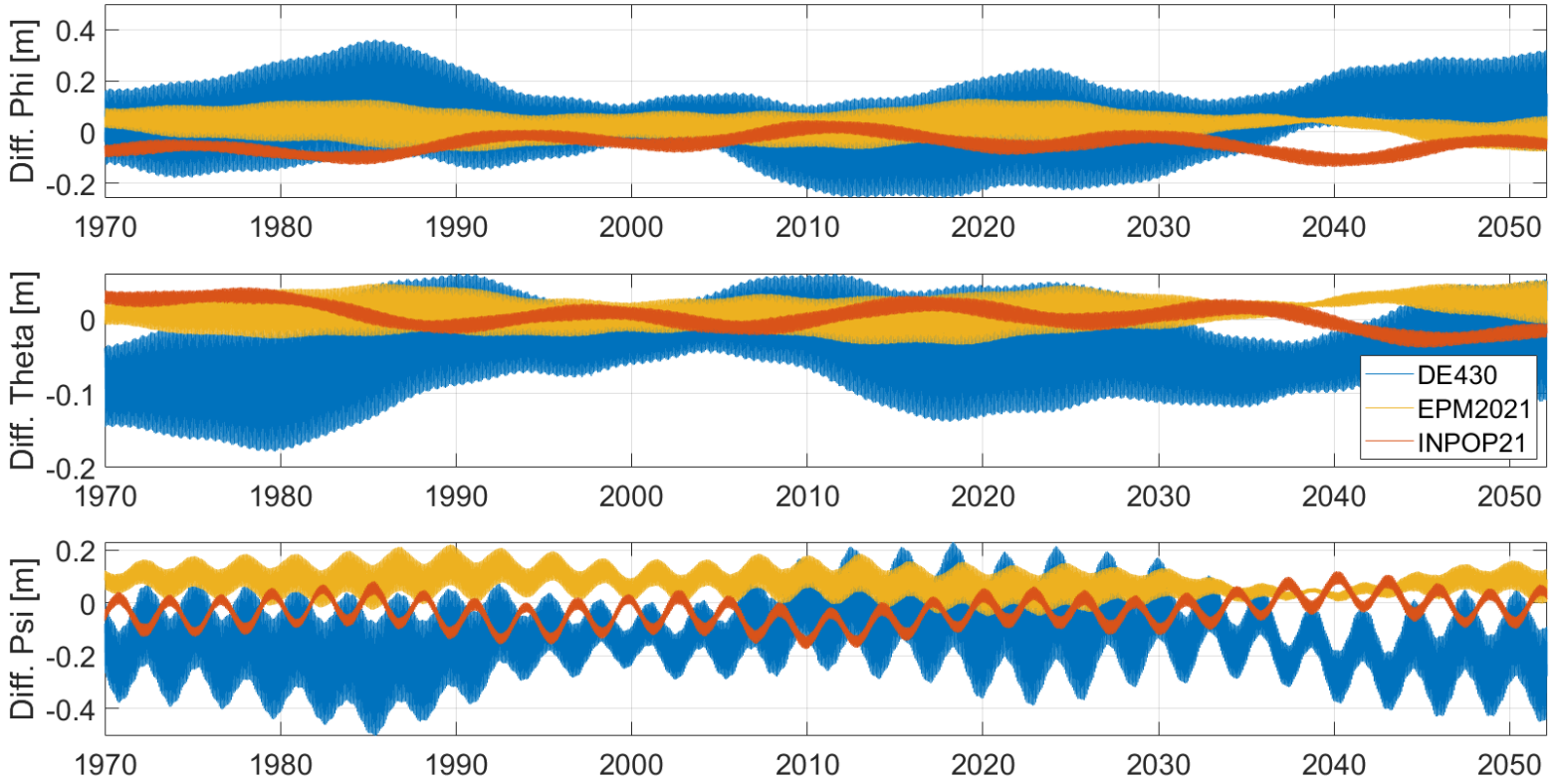
# Realization of the ILRF - Orientation

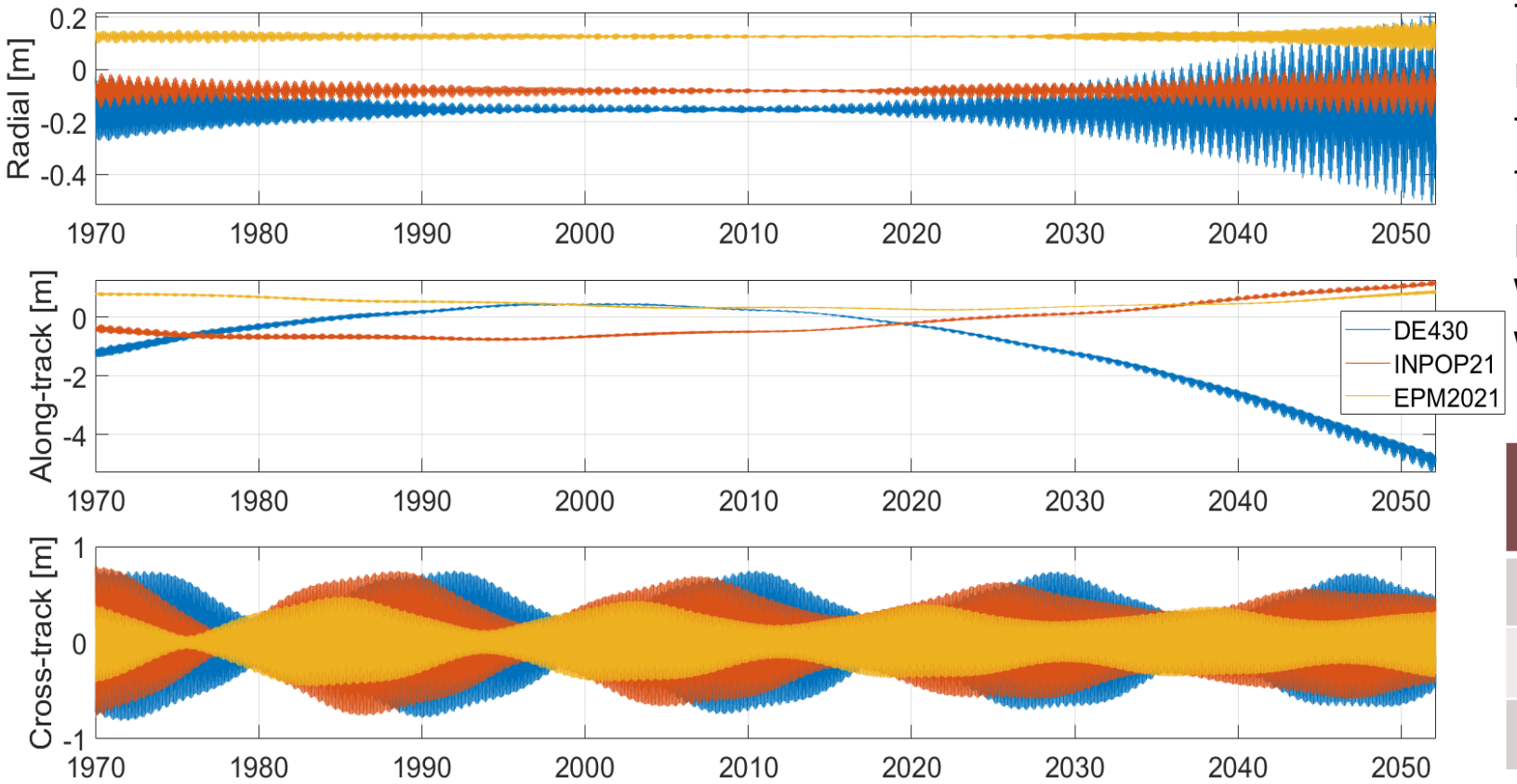
Orientation of the ILRF PA – residuals with respect to individual ephemerides (im meters on the lunar surface)




$$LRS = R_Z(\psi)\, R_X(\theta) R_Z(\varphi)\, LCRS$$

| RMS [m] | Phi | Theta | Psi |
|---|---|---|---|
| EPM21 | **0.054** | 0.022 | 0.092 |
| INPOP21a | 0.059 | **0.017** | **0.063** |
| DE430 | 0.139 | 0.067 | 0.189 |

# Realization of the ILRF - Origin




Origin of the RF – differences w.r.t. the combination

Origin of the ILRF system relative to the geocenter – residuals relative to individual ephemerides.
The offset for the radial component results from different scales of the frames. The largest differences occur for the along-track component in the prediction period. Cross-track is stable which means that the orbital plane is well-defined.

| RMS [m] | Radial | Along-track | Cross-track |
|---|---|---|---|
| EPM21 | 0.126 | **0.511** | **0.214** |
| INPOP21 | **0.084** | 0.599 | 0.350 |
| DE430 | 0.168 | 1.666 | 0.372 |

STD is the standard deviation w.r.t. the mean value. RMS is the root mean square error without removing the mean offset value.

# Realization of the ILRF – error assessment

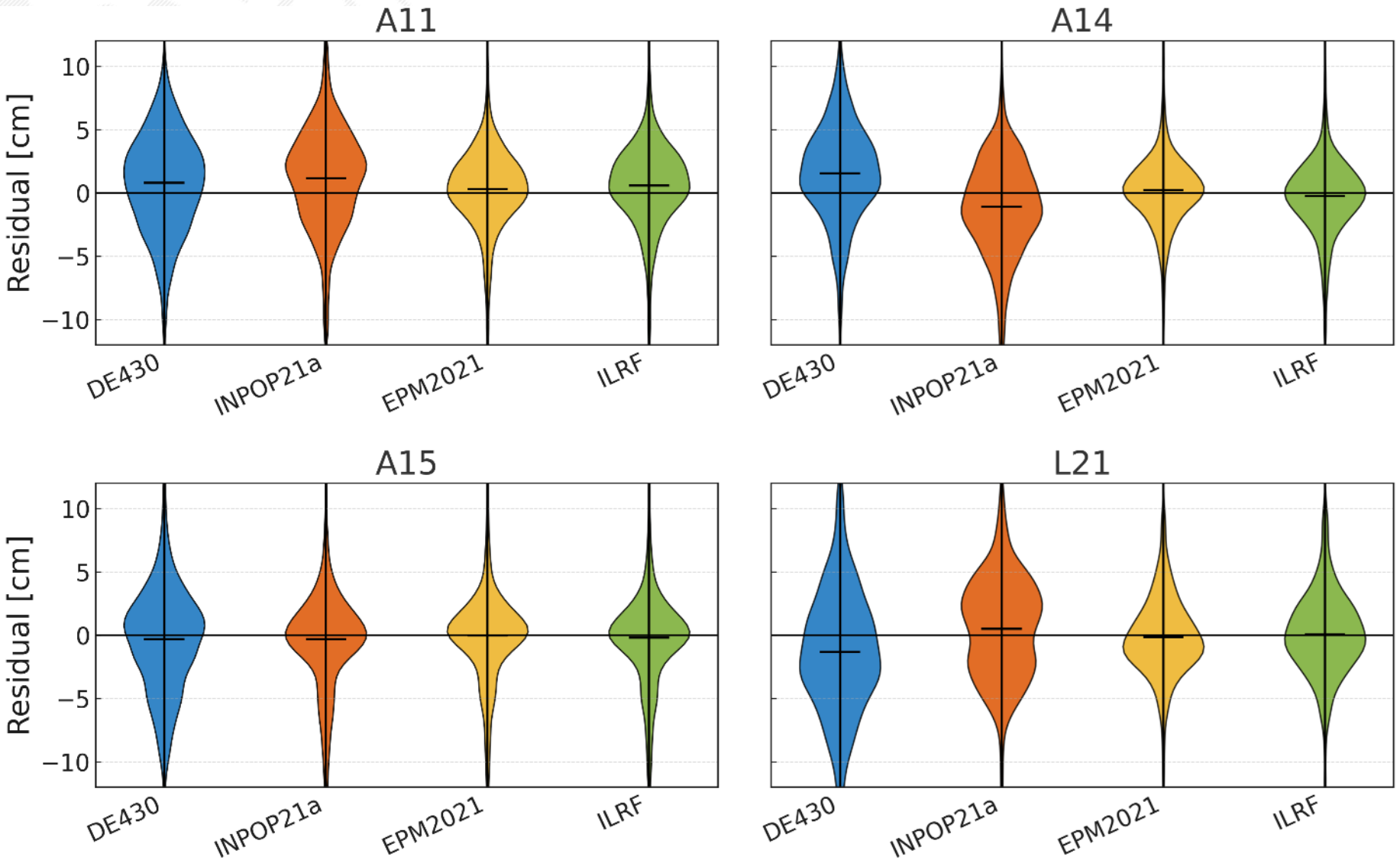
| Frame | weights |
|---|---|
| EPM21 | 0.451454072137127 |
| INPOP21a | 0.380949596178372 |
| DE430 | 0.167596331684500 |

Mean errors of the ILRF combination for individual epochs as derived from VCE with a distinction for the origin and orientation constituents.

The smallest combination errors are for the period **2010-2030** with a **total error of 17.6 cm** (dominated by the origin error of 15.3 cm with a contribution from orientation of 8.6 cm).
For the whole assessed period, the total error equals to **31.6 cm**.

| Mean Error [m] | 1970-1990 | 1990-2010 | 2010-2030 | 2030-2052 | Whole |
|---|---|---|---|---|---|
| **Origin** | 0.284 | 0.225 | **0.153** | 0.467 | 0.305 |
| **Orientation** | 0.085 | 0.061 | **0.086** | 0.068 | 0.082 |
| **Total** | 0.296 | 0.233 | **0.176** | 0.472 | 0.316 |

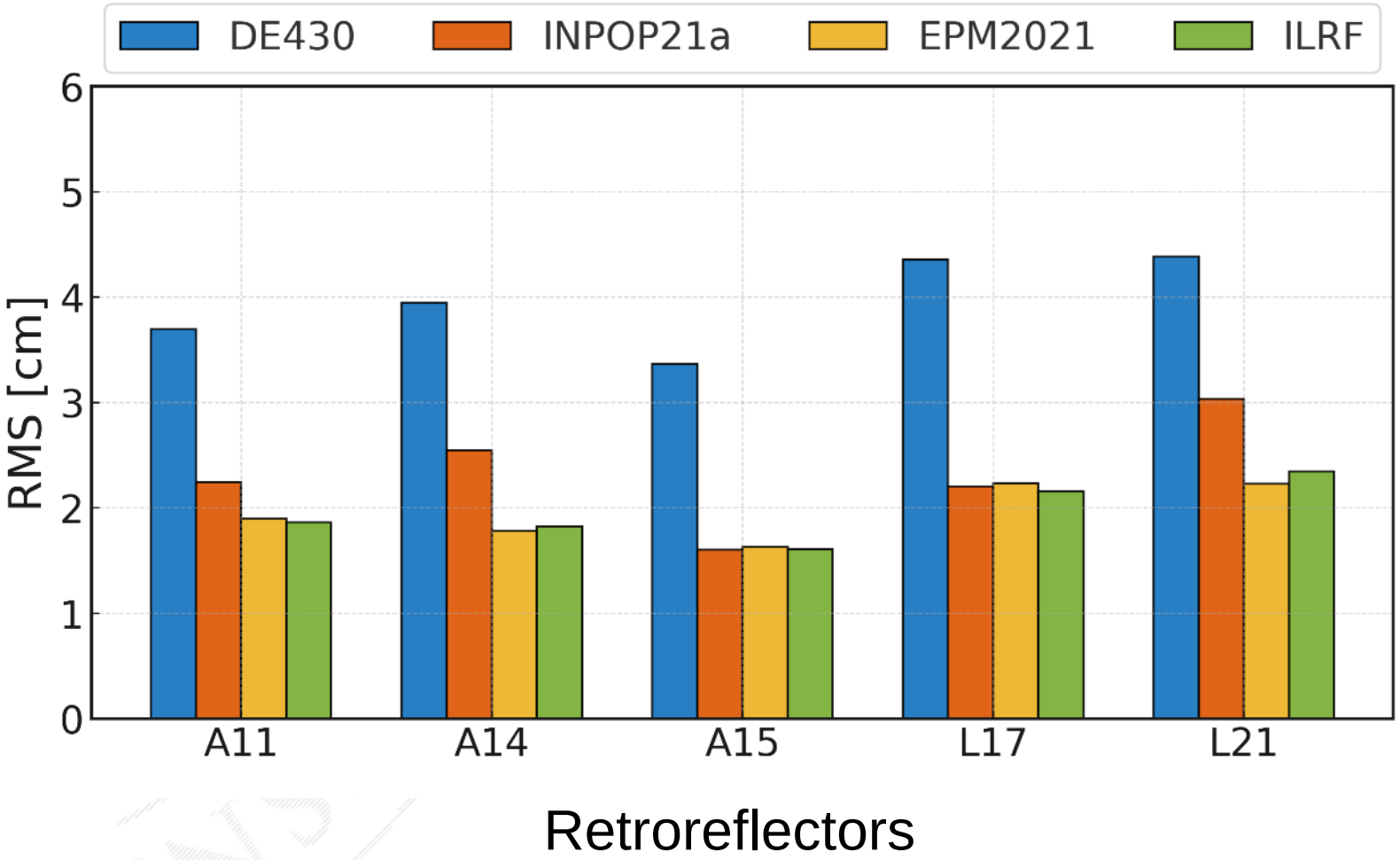
# Realization of the ILRF – validation using LLR




Error distribution for a solution without kinematic corrections (only 3 Euler angles are applied + tidal displacement with VCE-d $k_2$ value).

Period: 1984-2025

# Realization of the ILRF – validation using LLR




Mean errors for LLR observations (one way), 2015-2025*.

Mean retroflector positions and kinematic corrections were derived based on VCE weights

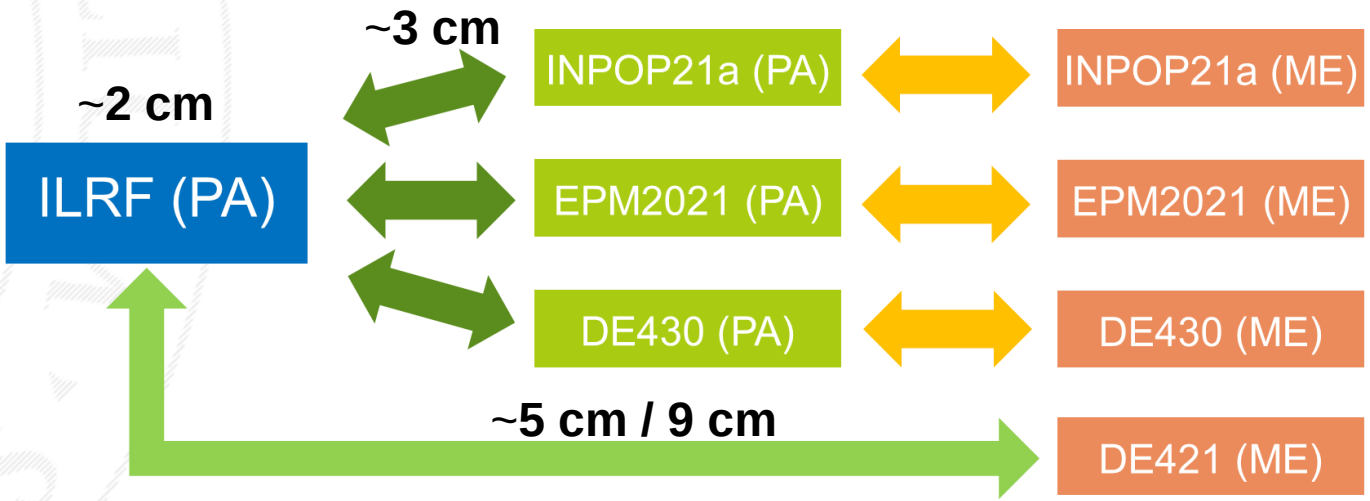
For ILRF, the errors are at the level of 2 cm (despite no parameter fit for retroreflector positions and kinematic corrections).

* DE430 was fitted up to 2012, other RFs - 2021.

# Transformation parameters to PA and Mean Earth (ME) frames

Transformation formula (7-par. Helmert) based on lunar retroreflectors:

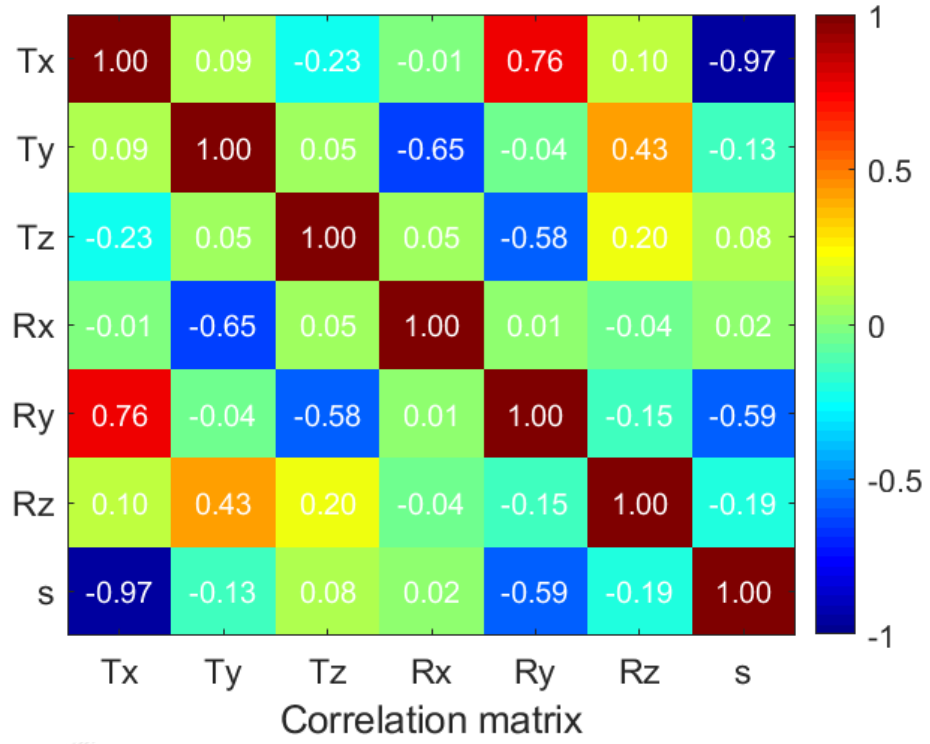
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} T_X \\ T_Y \\ T_z \end{bmatrix} + \begin{bmatrix} 1+Sc & R_z & -R_Y \\ -R_z & 1+Sc & R_X \\ R_Y & -R_X & 1+Sc \end{bmatrix} \cdot \begin{bmatrix} X_{ILRF} \\ Y_{ILRF} \\ Z_{ILRF} \end{bmatrix}$$

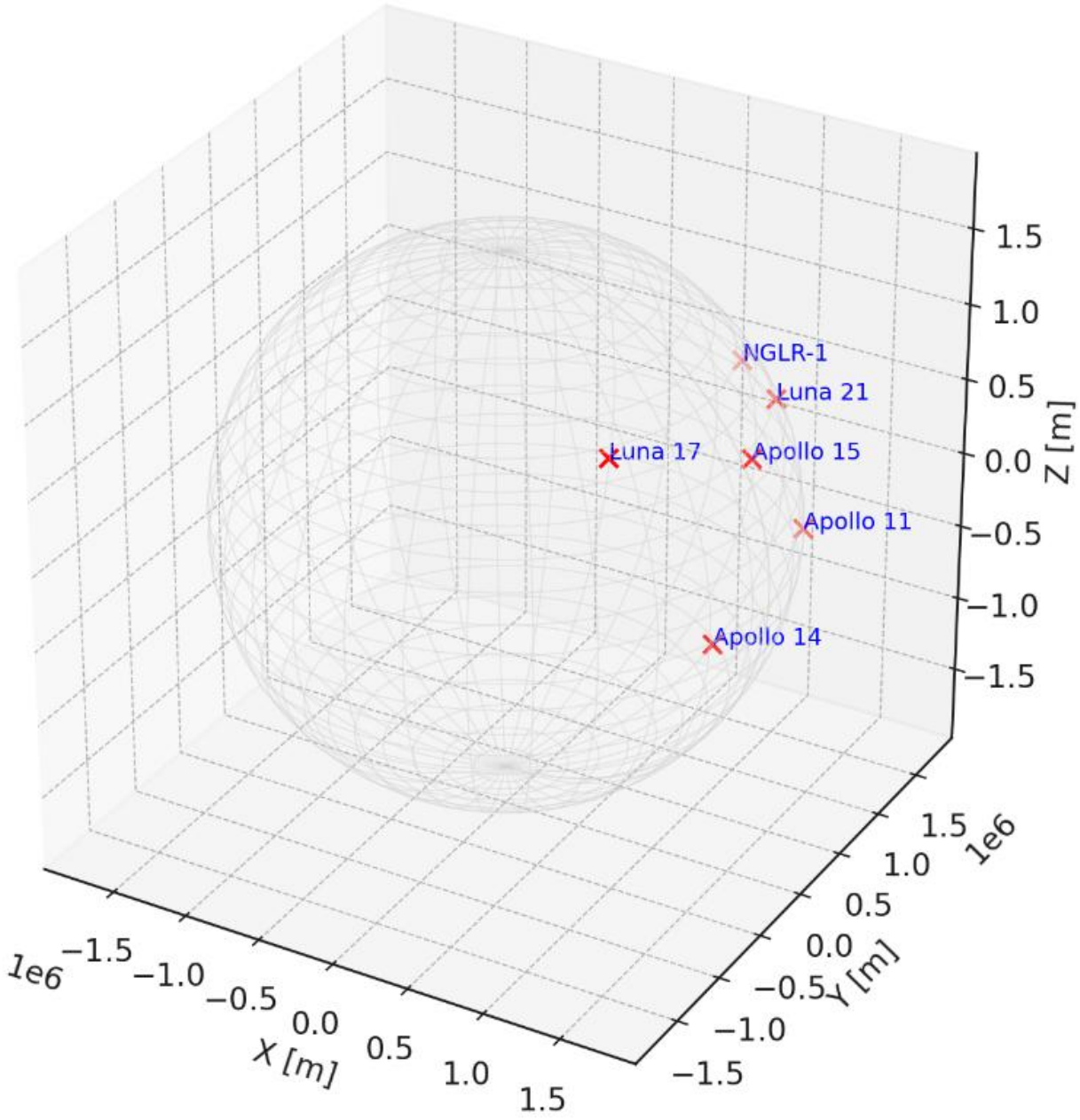



Mean errors of deriving transformation parameters are equal to **3.5, 2.8, and 1.5 cm** for DE430, INPOP21a, and EPM2021, respectively; therefore, the accuracy of the transformation procedure at the level of 2-3 cm is considered sufficient for most of the lunar applications. The tranformation error to ME is **5 cm** (**7-parameters**) or 9 cm (3-parameters – rotations only).

# Transformation parameters

Transformation parameters derived based on retroreflector positions in each RF.
The X-translation is highly correlated with the scale parameter due to the geometry of observations (r=-0.97).



# Summary

**Definitions for LRS and ILRF** have been proposed.
For the time system – many options for the definition are still possible.

The reference frame is based on a series of three contributing solutions: **DE430, INPOP21a, and EPM2021** combined using a modified version of **VCE** (for the 6D vectors including 3 translation parameters and 3 rotation, with constant weight to meet the requirement of a Keplerian motion according to Beutler et al., 1995).

The mean combination error is **17.6 cm for 2010-2030** with the dominating error coming from the origin (15.3 cm).

The LLR post-fit residuals in ILRF are at the level of **2 cm** for the last 10 years of LLR data.

The mean errors of the **transformation between ILRF** and other reference frame realizations in PA are at the level of **3 cm**, whereas the mean tranformation errors to ME frames is 5 cm.

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# Thank you for your attention

**Paper under review in Astronomy & Astrophysics**

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https://geodesy.science/com1/structure/com-1-working-groups/joint-working-group-1-1/