

On background radiation and effective source strength

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Any measurement of the activity of a radiation source has to be performed in an environment which is more or less contaminated by unwanted background radiation. In the majority of practical situations it is possible to take precautions which reduce its influence to such a degree that approximate corrections are sufficient. However, there are also cases (for instance in low-level counting or in some metrological applications) where the disturbing influence of a background has to be taken into account in a more accurate way.

It is well known that a constant background radiation can be thought of as being emitted by a secondary source which is independent of the main or primary source to be measured. If a disintegration in the source under study results in the emission of both a beta particle and a gamma ray, which can be separately measured by appropriate detectors, a certain complication is caused by the fact that in general the ratio of beta to gamma events is not the same for the primary and the secondary source; thus, the counting efficiencies seem to depend on the source considered. However, since we would like to characterize the sensitivity of a detector to a given type of radiation by a single number, namely its detection efficiency, we are led to a model which consists not only of two sources, but also of two detector systems [1]. Each radiation is then only "seen" by the appropriate detector, and the superposition of the beta (or gamma) pulses from the two sources is supposed to be performed afterwards.

In such a model, where dead-time losses are neglected (they will be taken into account at a later stage), the following simple relations hold for the beta, gamma and coincidence rates (for the notation used see [1], especially Fig. 1)

- for the primary source N_1 :

$$\begin{aligned} N_{\beta 1} &= N_1 \varepsilon_{\beta 1} , \\ N_{\gamma 1} &= N_1 \varepsilon_{\gamma 1} \quad \text{and} \\ N_{c1} &= N_1 \varepsilon_{\beta 1} \varepsilon_{\gamma 1} , \end{aligned} \tag{1}$$

hence $\varepsilon_{c1} = \varepsilon_{\beta 1} \varepsilon_{\gamma 1}$ (2)

and $N_1 = N_{\beta 1} N_{\gamma 1} / N_{c1}$;

- for the secondary source N_2 :

$$\begin{aligned} N_{\beta 2} &= N_2 \varepsilon_{\beta 2} \equiv B_{\beta} , \\ N_{\gamma 2} &= N_2 \varepsilon_{\gamma 2} \equiv B_{\gamma} \text{ and} \\ N_{c 2} &= N_2 \varepsilon_{\beta 2} \varepsilon_{\gamma 2} \equiv B_c , \end{aligned} \quad (3)$$

hence
$$\varepsilon_{c 2} = \varepsilon_{\beta 2} \varepsilon_{\gamma 2} \quad (4)$$

and
$$N_2 = B_{\beta} B_{\gamma} / B_c .$$

In what follows dashed count rates will always refer to a sum, for example $N_{\beta}' = N_{\beta 1} + B_{\beta 2}$, etc. It is interesting to note that the superposition of the two sources corresponds exactly to a single "effective" source with activity $N' = N_1 + N_2$, provided that we assume for the detector system the modified efficiencies

$$\begin{aligned} \varepsilon_{\beta}' &= \frac{N_{\beta}'}{N'} = \frac{\varepsilon_{\beta 1} N_1 + \varepsilon_{\beta 2} N_2}{N_1 + N_2} , \\ \varepsilon_{\gamma}' &= \frac{N_{\gamma}'}{N'} = \frac{\varepsilon_{\gamma 1} N_1 + \varepsilon_{\gamma 2} N_2}{N_1 + N_2} \text{ and} \\ \varepsilon_c' &= \frac{N_c'}{N'} = \frac{\varepsilon_{\beta 1} \varepsilon_{\gamma 1} N_1 + \varepsilon_{\beta 2} \varepsilon_{\gamma 2} N_2}{N_1 + N_2} . \end{aligned} \quad (5)$$

These new detection efficiencies may be considered as some kind of "weighted means" of the respective original efficiencies. It is easy to see that the effective source of activity N' , measured by a single counter arrangement with the efficiencies given in (5), indeed leads to the observed count rates, i.e.

$$\begin{aligned} N_{\beta}' &= N' \varepsilon_{\beta}' = N_{\beta} + B_{\beta} , \\ N_{\gamma}' &= N' \varepsilon_{\gamma}' = N_{\gamma} + B_{\gamma} \text{ and} \\ N_c' &= N' \varepsilon_c' = N_c + B_c , \end{aligned} \quad (6)$$

where we have identified, for simplicity, $N_{\beta 1}$ with N_{β} , etc.

However, a restriction concerning this effective source N' is worth noting, namely the fact that now

$$\epsilon'_c \neq \epsilon'_\beta \epsilon'_\gamma, \quad (7)$$

and hence also

$$N' \neq \frac{N'_\beta N'_\gamma}{N'_c}. \quad (8)$$

This is apparently the price we have to pay for the simplification achieved. Since ϵ'_c as well as N' can always be determined otherwise (as indicated above), the "loss" described by (7) and (8) is of little consequence, although the restriction should be kept in mind, of course.

One may rightly wonder now if the above elementary exercise in notation was really worth while. Indeed, at first sight all this seems trivial and useless, but after reflection this impression might change somewhat. To begin with, there remains the fact that the previous arrangement of two separate sources and detector systems has formally been reduced to a single one, and this has at least the advantage of simplicity. However, a more objective judgment may be based on possible practical consequences. For this purpose we would like to mention briefly two simple applications.

The first has to do with a recent attempt at incorporating background in a Monte-Carlo simulation of the selective sampling method [2]. In such an approach one first chooses at random exponentially-distributed time intervals which separate the instances in time at which the source N_1 produces a disintegration, and one then decides with the probabilities $\epsilon_{\beta 1}$ and $\epsilon_{\gamma 1}$ whether the respective detectors "see" a beta or a gamma event. Addition of a background requires one to perform the same operations for N_2 and the corresponding efficiencies. For the beta as well as for the gamma channel the two resulting pulse sequences then have to be brought into chronological order before the counting losses, produced by the appropriate dead times (or a series arrangement of them, in each channel), can be simulated. All this is certainly possible, but somewhat tedious to perform. By reasoning with a single source N' , the procedure can be much simplified and there is in particular no need for superimposing (twice) two series and then arranging them in time order.

As for the random choice of the nature of the detected events, a possible scheme applicable to the effective-source model is sketched in Fig. 1.

For the sake of completeness, we may also mention a possible "intermediate" model, where for each disintegration from N' we first decide at random if it originates from N_1 or from N_2 . Accordingly, the nature of the detected pulses would be based on the respective efficiencies (by means of a scheme similar to the one sketched in Fig. 1).

While delivering both the beta and gamma events in chronological order, this model requires additional random choices to be made which are avoided by the direct use of the effective efficiencies (5). Hence, this approach, although fully equivalent, is of no practical interest.

It may be worth mentioning that it is this Monte-Carlo simulation which has led us to look for a simple alternative to the model with two sources.

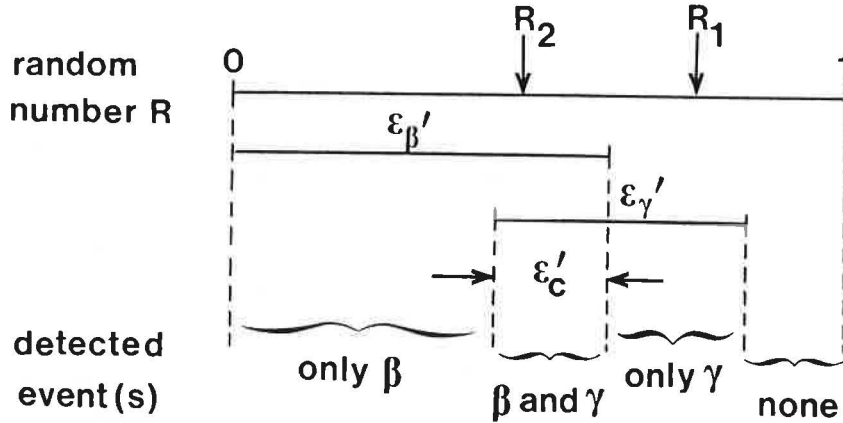


Fig. 1 - Schematic diagram illustrating how, by means of a uniformly distributed random number R , one can decide if a given disintegration leads to the detection of a beta or a gamma pulse, to both or to nothing. For instance R_1 gives rise to a gamma, whereas R_2 results in a (true) beta-gamma coincidence.

The second application is of a more practical nature, as it stems from the need to dispose of a background correction formula for the selective sampling method. As this problem has been treated before [1], the new derivation can be taken as a check of the previous one. The average channel contents, including background, for the two regions in the time spectrum of the registered gamma pulses is now obviously given by (κ is a constant)

$$G = \kappa N_{\gamma}' \quad \text{and} \quad g = \kappa (N_{\gamma}' - N_{\text{c}}'), \quad (9)$$

and substituting (6) we find readily for their ratio

$$\frac{g}{G} = \frac{N_{\gamma} + B_{\gamma} - N_{\text{c}} - B_{\text{c}}}{N_{\gamma} + B_{\gamma}}, \quad (10)$$

which is identical with eq. (3) in [1]. We are thus led to the same correction formula as before, which is a welcome confirmation.

Finally, we should like to speculate upon a possible further application, while eschewing for the present the elaboration of any details. It concerns the well-known solution derived by Cox and Isham [3] for the coincidence method. These authors begin by considering three independent Poisson processes, with count rates given (in their notation) by

$$\begin{aligned}\lambda_1 &= \lambda \varepsilon_1(1 - \varepsilon_2) , \\ \lambda_2 &= \lambda \varepsilon_2(1 - \varepsilon_1) \quad \text{and} \\ \lambda_{12} &= \lambda \varepsilon_1 \varepsilon_2 .\end{aligned}\tag{11}$$

The question now arises whether an experimental background can be taken into account rigorously in the framework of this approach. Remembering that the three count rates correspond, for instance, to the rate of detected single betas, single gammas and (true) coincidences, which we have denoted above by $N'_\beta - N'_c$, $N'_\gamma - N'_c$ and N'_c , the relations corresponding to (11), but with background included, may then be written as

$$\begin{aligned}\lambda_1 &= \lambda'(\varepsilon'_1 - \varepsilon'_3) , \\ \lambda_2 &= \lambda'(\varepsilon'_2 - \varepsilon'_3) \quad \text{and} \\ \lambda_3 &= \lambda' \varepsilon'_3 .\end{aligned}\tag{12}$$

The fact that now ε'_3 (identical with ε'_c) is no longer equal to the product $\varepsilon'_1 \varepsilon'_2$ seems to be of little consequence for the further developments [3], and in particular the total count rates for the two detectors are still given by

$$\rho_1 = \lambda_1 + \lambda_3 \quad \text{and} \quad \rho_2 = \lambda_2 + \lambda_3 .$$

It therefore seems that an exact inclusion of background should be possible, perhaps even in a rather straightforward manner.

APPENDIX

Some approximations

In order to get a good idea of the changes in the detection efficiencies due to the adoption of the effective-source model outlined above, some approximate relations may be useful, in particular for the important special case of a weak background.

Let us therefore now assume that

$$N_2/N_1 \equiv n \ll 1 \quad (\text{A1 a})$$

as well as

$$B_\beta/N_\beta \equiv b_\beta \ll 1 ,$$

$$B_\gamma/N_\gamma \equiv b_\gamma \ll 1 \quad \text{and} \quad (\text{A1 b})$$

$$B_c/N_c \equiv b_c \ll 1 .$$

From (5) and (6) we then find, for instance, that

$$\varepsilon'_\beta = \frac{N_\beta + B_\beta}{N_1 + N_2} = \frac{N_\beta(1 + b_\beta)}{N_1(1 + n)} \cong \varepsilon_\beta(1 + b_\beta - n) , \quad (\text{A2 a})$$

and similarly

$$\varepsilon'_\gamma \cong \varepsilon_\gamma(1 + b_\gamma - n) , \quad (\text{A2 b})$$

$$\varepsilon'_c \cong \varepsilon_c(1 + b_c - n) .$$

Therefore, the new (dashed) efficiencies can be larger or smaller than the old (undashed) ones. More specifically, since

$$b_\beta = \frac{N_2}{N_1} \frac{\varepsilon_{\beta 2}}{\varepsilon_{\beta 1}} = n \frac{\varepsilon_{\beta 2}}{\varepsilon_{\beta 1}} ,$$

it follows from (A2 a) that

$$\varepsilon'_\beta \begin{cases} > \varepsilon_\beta \\ < \varepsilon_\beta \end{cases} \quad \text{if} \quad \varepsilon_{\beta 2} \begin{cases} > \varepsilon_{\beta 1} \\ < \varepsilon_{\beta 1} \end{cases} , \quad (\text{A3})$$

and likewise for ε'_γ and ε'_c . This can also be seen directly from (5).

If the assumptions (A1) are justified, (7) may be replaced by an approximate equation. Since $\varepsilon_c = \varepsilon_\beta \varepsilon_\gamma$, we obtain from (A2)

$$\begin{aligned}\varepsilon'_c &\cong \varepsilon_\beta \varepsilon_\gamma (1 + b_c - n) \\ &\cong \varepsilon'_\beta (1 - b_\beta + n) \varepsilon'_\gamma (1 - b_\gamma + n) (1 + b_c - n) \\ &\cong \varepsilon'_\beta \varepsilon'_\gamma (1 - b_\beta - b_\gamma + b_c + n),\end{aligned}\tag{A4}$$

where we have written ε_β for $\varepsilon_{\beta 1}$, etc.

A similar treatment leads for (8) to

$$\begin{aligned}\frac{N'_\beta N'_\gamma}{N'_c} &\cong \frac{N_\beta N_\gamma}{N_c} (1 + b_\beta + b_\gamma - b_c) \quad \text{and} \\ N' &= \frac{N_\beta N_\gamma}{N_c} + \frac{B_\beta B_\gamma}{B_c} \cong \frac{N_\beta N_\gamma}{N_c} \left(1 + \frac{b_\beta b_\gamma}{b_c}\right),\end{aligned}$$

so that (8) can be strengthened to

$$N' \cong \frac{N'_\beta N'_\gamma}{N'_c} \left[\frac{1 + b_\beta b_\gamma / b_c}{1 + b_\beta + b_\gamma - b_c} \right].\tag{A5}$$

As a numerical example let us assume the following data

- for main source: $N_0 = 1\,000\text{ s}^{-1}$, $\varepsilon_\beta = 0.9$, $\varepsilon_\gamma = 0.2$;

- for background : $B_\beta = 5\text{ s}^{-1}$, $B_\gamma = 40\text{ s}^{-1}$, $B_c = 2\text{ s}^{-1}$.

These lead to the following results

	exact (eq. 5)	approximate (eq. A2)
ε'_β	0.823	0.815
ε'_γ	0.218	0.220
ε'_c	0.165	0.164
	exact	approximate (eq. A5)
N'	$1\,100.0\text{ s}^{-1}$	$1\,099.0\text{ s}^{-1}$

This shows that even for the relatively high background contribution assumed ($n = 0.1$) the indicated approximate formulae still give quite useful results. It is easy to verify that the signs of the differences $\varepsilon'_\beta - \varepsilon_\beta$, etc., are in agreement with the rule given in (A3).

References

- [1] J.W. Müller: "Background correction for SESAM", BIPM WPN-223 (1982)
- [2] id.: "Selective sampling - an alternative to coincidence counting", Nucl. Instr. and Methods 189, 449 (1981)
- [3] D.R. Cox, V. Isham: "A bivariate process connected with electronic counters", Proc. Roy. Soc. (London) A 356, 149 (1977)

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