

To scale or not to scale?

Issues with scaled coordinate times

Sergei A. Klioner

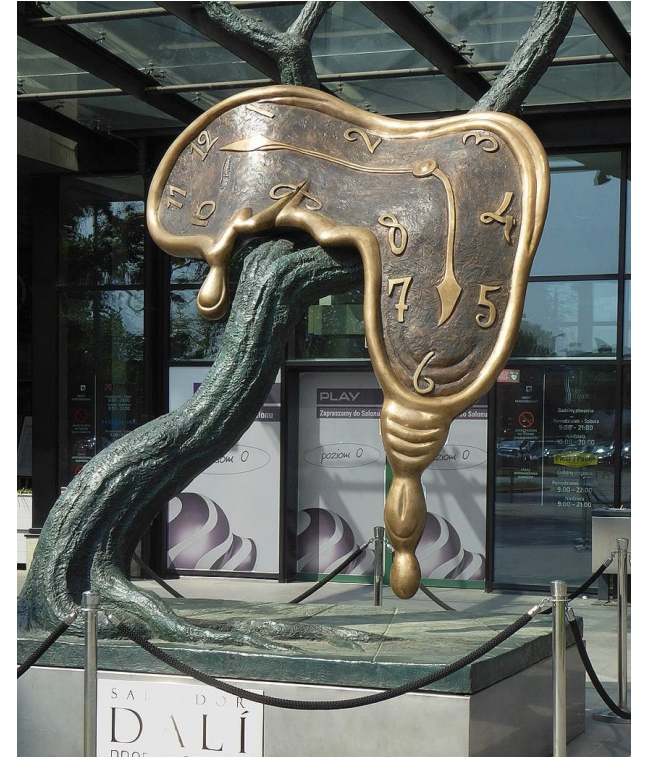
Lohrmann Observatory, Technische Universität Dresden



CCTF Workshop on Moon Time, 18 November 2025

Relativity and high-accuracy observations

- Since decades, the accuracy of many kinds of observational data is many orders of magnitude higher than the leading relativistic effects. This is especially true for atomic clocks!
- According to General Relativity, time and frequency are affected by
 - (a) the motion of clock
this is verified with very high accuracy
 - (b) the gravitational potential at the location of the clock
this is verified to 2.5×10^{-5} (Galileo satellites on eccentric orbits)
- General Relativity must be used to model the data!
 - ➔ Not just “small corrections” to Newtonian models: the whole relativistic paradigm should be used!



The IAU 2000 framework

A framework of a hierarchy of **relativistic reference systems** that define algorithms to model observations

IAU 2000: Resolutions B1.3, B1.4, B1.5, and B1.9: See <https://iau.org/iau/Publications/List-of-Resolutions>

Augmented by

IAU 2006: Resolution B2

Resolution B3

This is based on a series of publications of a number of experts in General Relativity over about 20 years:

Ashby, Bertotti, 1986

Kopeikin, 1988-

Brumberg, Kopeikin, 1988-1992

Damour, Soffel, Xu, 1991-1994

Klioner, Voinov, 1993

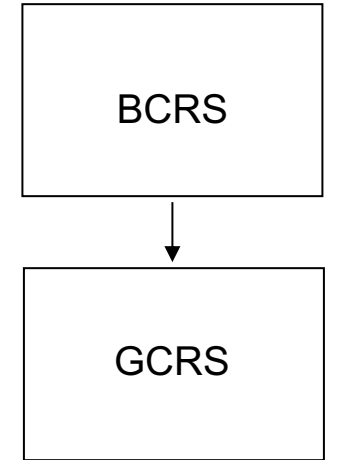
Klioner, Soffel, 2000

Accuracy for frequencies better than 10^{-20}

Explanatory supplement: [Soffel et al, 2003: AJ, 126, 2687](#)

The IAU 2000 framework

- Two standard astronomical reference systems were defined
 - **BCRS** (Barycentric Celestial Reference System)
 - **GCRS** (Geocentric Celestial Reference System)
- These reference systems are defined by
 - the form of the corresponding metric tensors
 - the 4D coordinate transformations between BCRS and GCRS



$$T = t - \frac{1}{c^2} [A(t) + v_E^i r_E^i] + \frac{1}{c^4} [B(t) + B^i(t) r_E^i + B^{ij}(t) r_E^i r_E^j + C(t, \mathbf{x})] + O(c^{-5})$$

$$X^a = \delta_{ai} \left\{ r_E^i + \frac{1}{c^2} \left[\frac{1}{2} v_E^i v_E^j r_E^j + w_{\text{ext}}(\mathbf{x}_E) r_E^i + r_E^i a_E^j r_E^j - \frac{1}{2} a_E^i r_E^2 \right] \right\} + O(c^{-4}) ,$$

$$\frac{d}{dt} A(t) = \frac{1}{2} v_E^2 + w_{\text{ext}}(\mathbf{x}_E) , \quad \frac{d}{dt} B(t) = -\frac{1}{8} v_E^4 - \frac{3}{2} v_E^2 w_{\text{ext}}(\mathbf{x}_E) + 4 v_E^i w_{\text{ext}}^i(\mathbf{x}_E) + \frac{1}{2} w_{\text{ext}}^2(\mathbf{x}_E) ,$$

$$B^i(t) = -\frac{1}{2} v_E^2 v_E^i + 4 w_{\text{ext}}^i(\mathbf{x}_E) - 3 v_E^i w_{\text{ext}}(\mathbf{x}_E) ,$$

$$B^{ij}(t) = -v_E^i \delta_{aj} Q^a + 2 \frac{\partial}{\partial x^j} w_{\text{ext}}^i(\mathbf{x}_E) - v_E^i \frac{\partial}{\partial x^j} w_{\text{ext}}(\mathbf{x}_E) + \frac{1}{2} \delta^{ij} \dot{w}_{\text{ext}}(\mathbf{x}_E) ,$$

$$C(t, \mathbf{x}) = -\frac{1}{10} r_E^2 (\dot{a}_E^i r_E^i) ; \quad Q^a = \delta_{ai} \left[\frac{\partial}{\partial x_i} w_{\text{ext}}(\mathbf{x}_E) - a_E^i \right] ;$$

The IAU 2000 framework

- More than two reference systems were defined!

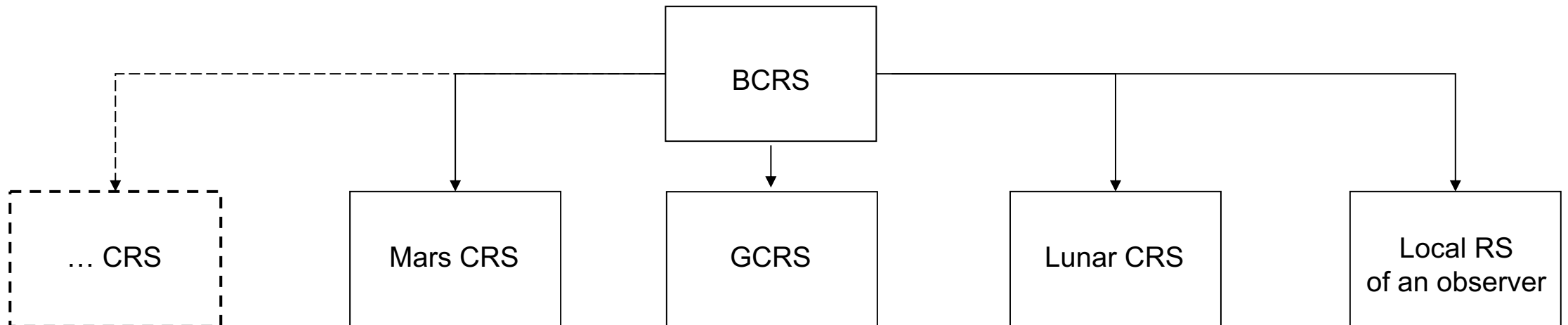
The same principles can and should be applied to any other body e.g. in the Solar system:

- **BCRS** (Barycentric Celestial Reference System)
- **GCRS** (Geocentric Celestial Reference System)
- XCRS: a physically optimal relativistic reference system for any body

in the Solar System (massive bodies or also massless observer): [IAU 2024 Resolution II](#)

“LCRS” for the Moon, “MarsCRS” for Mars, “MercuryCRS” for Mercury etc:

use exactly the same formulas as for the GCRS, but substitute body’s parameters instead of the Earth’s ones



Two kinds of times in General Relativity

1. Coordinate time:

one of the 4 coordinates of a 4-dimensional relativistic reference system

- defined for any 4D event in the region of space-time covered by that reference system
- different coordinate times can be transformed one into another (4D transformations!)
- examples: TCB (TDB) and TCG (TT, UTC, TAI)

$$T = t - \frac{1}{c^2} [A(t) + v_E^i r_E^i] + \frac{1}{c^4} [B(t) + B^i(t) r_E^i + B^{ij}(t) r_E^i r_E^j + C(t, \mathbf{x})]$$

BCRS position of the event

2. Proper time τ of an observer:

reading of an ideal clock located and moving together with the observer

- meaningful only for a specified observer: "obs"
- can be related to any coordinate time

$$\frac{d\tau}{dt} = \left(-g_{00}(t, \mathbf{x}_{obs}(t)) - \frac{2}{c} g_{0i}(t, \mathbf{x}_{obs}(t)) \dot{x}_{obs}^i(t) - \frac{1}{c^2} g_{ij}(t, \mathbf{x}_{obs}(t)) \dot{x}_{obs}^i(t) \dot{x}_{obs}^j(t) \right)^{1/2}$$

The reasons to introduce the scaled times TT and TDB

1. We scale TCG to TT to have "no drift" relative to an ideal clock **on the geoid**: $TT = (1-L_G) TCG$

$$\frac{d\tau}{dTT} = 1 - \frac{1}{c^2} \left(\text{"terms } \sim h, v^i \text{"} + \text{"tidal terms"} + \dots \right) + \dots$$

can be neglected in many cases

2. We scale TCB to TDB to have the same rate of TDB and TT **at the geocenter**: $TDB = (1-L_B) TCB + TDB_0$

At the geocenter one considers an *approximate* split: $TCG = TCB + \text{"secular drift"} + \text{"periodic terms"}$

Small differences "TT-TDB" considered at the geocenter or "close to it" can be ignored in many applications

Two time scales: **two** goals and **two** constants L_B and L_G to achieve these two goals...

Further consequences of scaled coordinate times

An attempt to trick the physics and “kill” the main relativistic effects (time dilation and the redshift):

dTT/dTDB is “almost 1” for events “close to” the geocenter

- this scaling must be accompanied by the scaling of spatial coordinates AND mass parameters:
TT-compatible and TDB-compatible values vs physical values (= TCB- or TCG-compatible)

See e.g. [Klioner, 2008, A&A, 478, 951](#)

- time

$$\text{BCRS} \\ \text{TDB} = t^* = F \cdot \text{TCB} + t_0^*$$

$$\text{GCRS} \\ T^{**} = TT = L \cdot \text{TCG}$$

- spatial coordinates

$$\mathbf{x}^* = F \cdot \mathbf{x}$$

$$X^{**} = L \cdot X$$

- masses ($\mu = GM$) of each body

$$\mu^* = F \cdot \mu$$

$$\mu^{**} = L \cdot \mu$$

$$F = 1 - L_B$$

$$L = 1 - L_G$$

Quite some risk to mess up things in high precision modelling!

Scaled times at work: three masses of the Earth

The mass parameters $\mu = GM$ are the **same in non-scaled** BCRS and GCRS, but not the same with the scaled versions

The mass of the Earth:

TCB/TCG-compatible $\mu_{\oplus}^{**} = (398600441.\underline{5} \pm 0.4) \times 10^6$

TT-compatible $\mu_{\oplus} = (1 - L_G)^{-1} \mu_{\oplus}^{**} = (398600441.\underline{8} \pm 0.4) \times 10^6$

TDB-compatible $\mu_{\oplus}^* = (1 - L_B) \mu_{\oplus} = (398600435.\underline{6} \pm 0.4) \times 10^6$

Each additional scaled coordinate time will add one more value of the Earth mass and other parameters as well as one more set of scaled spatial coordinates!

A lot of risk to mess up things in high precision modelling!

With three coordinate times linear drifts persist!

The ideas behind the scaling of the coordinate times TCG and TCB:

1. We scale TCG to TT to have "no drift" relative to an ideal clock at the geoid: $TT = (1-L_G) TCG$
2. We scale TCB to TDB to have the same rate of TDB and TT at the geocenter: $TDB = (1-L_B) TCB + TDB_0$

Two time scales, two goals and two constants L_B and L_G to achieve these two goals...

Consider one additional time TCL for the Moon. A scaled version would be: $TL = (1-L_L) TCL$

Whichever L_L is used, one cannot remove all linear drifts:

TL has a linear drift either relative to TDB or relative to the proper time of a clock on a lunar "geoid"

One cannot kill two drifts with one constant L_L !

BUT: If $L_L \neq 0$, one gets an additional set of non-physical values of constants and spatial coordinates!

A common misconception with the coordinate time scales

All these statements are **WRONG**:

- TCB is "the time in the barycenter of the solar system"
- TCG is "the time in the geocenter"
- TCL is "the time at the center of the Moon"
- TT is "the time on the geoid"

CORRECT: all these time scales are defined for any location (event) within the Solar system!

A real clock put at the barycenter of the solar system would not show TCB (also the rate would be different)!
The same is true for TCG and the geocenter as well as TCL and the center of the Moon.

Possible misconception for the units of measurements

- The **misconception**: Only with the scaled time TT we use the SI second, i.e. “SI second on a geoid”
- The root of this confusion is the formulation in the old CCDS Resolutions on TAI (1980-1981)

“TAI is a coordinate time scale defined in a geocentric reference frame with the SI second as realized on the rotating geoid as the scale unit”
- SI second is a unit of proper time, defined as a **procedure** to implement it and is independent on the position or velocity of the observer and equipment that implement it!

Think about an accurate clock on a spacecraft (e.g. ACES!): that clock also realizes the SI second!

Possible misconception for the units of measurements

We always use the SI units with TCG, TCB, TT, TCB and any other quantities!

Relativistic formulas like

$$T = t - \frac{1}{c^2} [A(t) + v_E^i r_E^i] + \frac{1}{c^4} [B(t) + B^i(t) r_E^i + B^{ij}(t) r_E^i r_E^j + C(t, \mathbf{x})]$$

relate the **quantities**, not **values**.

If the same formulas are used for numerical **values** the same units should be used on the left- and right-hand side of these equations!

See [Klioner, Capitaine, Folkner, Guinot et al. 2010](#)

One more possible misconception: “traceability to UTC”

No precise clock around the Moon

↓ (correct)

We must use Earth-bound clock to monitor the clocks near the Moon

↓ (wrong!)

We can only realize an Earth-bound time (TT, UTC, TCG) with those Moon clocks

- any clock measures its proper time (plus technical errors);
no clock measures coordinate times like TT (UTC, TAI), TCB, TCG, ...
- clock reading must be **recomputed** into some coordinate time, before we can compare two or more clocks at different locations (the concept of coordinate simultaneity – [Allan, Ashby 1979](#))
- To which coordinate time scale? To **ANY** we wish!

Conclusions for the Lunar Time

1. As proposed in the IAU 2024 Resolution 2:
 - define Lunar CRS (LCRS) in the IAU 2000 framework (all formulas are readily available)
 - TCL (Lunar Coordinate Time) is an integral part of LCRS
2. There is no scaling of TCL that removes linear drifts between all relevant coordinate and proper times
3. Introducing new ad-hoc scaling factors would imply new additional values for the mass parameters and spatial coordinates:
one additional set of constants and spatial coordinates for each scaled coordinate time!

➔ Don't scale TCL at all: $TL = TCL (+ \text{const})$

Backup slides

Linear drifts between time scales

Pair	Drift per year (seconds)	Difference at J2023 (seconds)
TT-TCG	0.021993	1.011678
TDB-TCB	0.489307	22.508122
TCB-TCG @ geocenter	0.467313	21.496398

Other coordinate times: to scale or not to scale?

One can imagine several solution for the other bodies like Moon, Mars or Mercury:

1. Scale TCX with the SAME scaling factor as used for TT: $TX = (1 - L_G) TCX$

- ✓ no additional values of mass parameters or spatial coordinates: TX-compatible $\equiv$ TT-compatible
- ✗ significant linear drift of TX relative to TDB (at the center of body X)
- ✗ TX has a linear drift relative to the clocks on the body-centered analogue of geoid

2. Scale TCX with the SAME scaling factor as used for TDB: $TX = (1 - L_B) TCX$

- ✓ no additional values of mass parameters or spatial coordinates: TX-compatible $\equiv$ TDB-compatible
- ✗ significant linear drift of TX relative to TDB (at the center of body X)
- ✗ TX has a linear drift relative to the clocks on the body-centered analogue of geoid

Godard, Budnik, Morley, Lopez, 2012 ← Analysis for BepiColombo around Mercury

Other coordinate times: to scale or not to scale?

3. Scale TCX with a special factor to remove the main part of the linear drift to TDB: $TX = (1 - L_X) TCX$

- ✗ new values of mass parameters and spatial coordinates: TX-compatible (one new for each body!)
- ✓ practically negligible linear drift of TX relative to TDB (at the center of body X)
- ✗ TX has a linear drift relative to the clocks on the body-centered analogue of geoid

4. Scale TCX with a special factor to remove the linear drift relative to the ideal clocks on the body-centered analogue of geoid: $TX = (1 - L_{XX}) TCX$

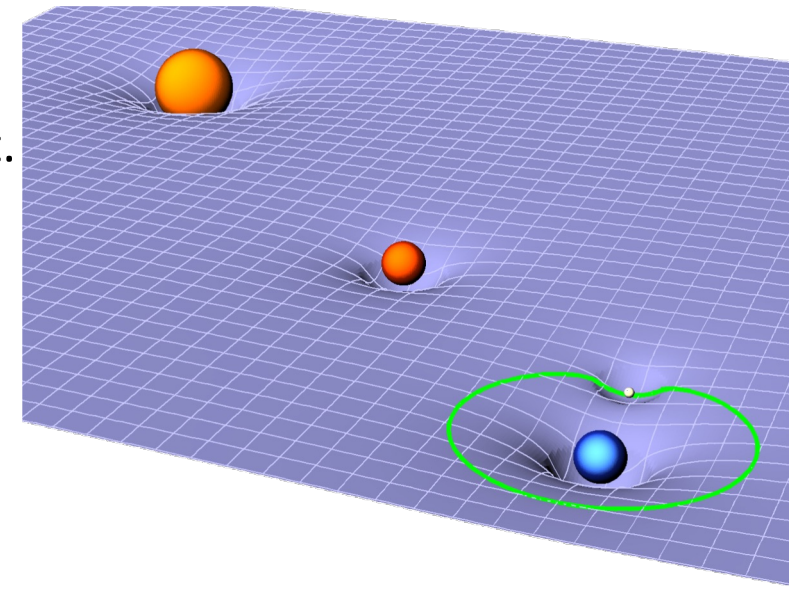
- ✗ new values of mass parameters and spatial coordinates: TX-compatible (one new for each body!)
- ✗ significant linear drift of TX relative to TDB (at the center of body X)
- ✓ TX has NO linear drift relative to the clocks on the body-centered analogue of geoid

5. Don't scale at all: $TX = TCX$

- ✓ no additional values of mass parameters or spatial coordinates: TX-compatible $\equiv$ TCB/TCG-compatible
- ✗ significant linear drift of TX relative to TDB (at the center of body X)
- ✗ TX has a linear drift relative to the clocks on the body-centered analogue of geoid

Reference Systems in General Relativity

- Relativistic reference systems are coordinate charts that can be used to assign 4 numbers
 - coordinate time and three spatial coordinates –to any event in the region of space-time covered by that reference system.
- Relativistic reference systems are defined in the mathematical language of General Relativity:
 - by the components of the metric tensor in these particular coordinates.
- One can use any reference system, but some of them are more convenient.
- One can find transformations between the coordinates in different reference systems.



Conclusions for the relativistic times

1. As proposed in the IAU 2024 Resolution 2:
 - Define Lunar CRS (LCRS) in the IAU 2000 framework (all formulas are readily available)
 - TCL (Lunar Coordinate Time) is an integral part of LCRS
2. Coordinate time scales for solar system any body can be easily computed numerically:
e.g. <https://gaia.geo.tu-dresden.de/TimeEphemerides/>
3. Proper time of any clock can be recomputed into any time scale (and back)
4. There is no scaling of TCL or TCX that removes linear drifts between all relevant coordinate times
5. Introducing new ad-hoc scaling factors will produce new additional values
for the mass parameters and spatial coordinates (one set of many constants for each body!)

Conclusions for the relativistic times

1. As proposed in the IAU 2024 Resolution 2:
 - Define Lunar CRS (LCRS) in the IAU 2000 framework (all formulas are readily available)
 - TCL (Lunar Coordinate Time) is an integral part of LCRS
2. Coordinate time scales for solar system any body can be easily computed numerically:
e.g. <https://gaia.geo.tu-dresden.de/TimeEphemerides/>
3. Proper time of any clock can be recomputed into any time scale (and back)
4. There is no scaling of TCL or TCX that removes linear drifts between all relevant coordinate times
5. Introducing new ad-hoc scaling factors will produce new additional values
for the mass parameters and spatial coordinates (one set of many constants for each body!)

Backup slides

Proper time scales and TCG

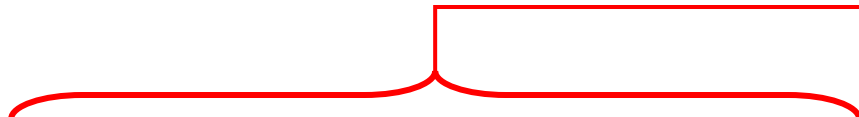
- Specially interesting case: an observer close to the Earth surface:

$$\frac{d\tau}{dT} = 1 - \frac{1}{c^2} \left(\frac{1}{2} \dot{X}_{obs}^2(T) + W_E(T, \mathbf{X}_{obs}) + \text{"tidal terms"} \right) + O(c^{-4})$$


$\sim 10^{-17}$

- Idea:

$$TT = (1 - L_G) TCG, \quad L_G \equiv 6.969290134 \times 10^{-10}$$


$$\frac{d\tau}{dTT} = 1 - \frac{1}{c^2} \left(\text{"terms"} \sim h, v^i + \text{"tidal terms"} + \dots \right) + \dots$$

can be neglected
in many cases

h is the height above the geoid

v^i is the velocity relative to the rotating geoid

Coordinate Time Scales: TT

- **Idea:** let us define a time scale linearly related to $T=TCG$, but which is numerically close to the proper time of an observer on the geoid:

$$TT = (1 - L_G) TCG, \quad L_G \equiv 6.969290134 \times 10^{-10}$$

$$\frac{d\tau}{dTT} = 1 - \frac{1}{c^2} \left(\text{"terms } \sim h, v^i \text{"} + \text{"tidal terms"} + \dots \right) + \dots$$

can be neglected
in many cases

- To avoid errors and changes in TT implied by changes/improvements in the geoid, the IAU (2000) has made L_G to be a **defined constant**:

$$L_G \equiv 6.969290134 \times 10^{-10}$$

- **TAI is a practical realization of TT** (up to a constant shift of 32.184 s)
- Older name TDT (introduced by IAU 1976) is fully equivalent to TT

Coordinate Time Scales: TDB

- **Idea:** to scale TCB in such a way that the “scaled TCB” remains close to TT

The IAU has [re-]defined TDB to be fixed linear function of TCB:

$$TDB = TCB - L_B \times (JD_{TCB} - T_0) \times 86400 + TDB_0$$

- TDB to be defined through a conventional relationship with TCB:
 - $T_0 = 2443144.5003725$ exactly,
 - $JD_{TCB} = T_0$ for the event 1977 Jan 1.0 TAI at the geocenter and increases by 1.0 for each 86400s of TCB,
 - $L_B \equiv 1.550519768 \times 10^{-8}$,
 - $TDB_0 \equiv -6.55 \times 10^{-5} \text{ s}$.

Scaled BCRS: not only time is scaled

- If one uses scaled version TCB – T_{eph} or TDB – one effectively uses three scaling:

- time

$$t^* = F \cdot TCB + t_0^*$$

- spatial coordinates

$$\mathbf{x}^* = F \cdot \mathbf{x}$$

- masses ($\mu = GM$) of each body

$$\mu^* = F \cdot \mu$$

$$F = 1 - L_B$$

WHY THREE SCALINGS?

Scaled BCRS

- These **three** scalings together leave the dynamical equations unchanged:

- for the motion of the solar system bodies:

$$\begin{aligned}\ddot{\mathbf{x}}_A = & - \sum_{B \neq A} \mu_B \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|^3} \\ & + \frac{1}{c^2} \sum_{B \neq A} \mu_B \frac{\mathbf{r}_{AB}}{|\mathbf{r}_{AB}|^3} \left\{ \sum_{C \neq B} \frac{\mu_C}{|\mathbf{r}_{BC}|} + 4 \sum_{C \neq A} \frac{\mu_C}{|\mathbf{r}_{AC}|} + \frac{3}{2} \frac{(\mathbf{r}_{AB} \cdot \dot{\mathbf{x}}_B)^2}{|\mathbf{r}_{AB}|^2} \right. \\ & \quad \left. - \frac{1}{2} \sum_{C \neq A, B} \mu_C \frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{BC}}{|\mathbf{r}_{BC}|^3} \right. \\ & \quad \left. - 2 \dot{\mathbf{x}}_B \cdot \dot{\mathbf{x}}_B - \dot{\mathbf{x}}_A \cdot \dot{\mathbf{x}}_A + 4 \dot{\mathbf{x}}_A \cdot \dot{\mathbf{x}}_B \right\} \\ & + \frac{1}{c^2} \sum_{B \neq A} \mu_B \frac{\dot{\mathbf{x}}_A - \dot{\mathbf{x}}_B}{|\mathbf{r}_{AB}|^3} \left\{ 4 \dot{\mathbf{x}}_A \cdot \mathbf{r}_{AB} - 3 \dot{\mathbf{x}}_B \cdot \mathbf{r}_{AB} \right\} \\ & - \frac{1}{c^2} \frac{7}{2} \sum_{B \neq A} \frac{\mu_B}{|\mathbf{r}_{AB}|} \sum_{C \neq A, B} \mu_C \frac{\mathbf{r}_{BC}}{|\mathbf{r}_{BC}|^3} + O(c^{-4}),\end{aligned}$$

- for light propagation:

$$\begin{aligned}c(t_2 - t_1) = & |\mathbf{x}_2 - \mathbf{x}_1| \\ & + \sum_A \frac{2\mu_A}{c^2} \ln \frac{|\mathbf{r}_{1A}| + |\mathbf{r}_{2A}| + |\mathbf{r}_{21}|}{|\mathbf{r}_{2A}| + |\mathbf{r}_{1A}| - |\mathbf{r}_{21}|} + O(c^{-4}),\end{aligned}$$

Scaled GCRS

- If one uses TT being a scaled version TCG one effectively uses three scaling:

- time

$$T^{**} = TT = L \cdot TCG$$

- spatial coordinates

$$X^{**} = L \cdot X$$

- masses of each body

$$\mu^{**} = L \cdot \mu$$

$$L = 1 - L_G$$

- International Terrestrial Reference Frame (ITRF) uses such scaled GCRS coordinates and quantities
- Note that the masses are the **same in non-scaled** BCRS and GCRS...

Scaled masses

The mass [parameters] $\mu = GM$ are the **same in non-scaled** BCRS and GCRS, but not the same with the scaled versions

scaled BCRS (with TDB) $\mu^* = F \cdot \mu, \quad F = 1 - L_B$

scaled GCRS (with TT) $\mu^{**} = L \cdot \mu, \quad L = 1 - L_G$

The mass of the Earth:

TT-compatible $\mu_{\oplus}^{**} = (\underline{398600441.5 \pm 0.4}) \times 10^6$

TCB/G-compatible $\mu_{\oplus} = (1 - L_G)^{-1} \mu_{\oplus}^{**} = (\underline{398600441.8 \pm 0.4}) \times 10^6$

TDB-compatible $\mu_{\oplus}^* = (1 - L_B) \mu_{\oplus} = (\underline{398600435.6 \pm 0.4}) \times 10^6$

Backup slides

The IAU 1991 Resolution A4

XXIst General Assembly, Buenos Aires, Argentina

- Discussing relativistic reference systems for the Moon and other bodies we don't do anything new!
- IAU 1991 Resolution A4:
 - laid foundation for relativistic modelling in astronomy
 - was then drastically extended by the IAU 2000 framework
 - already discussed relativistic Barycentric and Geocentric Reference Systems and “analogous coordinate systems for other planets and for the Moon”
 - contain very precise formulations for the units of measurements of the spatial and time coordinates (this will be discussed separately)

Relativistic reference system in “cislunar space”

- “CISLUNAR” is lying between the Earth and the Moon or the Moon's orbit
- Which relativistic reference system to use?

In principle, BCRS, GCRS and LCRS all cover the cislunar space, but

- the Geocentric CRS (GCRS) is adequate for the vicinity of the Earth (physics of the Earth, Earth satellites)
 - the Lunar CRS (LCRS) is adequate for the vicinity of the Moon (physics of the Moon, Moon satellites)
 - the BCRS can be used safely (and is being used to model the geocentric motion of the Moon!)
(the normal Newtonian shift of the origin can of course be done)
- Is it possible to define something “better”, more specific for the cislunar space, as BCRS.

In principle “yes”, but we don’t do this and we don’t need this:

this would be against established procedures in solar system ephemerides and space dynamics!

Transformations of coordinate time scales in General Relativity

Coordinate time is one of the 4 coordinates of some 4-dimensional relativistic reference system

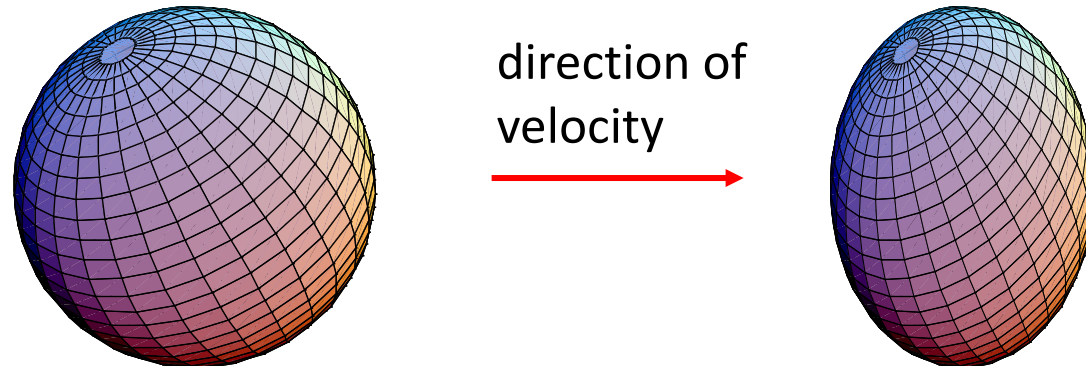
- defined for any 4-dimensional event in the region of space-time where the reference system is defined
- examples: TCB (TDB) and TCX (any TCG-like: TCG, TCL, TT, UTC, TAI,)

$$T = t - \frac{1}{c^2} [A(t) + v_E^i r_E^i] + \frac{1}{c^4} [B(t) + B^i(t) r_E^i + B^{ij}(t) r_E^i r_E^j + C(t, \mathbf{x})]$$

BCRS position of the event

What if one ignores/neglects the position-dependent terms?

E.g. the Lorentz-contraction effects and their gravitational counterparts would make the shape of the central body variate depending on its barycentric velocity!



Numerical computation of the TCG-like time ephemerides

For any GCRS-like reference system for an arbitrary body 'X' ('XCRS') the transformation between 'TCX' and TCB evaluated at the origin of XCRS is the given by the same formulas:

u = TCX is the TCG-like time scale for a body 'X' **at its center of mass**

t = TCB

$$\frac{du}{dt} = 1 + f(t)$$

$$f = -\frac{1}{c^2} \dot{A}(t) + \frac{1}{c^4} \dot{B}(t)$$

$$\begin{aligned} \dot{A}(t) &= \frac{1}{2} v_X^2 + \sum_{A \neq X} \frac{GM_A}{r_{XA}} \\ \dot{B}(t) &= -\frac{1}{8} v_X^4 + \frac{1}{2} \left(\sum_{A \neq X} \frac{GM_A}{r_{XA}} \right)^2 + \sum_{A \neq X} \left(\frac{GM_A}{r_{XA}} \sum_{B \neq A, X} \frac{GM_B}{r_{AB}} \right) \\ &\quad + \sum_{A \neq X} \frac{GM_A}{r_{XA}} \left(4 v_A^i v_X^i - \frac{3}{2} v_X^2 - 2 v_A^2 + \frac{1}{2} a_A^i r_{XA}^i \right. \\ &\quad \left. + \frac{1}{2} (v_A^i r_{XA}^i / r_{XA})^2 \right) \end{aligned}$$

$$\mathbf{r}_{XA} = \mathbf{x}_X - \mathbf{x}_A$$

Numerical computation of the TCG-like time ephemerides

u = TCX is the TCG-like time scale for a body 'X' at its center of mass

t = TCB

$$\begin{aligned} \frac{du}{dt} &= 1 + f(t) \\ f &= -\frac{1}{c^2} \dot{A}(t) + \frac{1}{c^4} \dot{B}(t) \end{aligned} \Rightarrow \begin{aligned} u &= t + \delta t(t) \\ t &= u - \delta u(u) \end{aligned} \Rightarrow \begin{aligned} \frac{d\delta t}{dt} &= f(t) \\ \frac{d\delta u}{du} &= \frac{f(u - \delta u(u))}{1 + f(u - \delta u(u))} \\ \delta t(t_0) &= 0 \\ \delta u(u_0) &= 0 \end{aligned}$$

Two ordinary differential equations with the initial conditions (e.g. $t_0 = 1$ Jan 1977).

Very easy to integrate numerically given a solar system ephemeris and store the result as a set of Chebyshev polynomials: the time ephemerides for each body 'X'

Time Ephemerides TCX19a: <https://gaia.geo.tu-dresden.de/TimeEphemerides/>

- The numerical integrations were computed using the exported version of INPOP19a over 200 years centered on J2000
- For completeness the transformations were computed and published for all bodies from INPOP19a:

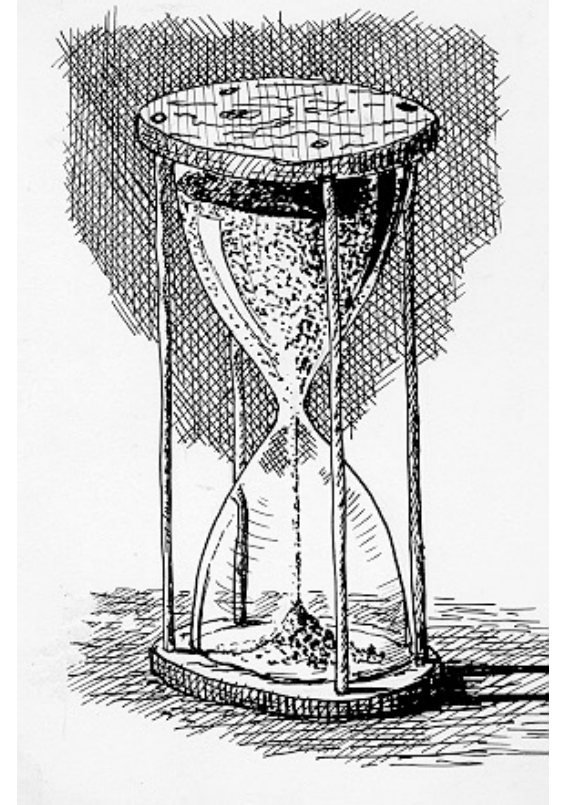
Sun, Mercury, Venus, Earth, Moon, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto

- The transformation from TCX to TCB and back are given for each body:

deltaTCB(TCB) as in $TCX = TCB + \text{deltaTCB}(TCB)$

deltaTCX(TCX) as in $TCB = TCX + \text{deltaTCX}(TCX)$

- The standard ASCII files with Chebyshev polynomials are available



Time Ephemerides TCX19a: <https://gaia.geo.tu-dresden.de/TimeEphemerides/>

Time ephemerides for the major solar system bodies

Sergei A. Klioner
Lohrmann Observatory, Technische Universität Dresden, 01062 Dresden, Germany

This web page contains numerical time ephemerides for the Moon and the major planets of the Solar System.

The time ephemerides for each body represent the time transformation between the coordinate time TCX of the local GCRS-like relativistic reference system of the corresponding body X and the coordinate time TCB of the BCRS.

The original GCRS (Geocentric Celestial Reference System) was defined for the Earth and adopted by the IAU back in 2000. For the Moon, the GCRS-like coordinate system is called LCRS (Lunar Celestial Reference System) and it was recommended by the IAU Resolution II of 2024 "to establish a standard Lunar Celestial Reference System (LCRS) and Lunar Coordinate Time (TCL)": <https://iau.org/Iau/Publications/List-of-Resolutions>. For the Moon then, the time ephemeris published here represents a numerical transformation between TCL and TCB evaluated in the center of gravity of the Moon.

As it was always intended for the GCRS and as the cited IAU Resolution states, similar GCRS-like relativistic reference systems and coordinate times can be defined for any body of the Solar System. This framework is assumed here. Analogous to TCG and TCL, the numerical transformations between between the coordinate times TCX of other solar system bodies are computed. The notation used for these coordinate time are given in the tables below.

The physical background of the relativistic reference systems on the example of BCRS and GCRS can be found in ([Soffel, Klioner, Petit et al. 2003](#)) and references therein. The description of the formulas used in the calculations of the time ephemerides are published in (Klioner, Gerlach, Soffel, 2010, Section 6) or, in much more details, in the publicly accessible Technical Note ([Klioner, 2025](#)) discussing the time framework of *Gaia*. In particular, two functions for each body are given here:

- Function $\text{deltaTCB}(\text{TCB})$ from the transformation from TCB to TCX: $\text{TCX} = \text{TCB} + \text{deltaTCB}(\text{TCB})$
- Function $\text{deltaTCX}(\text{TCX})$ from the transformation from TCX to TCB: $\text{TCB} = \text{TCX} - \text{deltaTCX}(\text{TCX})$

The time ephemerides published here were created using the export version of the INPOP19a ephemeris that can be downloaded from

From TCB to TCX at the gravity center of the body X ([the whole directory](#)):

body	notation	time ephemeris file	plot
Sun	TCSun	tcx19a_TCSun_m100_p100_asc_pos_TCB.asc	deltaTCB-Sun.png
Mercury	TCMer	tcx19a_TCMer_m100_p100_asc_pos_TCB.asc	deltaTCB-Mer.png
Venus	TCVen	tcx19a_TCVen_m100_p100_asc_pos_TCB.asc	deltaTCB-Ven.png
Earth	TCG	tcx19a_TCG_m100_p100_asc_pos_TCB.asc	deltaTCB-G.png
Moon	TCL	tcx19a_TCL_m100_p100_asc_pos_TCB.asc	deltaTCB-L.png
Mars	TCMar	tcx19a_TCMar_m100_p100_asc_pos_TCB.asc	deltaTCB-Mar.png
Jupiter	TCJup	tcx19a_TCJup_m100_p100_asc_pos_TCB.asc	deltaTCB-Jup.png
Saturn	TCSat	tcx19a_TCSat_m100_p100_asc_pos_TCB.asc	deltaTCB-Sat.png
Uranus	TCUra	tcx19a_TCUra_m100_p100_asc_pos_TCB.asc	deltaTCB-Ura.png
Neptune	TCNep	tcx19a_TCNep_m100_p100_asc_pos_TCB.asc	deltaTCB-Nep.png
Pluto	TCPlu	tcx19a_TCPlu_m100_p100_asc_pos_TCB.asc	deltaTCB-Plu.png

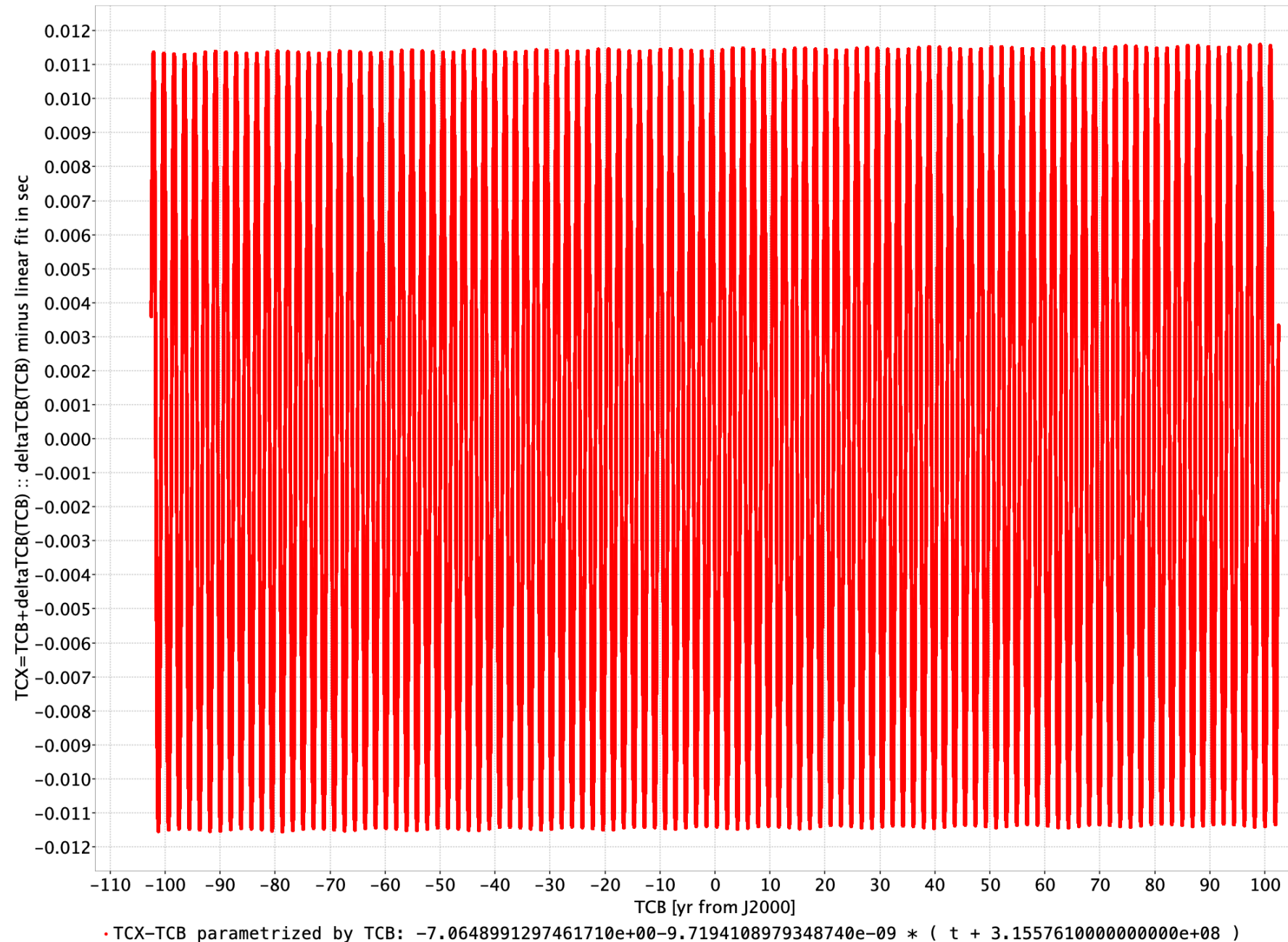
From TCX to TCB at the gravity center of the body X ([the whole directory](#)):

body	notation	time ephemeris file	plot
Sun	TCSun	tcx19a_TCB_m100_p100_asc_pos_TCSun.asc	deltaTCSun.png
Mercury	TCMer	tcx19a_TCB_m100_p100_asc_pos_TCMer.asc	deltaTCMer.png
Venus	TCVen	tcx19a_TCB_m100_p100_asc_pos_TCVen.asc	deltaTCVen.png
Earth	TCG	tcx19a_TCB_m100_p100_asc_pos_TCG.asc	deltaTCG.png
Moon	TCL	tcx19a_TCB_m100_p100_asc_pos_TCL.asc	deltaTCL.png
Mars	TCMar	tcx19a_TCB_m100_p100_asc_pos_TCMar.asc	deltaTCMar.png
Jupiter	TCJup	tcx19a_TCB_m100_p100_asc_pos_TCJup.asc	deltaTCJup.png
Saturn	TCSat	tcx19a_TCB_m100_p100_asc_pos_TCSat.asc	deltaTCSat.png
Uranus	TCUra	tcx19a_TCB_m100_p100_asc_pos_TCUra.asc	deltaTCUra.png
Neptune	TCNep	tcx19a_TCB_m100_p100_asc_pos_TCNep.asc	deltaTCNep.png
Pluto	TCPlu	tcx19a_TCB_m100_p100_asc_pos_TCPlu.asc	deltaTCPlu.png

Example: Mars and its “TCMar”

For each body we also publish plots of the differences between TCX and TCB.

Both the results of a linear fit and the quasi-periodic rest are shown:



The main characteristics of the TCX for other bodies

Body	Approximate linear drift [$\times 10^{-8}$]	Maximal amplitude of periodic terms [ms]
Sun	0.000222	0.0076
Mercury	3.825	12.7
Venus	2.047	0.60
Earth	1.481 ($\approx L_C$)	1.69
Moon	1.483	1.80
Mars	0.972	11.5
Jupiter	0.285	11
Saturn	0.155	18
Uranus	0.077	22
Neptune	0.049	5.5
Pluto	0.043	~ 100

Results from numerical
integrations using
INPOP19a

over about 200 years:

- very different drifts
and periodic terms
for different bodies!

- no way to define the linear
drift exactly!

it is always an approximation
depending on the exact
receipt how the linear fit
is computed!

TCG from our numerical integration and INPOP19a

The INPOP19a ephemeris includes the TCG-TCB transformation computed with the full dynamical model.

Here we can see the limitation of our simplified dynamical model consisting only of the 11 (major) bodies in the export version of INPOP19a: <https://www.imcce.fr/recherche/equipes/asd/inpop/download19a>

Direct numerical comparison shows

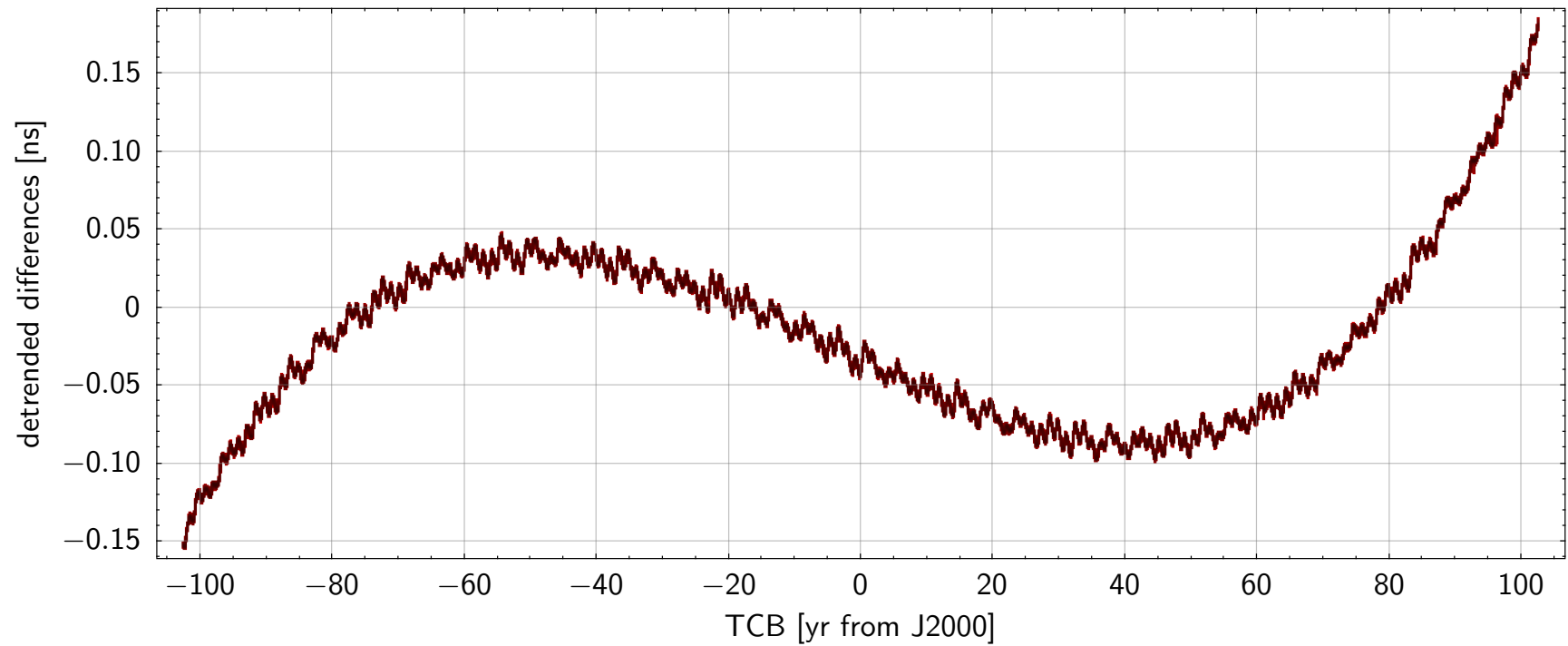
- the deviation in the linear drift of $\sim \mathbf{6.19 \times 10^{-18}}$ (no constant term is fitted!)
- the long-term quasi-periodic effect with an amplitude of $\sim \mathbf{0.15 \text{ nanosecond}}$ and a period around 180 yr
- the quicker quasi-periodic terms with an amplitude of a few picoseconds

Note:

The fitted linear drift differs from

$$L_C \equiv 1 - (1 - L_B)/(1 - L_G)$$

by $\sim \mathbf{2.8 \times 10^{-15}}$



Time ephemerides for arbitrary ideal clocks

Proper time τ of any clock can be recomputed into any coordinate time scale (and back) as soon as the trajectory of a clock is known!

The very same formulas for “XCRF” can be used to this transformation!

Assuming the trajectory in BCRS is known: $\mathbf{x}_o$ and $\mathbf{v}_o$ are the position and velocity; $t = \text{TCB}$

$$\frac{d\tau}{dt} = 1 + f(t)$$
$$f = \frac{1}{c^2} \alpha(t) + \frac{1}{c^4} \beta(t)$$

In general: $f(t)$ is a function of the metric tensor of the corresponding XCRS and the trajectory of the clock.

$$\alpha = -\frac{1}{2} v_o^2 - \sum_A \frac{GM_A}{r_{oA}}$$
$$\beta = -\frac{1}{8} v_o^4 + \frac{1}{2} \left(\sum_A \frac{GM_A}{r_{oA}} \right)^2 + \sum_A \left(\frac{GM_A}{r_{oA}} \sum_{B \neq A} \frac{GM_B}{r_{AB}} \right)$$
$$+ \sum_A \frac{GM_A}{r_{oA}} \left(4v_A^i v_o^i - \frac{3}{2} v_o^2 - 2v_A^2 + \frac{1}{2} a_A^i r_{oA}^i \right. \\ \left. + \frac{1}{2} (v_A^i r_{oA}^i / r_{oA})^2 \right)$$

$$\mathbf{r}_{oA} = \mathbf{x}_o - \mathbf{x}_A$$

Time ephemerides for arbitrary (ideal clock)

The very same formulas can be used to compute the relation of the “proper time” τ (reading of an ideal clock) as soon as the trajectory of a clock in BCRS, GCRS or any XCRS is known.

Assuming the trajectory in BCRS is known: $\mathbf{X}_O$ and $\mathbf{V}_O$ are the position and velocity; $t = \text{TCB}$

$$\frac{d\tau}{dt} = 1 + f(t)$$
$$f = \frac{1}{c^2} \alpha(t) + \frac{1}{c^4} \beta(t)$$

$\Rightarrow$

$$\tau = t + \delta t(t)$$
$$t = \tau - \delta \tau(\tau)$$

$\Rightarrow$

$$\frac{d\delta t}{dt} = f(t)$$
$$\frac{d\delta \tau}{d\tau} = \frac{f(\tau - \delta \tau(\tau))}{1 + f(\tau - \delta \tau(\tau))}$$

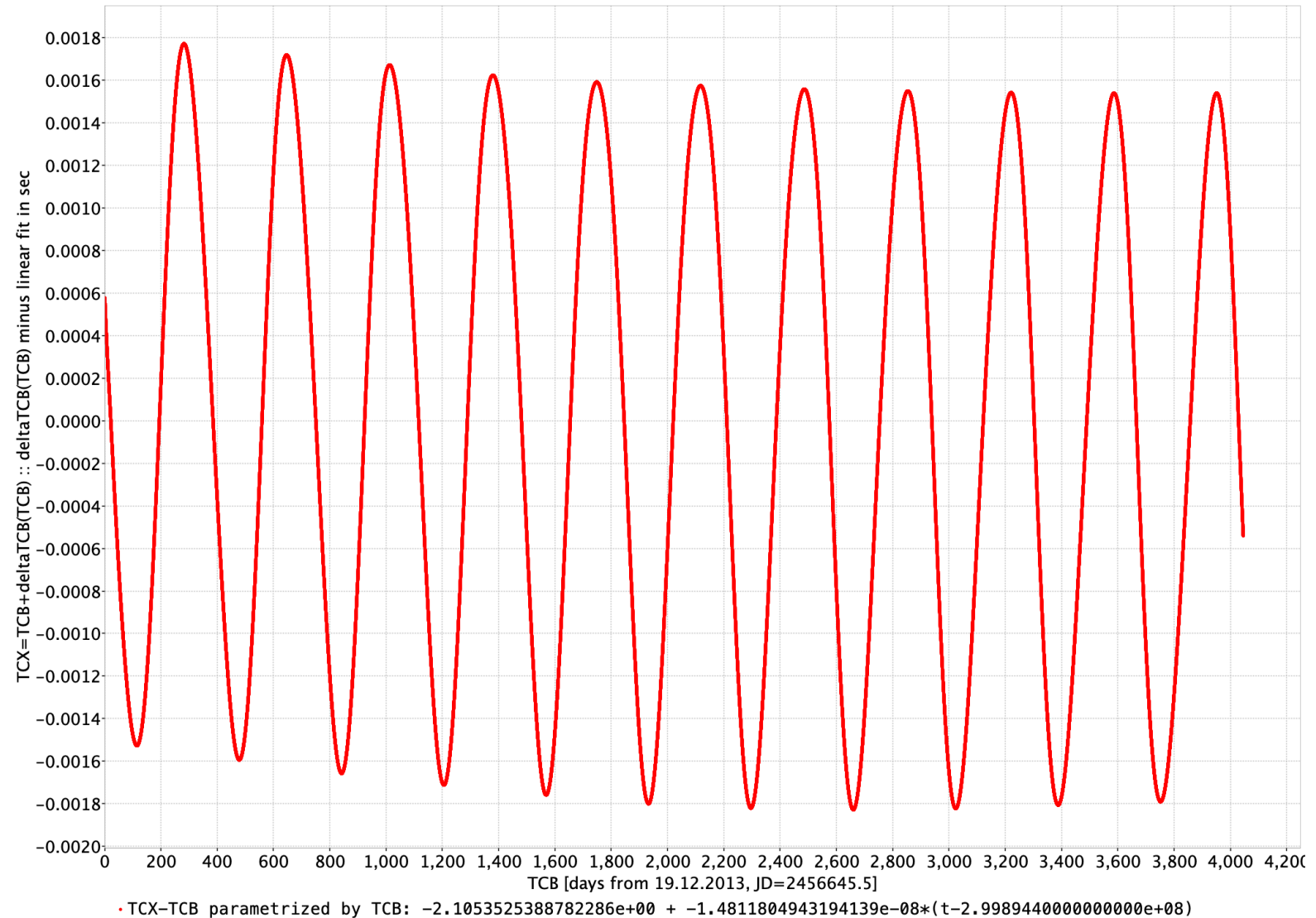
$$\delta t(t_0) = 0$$

$$\delta \tau(\tau_0) = 0$$

Two ordinary differential equations with the initial conditions which should be given at some t_0 for which the ephemeris of clock is known: $\tau(t_0) = \tau_0 = t_0$

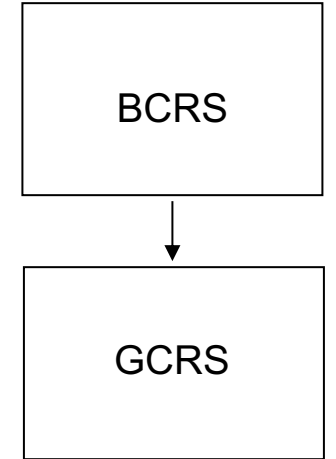
Time ephemeris for Gaia Rb clock

- TG is the proper time of Gaia satellite:
TG="TCX", for X=Gaia
- the same INPOP19a ephemeris is used
- the orbit is known in the period from 2014 to 2025 (a Lissajous orbit around L_2 of the Sun-Earth system)
- TG is defined to coincide with TCB at JD2457023.5 TCB
- the same representation as for all TCX



The IAU 2000 framework

- Two standard astronomical reference systems were defined
 - **BCRS** (Barycentric Celestial Reference System)
 - **GCRS** (Geocentric Celestial Reference System)
- These reference systems are defined by
 - the form of the corresponding metric tensors
 - the 4D coordinate transformations between BCRS and GCRS



$$\begin{aligned}
 g_{00} &= -1 + \frac{2}{c^2} w(t, \mathbf{x}) - \frac{2}{c^4} w^2(t, \mathbf{x}) + O(c^{-5}), & G_{00} &= -1 + \frac{2}{c^2} W(T, \mathbf{X}) - \frac{2}{c^4} W^2(T, \mathbf{X}) + O(c^{-5}), \\
 g_{0i} &= -\frac{4}{c^3} w^i(t, \mathbf{x}) + O(c^{-5}), & G_{0a} &= -\frac{4}{c^3} W^a(T, \mathbf{X}) + O(c^{-5}), \\
 g_{ij} &= \delta_{ij} \left(1 + \frac{2}{c^2} w(t, \mathbf{x}) \right) + O(c^{-4}). & G_{ab} &= \delta_{ab} \left(1 + \frac{2}{c^2} W(T, \mathbf{X}) \right) + O(c^{-4}).
 \end{aligned}$$

$$\begin{aligned}
 W_E &= G \sum_{l=0}^{\infty} \frac{(-1)^l}{l!} \left[M_L \partial_L \frac{1}{|\mathbf{X}|} + \frac{1}{2c^2} \ddot{M}_L \partial_L |\mathbf{X}| \right] + \frac{4}{c^2} \lambda_{,T} + O(c^{-4}), \\
 W_E^a &= -G \sum_{l=1}^{\infty} \frac{(-1)^l}{l!} \left[\dot{M}_{aL-1} \partial_{L-1} \frac{1}{|\mathbf{X}|} + \frac{l}{l+1} \epsilon_{abc} S_{cL-1} \partial_{bL-1} \frac{1}{|\mathbf{X}|} \right] - \lambda_{,a} + O(c^{-2})
 \end{aligned}$$

$$w(t, \mathbf{x}) = G \int d^3 x' \frac{\sigma(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} + \frac{1}{2c^2} G \frac{\partial^2}{\partial t^2} \int d^3 x' \sigma(t, \mathbf{x}') |\mathbf{x} - \mathbf{x}'|,$$

$$w^i(t, \mathbf{x}) = G \int d^3 x' \frac{\sigma^i(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|},$$

$$\sigma = (T^{00} + T^{kk}) / c^2, \quad \sigma^i = T^{0i} / c, \quad T^{\mu\nu} \text{ is the BCRS energy-momentum tensor}$$

$$M_L(T) \equiv \int_E d^3 X \hat{X}^L \Sigma + \frac{1}{2(2l+3)c^2} \frac{d^2}{dT^2} \int_E d^3 X \hat{X}^L \mathbf{X}^2 \Sigma - \frac{4(2l+1)}{(l+1)(2l+3)c^2} \frac{d}{dT} \int_E d^3 X \hat{X}^{aL} \Sigma^a,$$

$$S_L(T) \equiv \int_E d^3 X \epsilon^{ab < c_l} \hat{X}^{L-1 > a} \Sigma^b,$$

$$\Sigma = (\mathfrak{T}^{00} + \mathfrak{T}^{aa}) / c^2, \quad \Sigma^a = \mathfrak{T}^{0a} / c,$$

$$\mathfrak{T}^{\alpha\beta} \text{ is the GCRS energy-momentum tensor}$$

The proposal for the IAU 2024 Resolution 2

Resolution to establish a standard Lunar Celestial Reference System (LCRS) and Lunar Coordinate Time (TCL)

The XXXIInd International Astronomical Union General Assembly,

Considering

1. that the Resolution A4 of the XXIst General Assembly (1991), modified by Resolutions B1.3-B1.5 of the XXIVth General Assembly (2000), has defined the Barycentric Celestial Reference System (BCRS), and the Geocentric Celestial Reference System (GCRS), within the framework of General Relativity,
2. that the time coordinates of the BCRS and GCRS are designated Barycentric Coordinate Time (TCB) and Geocentric Coordinate Time (TCG), respectively,
3. that an analogous system of space-time coordinates for use in the lunar environment has not been defined,
4. that the high precision of many types of scientific measurements, including those involving astrometry, geodesy, time, and frequency, will require the practical application of such a framework in the lunar environment, and
5. that the theoretical work of an extension of this framework to the lunar environment has already been performed,

Recommends

1. the same techniques used to construct the GCRS be used to construct an analogous Lunar Celestial Reference System (LCRS), with its time coordinate designated Lunar Coordinate Time (TCL),
2. the LCRS metric tensor, the LCRS gravitational potentials, and the transformation between the BCRS and the LCRS be defined exactly as given in the relations in IAU Resolution B1.3 of the XXIVth General Assembly (2000), with the substitution of quantities related to the Moon for those related to the Earth,
3. the unit of measurement of TCL be consistent with the SI second,
4. the reading of TCL be 1977 January 1, $0^h 0^m 32.184^s$ exactly when the reading of TCB be 1977 January 1, $0^h 0^m 32.184^s$ at the center of the Moon, and

The principles of the IAU 2000 Framework are applied to the Moons to define:

- Lunar Celestial Reference System (LCRS)
- Lunar Coordinate Time (TCL)

5. the transformation between TCL and TCB be given by the relations in IAU Resolution B1.5 of the XXIVth General Assembly (2000), with the substitution of quantities related to the Moon for those related to the Earth.

Notes

The same formulae can apply for any celestial reference system in the Solar System by substituting quantities referred to the appropriate celestial body.

Note that 1977 January 1, $0^h 0^m 32.184^s$ is the common value for coordinate times Terrestrial Time (TT), TCG, TCB and now TCL, arbitrarily set to coincide with that of TT at the geocenter on 1977 January 1, $0^h 0^m 0^s$ TAI. The value of TT at that instant was set to ensure an approximate continuity with the previous time argument of ephemerides, Ephemeris Time (ET) (Resolution A4, Recommendations III and IV of the XXIst General Assembly (1991)). TCL has no historical relation to the other time scales so its initial epoch is entirely arbitrary and is set here for specificity.

References

M. Soffel, S. A. Klioner, G. Petit, P. Wolf, S. M. Kopeikin, P. Bretagnon, V. A. Brumberg, N. Capitaine, T. Damour, T. Fukushima, B. Guinot, T.-Y. Huang, L. Lindegren, C. Ma, K. Nordtvedt, J. C. Ries, P. K. Seidelmann, D. Vokrouhlický, C. M. Will, and C. Xu. *The IAU 2000 Resolutions for Astrometry, Celestial Mechanics, and Metrology in the Relativistic Framework: Explanatory Supplement*. Astron. J., **126**: 2687–2706, December 2003. doi:10.1086/378162.

Numerical computation of the TCG-like time scales

u = TCX is the TCG-like time scale for a body 'X' at its center of mass

t = TCB

$$\frac{du}{dt} = 1 + f(t)$$

$$f = \frac{1}{c^2} \alpha(t) + \frac{1}{c^4} \beta(t)$$

$\Rightarrow$

$$u = t + \delta t(t)$$

$$t = u - \delta u(u)$$

$\Rightarrow$

$$\frac{d\delta t}{dt} = f(t)$$

$$\frac{d\delta u}{du} = \frac{f(u - \delta u(u))}{1 + f(u - \delta u(u))}$$

$$\delta t(t_0) = 0$$

$$\delta u(u_0) = 0$$

$$\alpha = -\frac{1}{2} v_X^2 - \sum_{A \neq X} \frac{GM_A}{r_{XA}}$$

$$\begin{aligned} \beta = & -\frac{1}{8} v_X^4 + \frac{1}{2} \left(\sum_{A \neq X} \frac{GM_A}{r_{XA}} \right)^2 + \sum_{A \neq X} \left(\frac{GM_A}{r_{XA}} \sum_{B \neq A, X} \frac{GM_B}{r_{AB}} \right) \\ & + \sum_{A \neq X} \frac{GM_A}{r_{XA}} \left(4 v_A^i v_X^i - \frac{3}{2} v_X^2 - 2 v_A^2 + \frac{1}{2} a_A^i r_{XA}^i \right. \\ & \left. + \frac{1}{2} (v_A^i r_{XA}^i / r_{XA})^2 \right) \end{aligned}$$

$$\mathbf{r}_{XA} = \mathbf{x}_X - \mathbf{x}_A$$

Two ordinary differential equations with the initial conditions (e.g. $t_0 = 1$ Jan 1977).

For this what is done for Gaia in Gaia DPAC.

Very easy to integrate numerically given a solar system ephemeris and store the result as a set of Chebyshev polynomials: the time ephemerides for each body

The IAU 2000 framework

Explanatory supplement: Soffel et al, 2003: AJ, 126, 2687

THE IAU 2000 RESOLUTIONS FOR ASTROMETRY, CELESTIAL MECHANICS, AND METROLOGY IN THE RELATIVISTIC FRAMEWORK: EXPLANATORY SUPPLEMENT

M. SOFFEL,¹ S. A. KLIONER,¹ G. PETIT,² P. WOLF,² S. M. KOPEIKIN,³ P. BRETAGNON,⁴ V. A. BRUMBERG,⁵ N. CAPITAIN,⁶
T. DAMOUR,⁷ T. FUKUSHIMA,⁸ B. GUINOT,⁶ T.-Y. HUANG,⁹ L. LINDEGREN,¹⁰ C. MA,¹¹ K. NORDTVEDT,¹² J. C. RIES,¹³
P. K. SEIDELMANN,¹⁴ D. VOKROUHLICKÝ,¹⁵ C. M. WILL,¹⁶ AND C. XU¹⁷

Received 2002 August 9; accepted 2003 July 2

ABSTRACT

We discuss the IAU resolutions B1.3, B1.4, B1.5, and B1.9 that were adopted during the 24th General Assembly in Manchester, 2000, and provides details on and explanations for these resolutions. It is explained why they present significant progress over the corresponding IAU 1991 resolutions and why they are necessary in the light of present accuracies in astrometry, celestial mechanics, and metrology. In fact, most of these resolutions are consistent with astronomical models and software already in use. The metric tensors and gravitational potentials of both the Barycentric Celestial Reference System and the Geocentric Celestial Reference System are defined and discussed. The necessity and relevance of the two celestial reference systems are explained. The transformations of coordinates and gravitational potentials are discussed. Potential coefficients parameterizing the post-Newtonian gravitational potentials are expounded. Simplified versions of the time transformations suitable for modern clock accuracies are elucidated. Various approximations used in the resolutions are explicated and justified. Some models (e.g., for higher spin moments) that serve the purpose of estimating orders of magnitude have actually never been published before.

Key words: astrometry — celestial mechanics — reference systems — time

1. INTRODUCTION

It is clear that, beyond some threshold of accuracy, any astronomical problem has to be formulated within the

lunar laser ranging measures the distance to the Moon with a precision of a few centimeters, thereby operating at the 10^{-10} level. At this level, several relativistic effects are significant and observable. Relativistic effects related to the

Numerical computation of the TCG-like time scales

τ is the TCG-like time scale for a body 'o' at its center of mass

$t = TCB$

$$\frac{d\tau}{dt} = 1 + f(t)$$

$$f = \frac{1}{c^2} \alpha(t) + \frac{1}{c^4} \beta(t)$$

$$\Rightarrow \begin{aligned} \tau &= t + \delta t(t) \\ t &= \tau - \delta \tau(\tau) \end{aligned} \Rightarrow$$

$$\frac{d\delta t}{dt} = f(t)$$

$$\frac{d\delta \tau}{d\tau} = \frac{f(\tau - \delta \tau(\tau))}{1 + f(\tau - \delta \tau(\tau))}$$

$$\delta t(t_0) = 0$$

$$\delta \tau(\tau_0) = 0$$

$$\alpha = -\frac{1}{2} v_o^2 - \sum_A \frac{GM_A}{r_{oA}}$$

$$\begin{aligned} \beta = & -\frac{1}{8} v_o^4 + \frac{1}{2} \left(\sum_A \frac{GM_A}{r_{oA}} \right)^2 + \sum_A \left(\frac{GM_A}{r_{oA}} \sum_{B \neq A} \frac{GM_B}{r_{AB}} \right) \\ & + \sum_A \frac{GM_A}{r_{oA}} \left(4 v_A^i v_o^i - \frac{3}{2} v_o^2 - 2 v_A^2 + \frac{1}{2} a_A^i r_{oA}^i \right. \\ & \left. + \frac{1}{2} (v_A^i r_{oA}^i / r_{oA})^2 \right) \end{aligned}$$

$$\mathbf{r}_{oA} = \mathbf{x}_o - \mathbf{x}_A$$

Two ordinary differential equations with the initial conditions (e.g. $t_0 = 1$ Jan 1977).

Very easy to integrate numerically given a solar system ephemeris and store the result as a set of Chebyshev polynomials: the time ephemerides for each body

The main characteristics of the TCG-like for other bodies

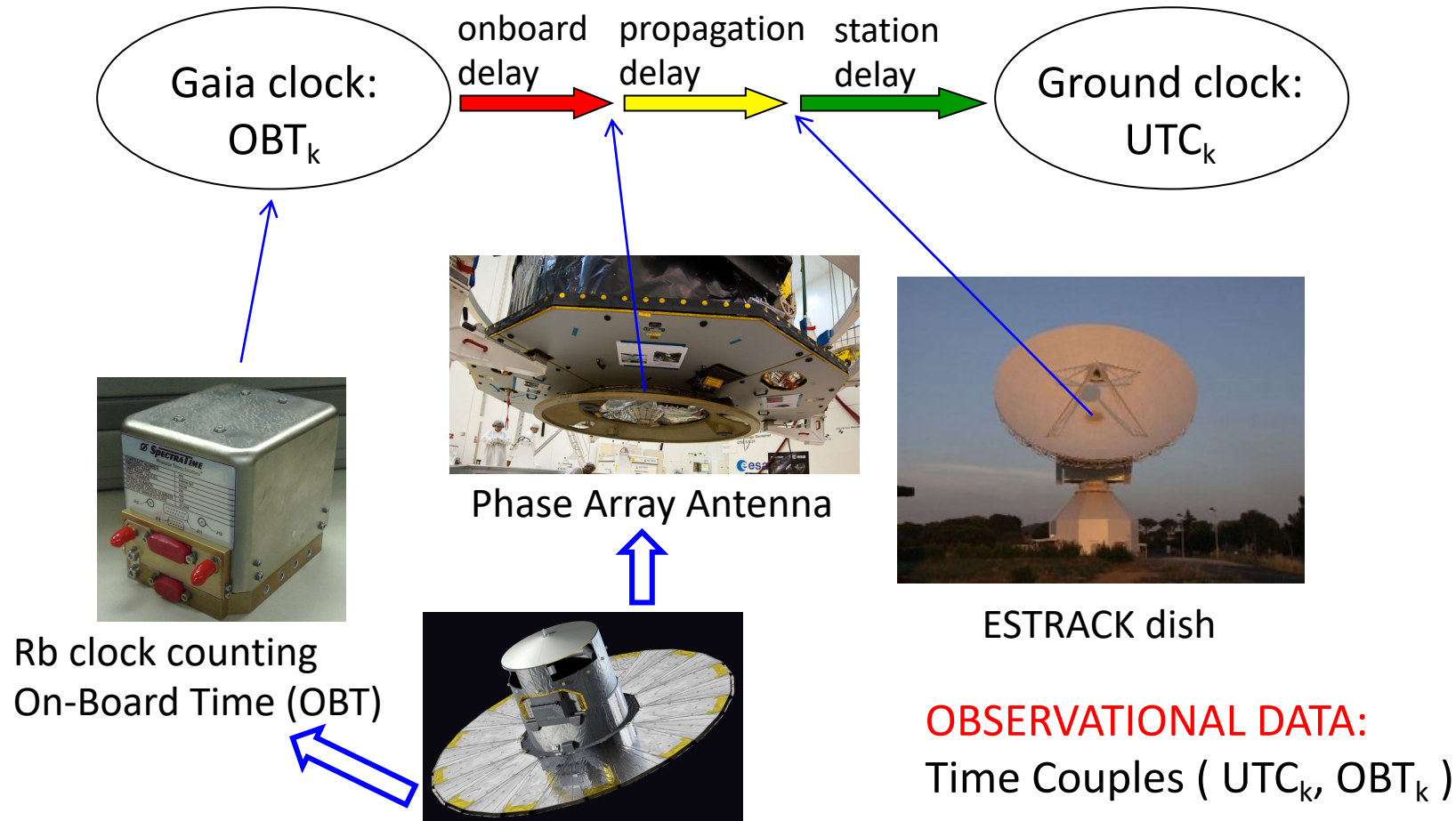
Body	Approximate linear drift [$\times 10^{-8}$]	Amplitude of the main periodic term [ms]
Mercury	3.825	12.7
Venus	2.047	0.57
Earth	1.481 ($\approx L_C$)	1.66
Moon	1.483	1.75
Mars	0.972	11.5
Jupiter	0.285	11.0

Results of numerical
integration using
INPOP19a
over about 60 years.

1. The linear drifts are different for different bodies: from $0.19 L_C$ to $2.59 L_C$.
2. The amplitude of the main period term comes from the elliptic motion of the body around the Sun and proportional to $e\sqrt{a}$ where a is the semi-major axis and e is the eccentricity of the heliocentric orbit: the amplitudes ranges from 0.3 to 7.7 of the effect for the Earth (1.6 ms).

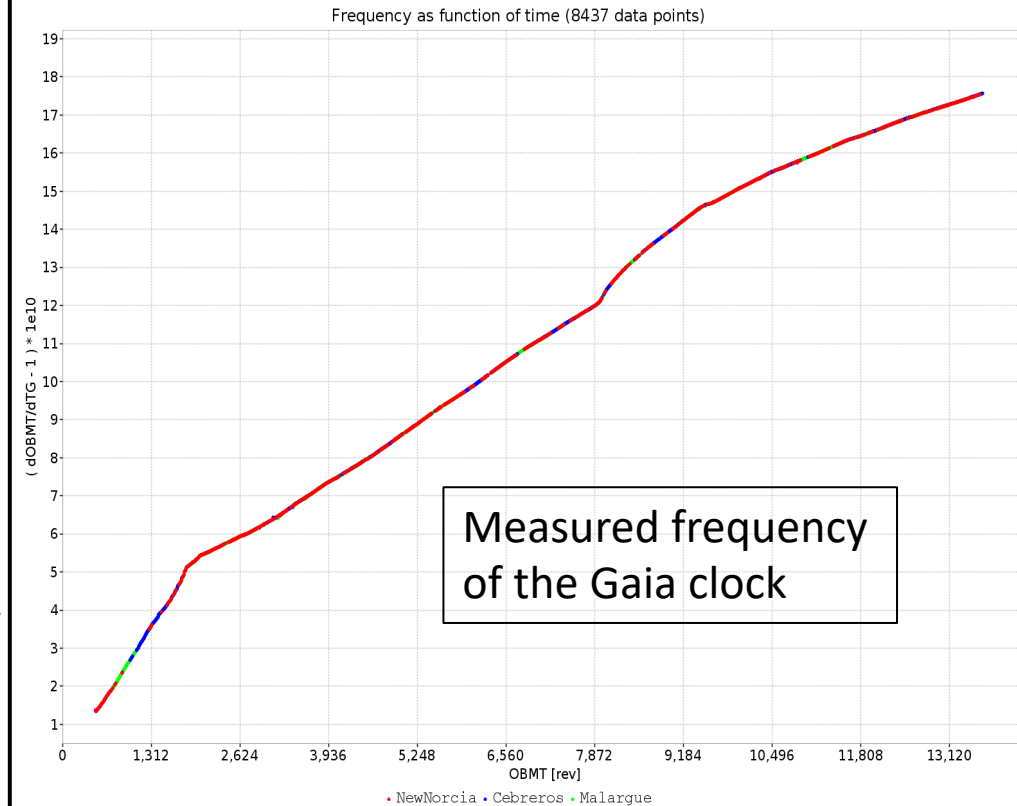
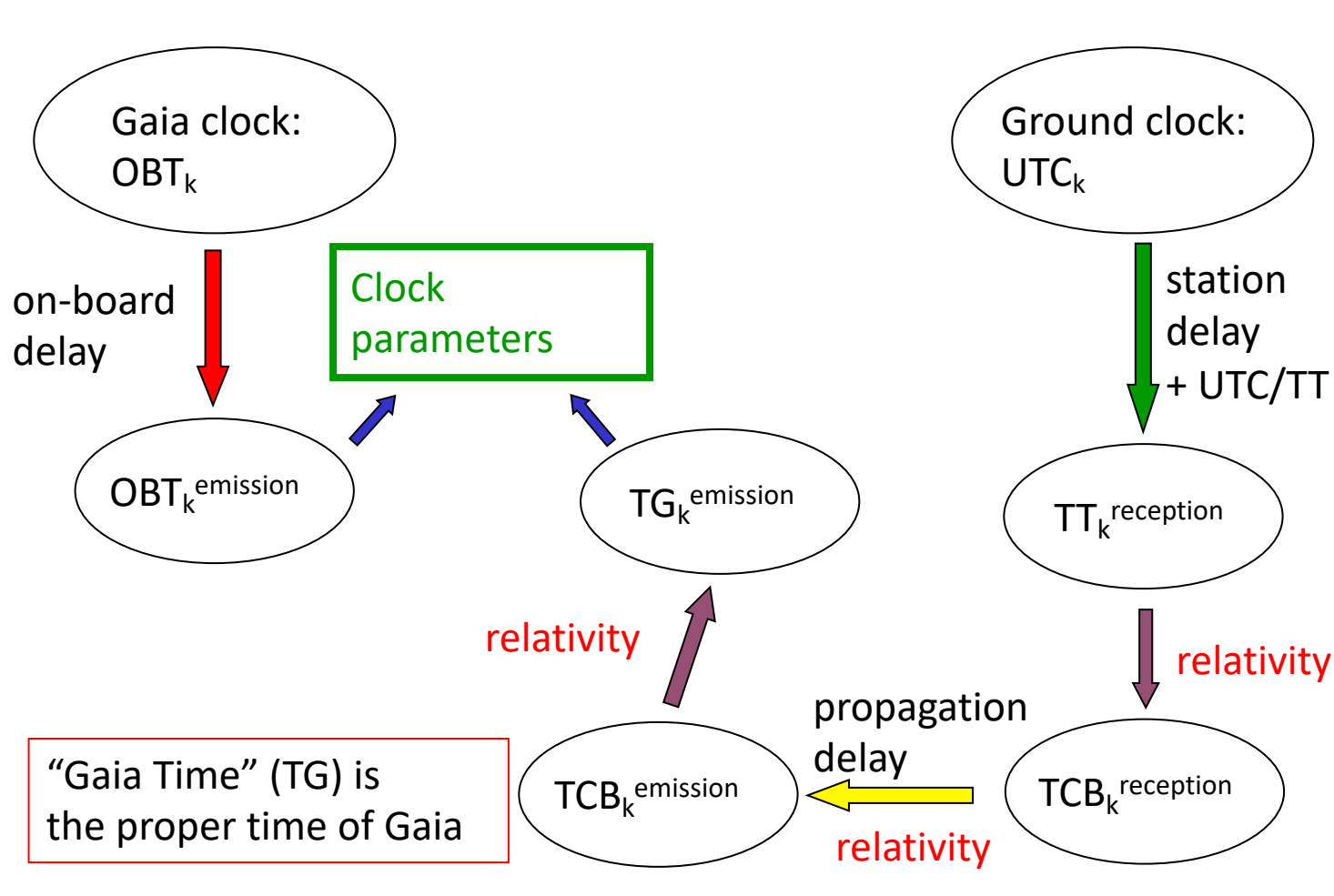
Example: the timing system of Gaia

- Gaia satellite has an atomic Rb clock on board (1.5 million km from the Earth)
- That clock is a free oscillator that needs to be synchronized to TCB (the time scale used in Gaia)
- Clock calibration for Gaia: one-way time transfer:



Example: the timing system of Gaia

- A relativistic modelling of the observable time couples:



- Gaia clock are synchronized with TCB and realized TCB on board of Gaia