On the influence of the gamma sensitivity of the proportional counter in the selective sampling method

by Jörg W. Müller

Bureau International des Poids et Mesures, F-92310 Sèvres

In the well-known $4\pi\beta-\gamma$ coincidence method for measuring source activities, the problems related to the measurement of and the correction for the sensitivity of the beta counter to gamma rays have been treated from various points of view in the past (for some relevant references, see [1]). The recent introduction of the selective sampling method [2] raises the question of how this deviation from the ideal counting situation may influence the results obtained by the new approach, where the detector efficiencies can be determined directly, thereby rendering unnecessary the measurement of coincidences.

Let us start with the general relations valid for the individual count rates as needed in the traditional approach. If we restrict ourselves to the case of a single decay branch, the respective expressions are, in the usual notation (see e.g. [3]),

$$N_\beta = N_o \left[\varepsilon_\beta + (1 - \varepsilon_\beta)\left(\frac{\alpha\,\varepsilon_{ce} + \varepsilon_{\beta\gamma}}{1 + \alpha}\right)\right],$$

$$N_\gamma = N_o \frac{\varepsilon_\gamma}{1 + \alpha} \quad \text{and} \tag{1}$$

$$N_c = N_o \left[\frac{\varepsilon_\beta\,\varepsilon_\gamma}{1 + \alpha} + (1 - \varepsilon_\beta)\,\varepsilon_c\right],$$

where ε_{ce} and $\varepsilon_{\beta\gamma}$ are the beta-counter detection efficiencies for conversion electrons and gamma rays respectively and α is the conversion coefficient, while ε_c denotes the probability for obtaining a coincidence when no beta particle has been detected.

N_β, N_γ and N_c stand for the observed count rates in the respective channels, but corrected for dead time, background and accidental coincidences. If it can be assumed that $\alpha = \varepsilon_c = 0$, as we shall do in what follows, the system (1) reduces to

$$N_\beta = N_o \left[\varepsilon_\beta + \varepsilon_{\beta\gamma} (1 - \varepsilon_\beta)\right] ,$$

$$N_\gamma = N_o \, \varepsilon_\gamma \quad \text{and} \qquad (2)$$

$$N_c = N_o \, \varepsilon_\beta \, \varepsilon_\gamma .$$

Since $N_o \, \varepsilon_\beta = N_\beta - N_o \, \varepsilon_{\beta\gamma} (1 - \varepsilon_\beta)$, (2')

the source is then given by

$$N_o = \frac{N_\gamma}{N_c} \left[N_\beta - N_o \, \varepsilon_{\beta\gamma} (1 - \varepsilon_\beta)\right].$$

In the selective sampling method, one measures directly (for instance) the gamma density ratio $R_\gamma = g/G$, where

$$G = \kappa \, N_\gamma \quad \text{and} \quad g = \kappa \, (N_\gamma - N_c) ,$$

κ being a constant which is independent of the count rates. Therefore, as for the case $\varepsilon_{\beta\gamma} = 0$, we still have

$$R_\gamma = \frac{N_\gamma - N_c}{N_\gamma} = \frac{N_o \, \varepsilon_\gamma - N_o \, \varepsilon_\beta \, \varepsilon_\gamma}{N_o \, \varepsilon_\gamma} = 1 - \varepsilon_\beta . \qquad (3)$$

With (2') this leads for the source activity to the expression

$$N_o = \frac{N_\beta}{\varepsilon_\beta + \varepsilon_{\beta\gamma} (1 - \varepsilon_\beta)} = \frac{N_\beta}{1 - R_\gamma + \varepsilon_{\beta\gamma} R_\gamma} = \frac{N_\beta}{1 - R_\gamma (1 - \varepsilon_{\beta\gamma})} . \qquad (4)$$

Hence, the inclusion of $\varepsilon_{\beta\gamma}$ diminishes somewhat the calculated value of N_o.

Let us now have a look at the "reversed" situation, where the registration cycle is initiated by a gamma and where we observe the arrival density of the betas. Considering the problems which arise from the need to extrapolate to $\varepsilon_\beta = 1$, one might expect that the situation is worse if the measured quantity is ε_γ, which is usually very far from unity. It will be easy to realize, however, that such an apprehension is unfounded. Since the observed gamma count rate is not influenced by $\varepsilon_{\beta\gamma}$, as can be seen from (2), no extrapolation will be required for the gamma efficiency.

In this case we have

$$R_\beta = \frac{b}{B} = \frac{N_\beta - N_c}{N_\beta} = 1 - \frac{N_o \, \varepsilon_\beta \, \varepsilon_\gamma}{N_o \left[\varepsilon_\beta + \varepsilon_{\beta\gamma} \, (1 - \varepsilon_\beta)\right]}$$

$$= 1 - \frac{\varepsilon_\gamma}{1 + \varepsilon_{\beta\gamma} \, (\frac{1 - \varepsilon_\beta}{\varepsilon_\beta})} , \qquad (5)$$

which may be compared with the expression $R_\beta = 1 - \varepsilon_\gamma$ which neglects the contribution of $\varepsilon_{\beta\gamma}$.

Since now

$$\varepsilon_\gamma = (1 - R_\beta) \left[1 + \varepsilon_{\beta\gamma} \, (\frac{1 - \varepsilon_\beta}{\varepsilon_\beta})\right] ,$$

we find for the source activity

$$N_o = \frac{N_\gamma}{\varepsilon_\gamma} = \frac{N_\gamma}{(1 - R_\beta) \left[1 + \varepsilon_{\beta\gamma} \, (\frac{1 - \varepsilon_\beta}{\varepsilon_\beta})\right]} . \qquad (6)$$

An elementary rearrangement of (4) shows that the corrected value of the activity can be written for both experimental counting conditions as

$$N_o = N_{oo} \, / \, C_\beta ,$$

where the correction factor C_β is given by

$$C_\beta = 1 + \varepsilon_{\beta\gamma} \, (\frac{1 - \varepsilon_\beta}{\varepsilon_\beta}) \qquad (7)$$

and N_{oo} is the activity obtained when $\varepsilon_{\beta\gamma}$ is neglected.

Assuming, for example, a value of $\varepsilon_{\beta\gamma} = 0.005$, we find numerically the approximate correction factors given in the following table.

ε_β	$C_\beta - 1$
0.95	2.6×10^{-4}
0.9	5.6×10^{-4}
0.8	1.2×10^{-3}
0.5	5.0×10^{-3}
0.2	2.0×10^{-2}.

For nuclides which can be measured with a high beta efficiency, the effect of $\varepsilon_{\beta\gamma}$ is therefore of little importance, but it has clearly to be taken into account if ε_β is only about 80% or less.

It also follows from the above considerations that the selective sampling method cannot be used for an independent measurement of $\varepsilon_{\beta\gamma}$, as one might perhaps have hoped at the outset.

References

[1] X.Z. Wu, C. Veyradier, C. Colas, P. Bréonce, A. Rytz: "Report on some experiments carried out with the 4πβ-γ coincidence counting equipment of BIPM", Rapport BIPM-80/5 (1980)

[2] J.W. Müller: "Selective sampling - an alternative to coincidence counting" (1981, submitted for publication)

[3] A.P. Baerg: "Measurement of radioactive disintegration rate by the coincidence method", Metrologia 2, 23 (1966), eq. 7

(April 1981)