First realization of a fully-primary optical pressure scale with two-color measurement

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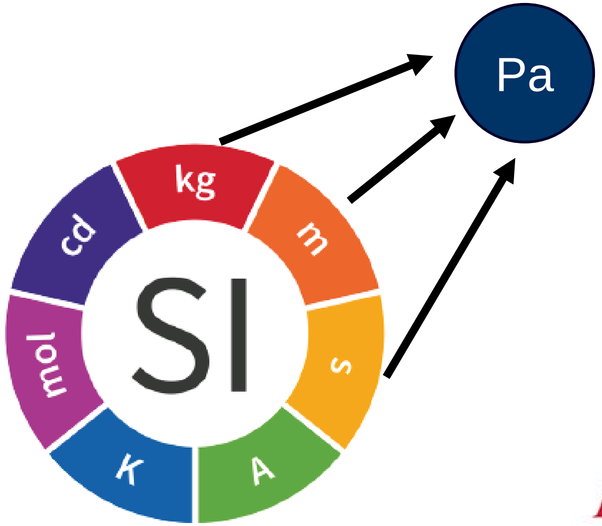
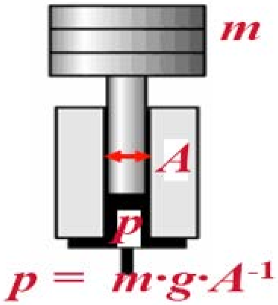
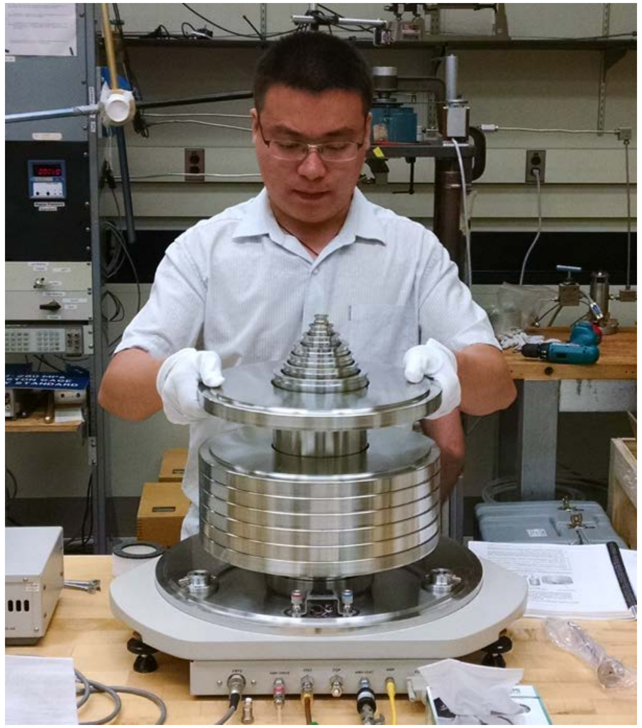
Y. Yang, J.A. Stone, P.F. Egan, Demonstration of dispersion gas barometry, Physical Review Applied, 23, 064041 (2025).*

Traditional pressure standards

$1\ \text{Pa} = 1\ \text{kg m}^{-1}\ \text{s}^{-2}$

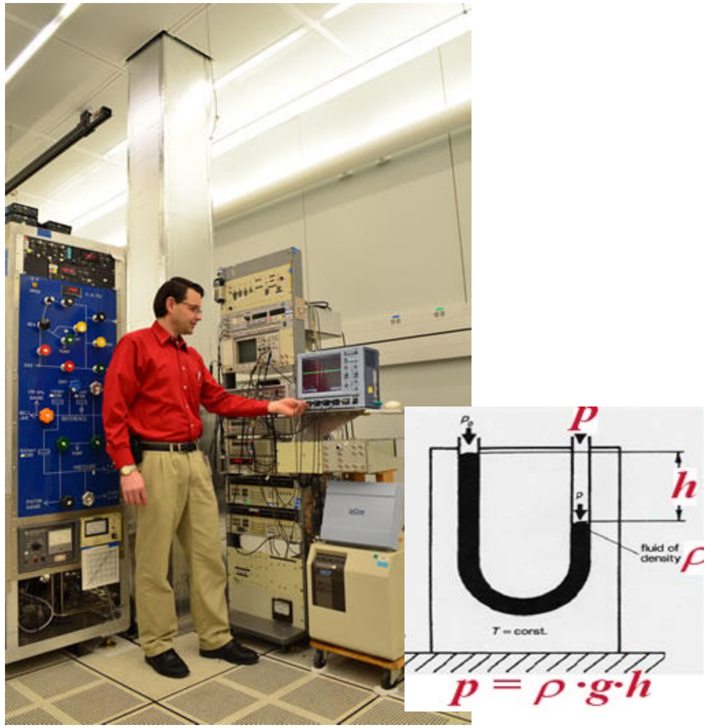






2015 at NIST

State of the art pressure balance with **100 kg** weights



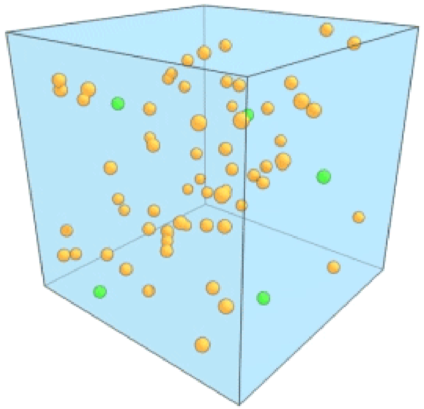
Courtesy of NIST website

NIST **3-meter-tall** manometer with **500 lb** mercury

Pressure: force per unit area

Optical pressure standard

SI brochure 9th ed.: A user is now free to choose any convenient equation of physics that links the defining constants to the quantity intended to be measured.



Pressure: thermal energy per unit volume

1 Pa = 1 J m^{-3}

Equation of state of gas

$$p = \rho RT(1 + B_\rho \rho + C_\rho \rho^2 + \cdots)$$

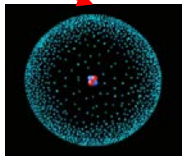
Lorentz-Lorenz relation

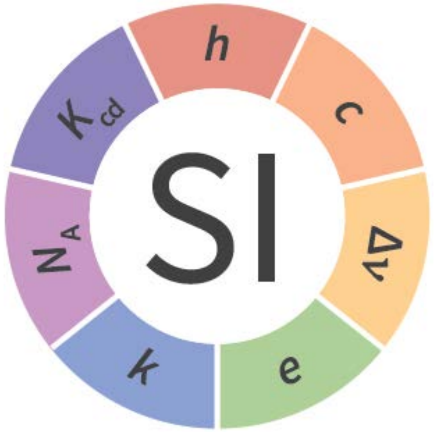
$$\frac{n^2 - 1}{n^2 + 2} = A_R\rho + B_R\rho^2 + C_R\rho^3 + \ldots$$

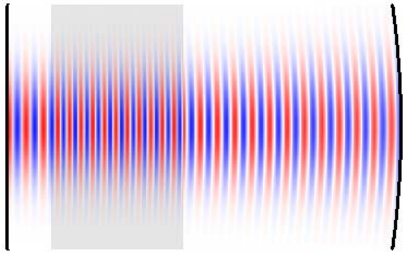
$$p = \frac{2N_A kT}{3A_R}(n-1) + \cdots$$

$$n - 1 \approx \frac{\Delta f}{\nu}$$

N_A | k | K | Molar polarizability | $\Delta\nu$







Optical measurement of refractivity

Helium properties

Magnetic susceptibility

$$A_R(\nu) = A_\varepsilon + A_\mu + A_2\nu^2 + A_4\nu^4 + \cdots$$

Static polarizability

Cauchy coefficients

relativistic+QED dispersion

review article

Luther et al., Metrologia 33, 341-352, 1996.

20 ppm

relativistic

Cencek et al., PRL 86, 5675-5678, 2001

2 ppm

relativistic + QED

Lach et al., PRL 92, 233001, 2004

0.2 ppm

9.1 ppm

Schmidt et al., PRL 98, 254504, 2007

Experiment
Microwave resonator

Puchalski et al., PRA 93, 032515, 2016

0.12 ppm

2 ppm

Gaiser and Fellmuth, PRL 120, 123203, 2018

Experiment
DCGT

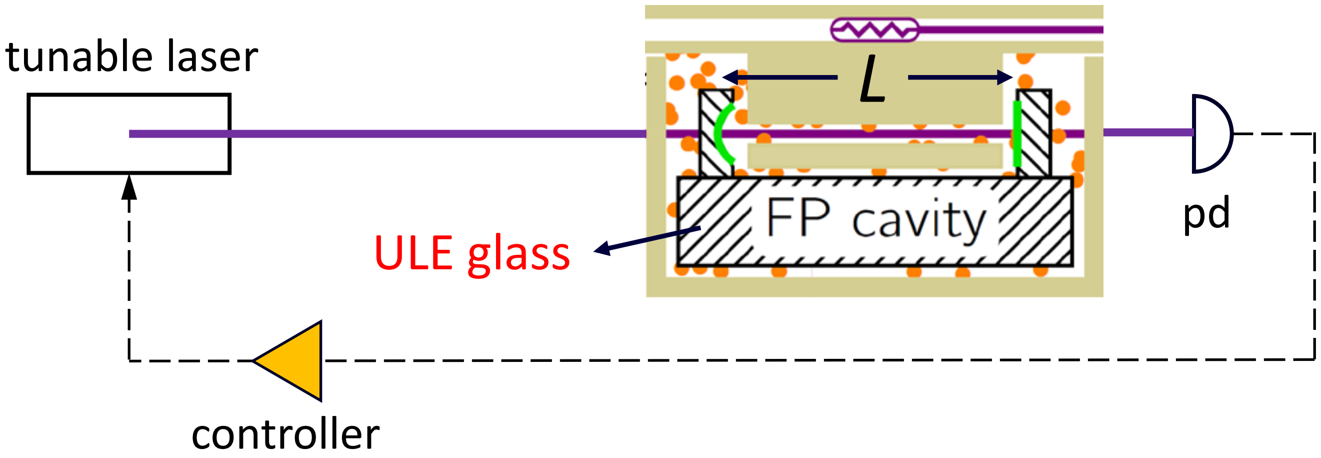
relativistic + QED

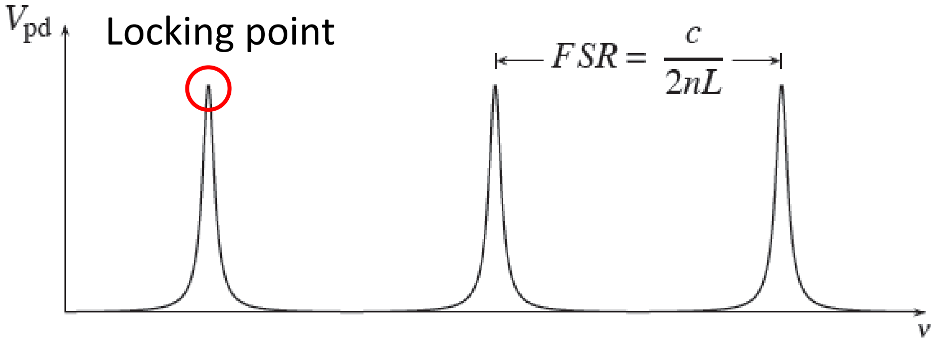
Puchalski et al., PRA 101, 022505, 2020

0.1 ppm

How to realize an optical pressure standard

The core part is a Fabry-Perot cavity: a pair of HR coating mirrors separated with a fixed length.





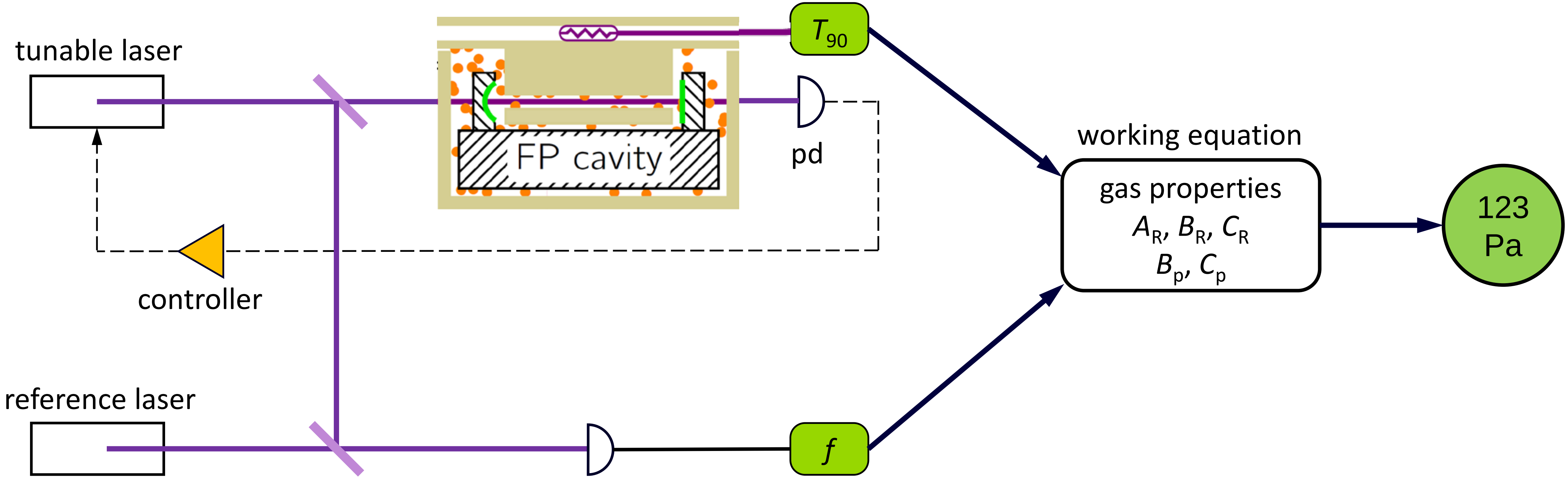
Vacuum: $\nu_0 = m\frac{c}{2L}$

Fill gas: $\nu_p = (m + \Delta m)\frac{c}{2nL}$

$$n - 1 = \frac{\nu_0 - \nu_p + \Delta m\frac{c}{2L}}{\nu_p} \equiv \frac{\Delta f}{\nu}$$

How to realize an optical pressure standard

The core part is a Fabry-Perot cavity: a pair of HR coating mirrors separated with a fixed length.

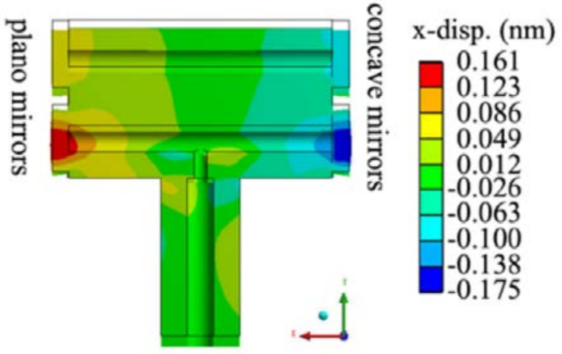


But the cavity length changes under pressure

$$n - 1 = \frac{\Delta f}{\nu} + n\kappa p$$

$\kappa \approx 10^{-11}\ \mathrm{Pa}^{-1}$ for ULE

- Targeting pressure uncertainty of 10^{-6}, κ should be known within 10^{-4}, i.e. 10^{-15} /Pa.
- Elastic property of ULE glass is not well known. FEA has large uncertainty.
- A dual cavity has been adopted by NIST and NIM. Effective κ reduces to $10^{-12}\ \mathrm{Pa}^{-1}$, still high.
- Determination of κ relied on traditional pressure standards. Not primary.



$$p\frac{3A_{\mathrm{R}}}{2RT} = n - 1 = \frac{\Delta f}{\nu} + n\kappa p$$

Two gas method

Helium

$$p\frac{3A_{\mathrm{R}}(\mathrm{He})}{2RT}=\left(\frac{\Delta f}{\nu}\right)_{\mathrm{He}}+n_{\mathrm{He}}\kappa p$$

gas: N_2 or Ar

$$p\frac{3A_{\mathrm{R}}(\mathrm{gas})}{2RT}=\left(\frac{\Delta f}{\nu}\right)_{\mathrm{gas}}+n_{\mathrm{gas}}\kappa p$$

Two equations for two unknowns

$A_{\mathrm{R}}(\mathrm{N_2})$ or $A_{\mathrm{R}}(\mathrm{Ar})$ is about 8 times of $A_{\mathrm{R}}(\mathrm{He})$.

Other gas properties

Most accurate results

Two gas method is semi-primary!

gas	A_R/(cm³mol)	u_r	traceability	reference
nitrogen	4.446139 (633nm, 303 K)	3.6 ppm	NIST UIM	*Egan et al., Opt Lett, 40 (2015) 3945-3948.*
nitrogen	4.446175 (633nm, 303 K)	3.1 ppm	NIST PG	*Egan and Yang, Int. J. Thermophys., 44 (2023) 181.*
nitrogen	4.396718 (1542nm, 303 K)	3.1 ppm	NIST PG	*Egan and Yang, Int. J. Thermophys., 45 (2024) 120.*
argon	4.195735 (633 nm)	3.1 ppm	NIST PG	*Egan and Yang, Int. J. Thermophys., 44 (2023) 181.*
argon	4.149749 (1542nm)	3.1 ppm	NIST PG	*Egan and Yang, Int. J. Thermophys., 45 (2024) 120.*
argon	$A_\varepsilon + A_\mu + A_2\nu^2 + \cdots$	10 ppm	A_ε: PTB PG A_μ, A_2: theory	*Gaiser and Fellmuth, PRL 120, 123203, 2018.* *Lesiuk et al., PRA, 107 (2023) 042805.*

Two color method

Helium, 633nm $$p\frac{3A_R^{He}(633)}{2RT} = \left(\frac{\Delta f}{\nu}\right)_{633} + n_{He}\kappa p$$

fully-primary!

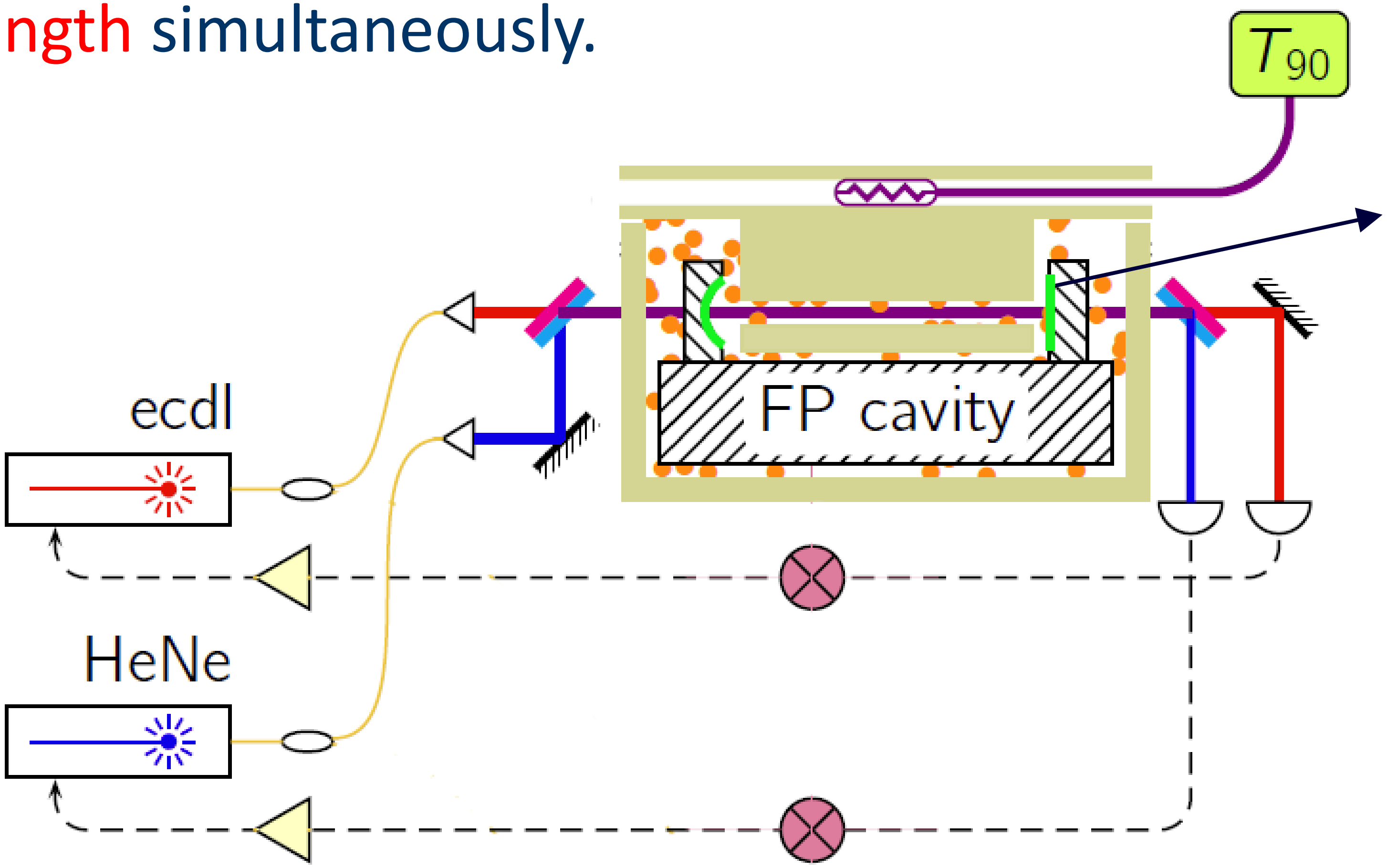
Helium, 1542nm $$p\frac{3A_R^{He}(1542)}{2RT} = \left(\frac{\Delta f}{\nu}\right)_{1542} + n_{He}\kappa p$$

Two equations for two unknowns

But $A_R^{He}(633)$ and $A_R^{He}(1542)$ only differs by 0.5%!
Separate measurement like the two gas method doesn't work.

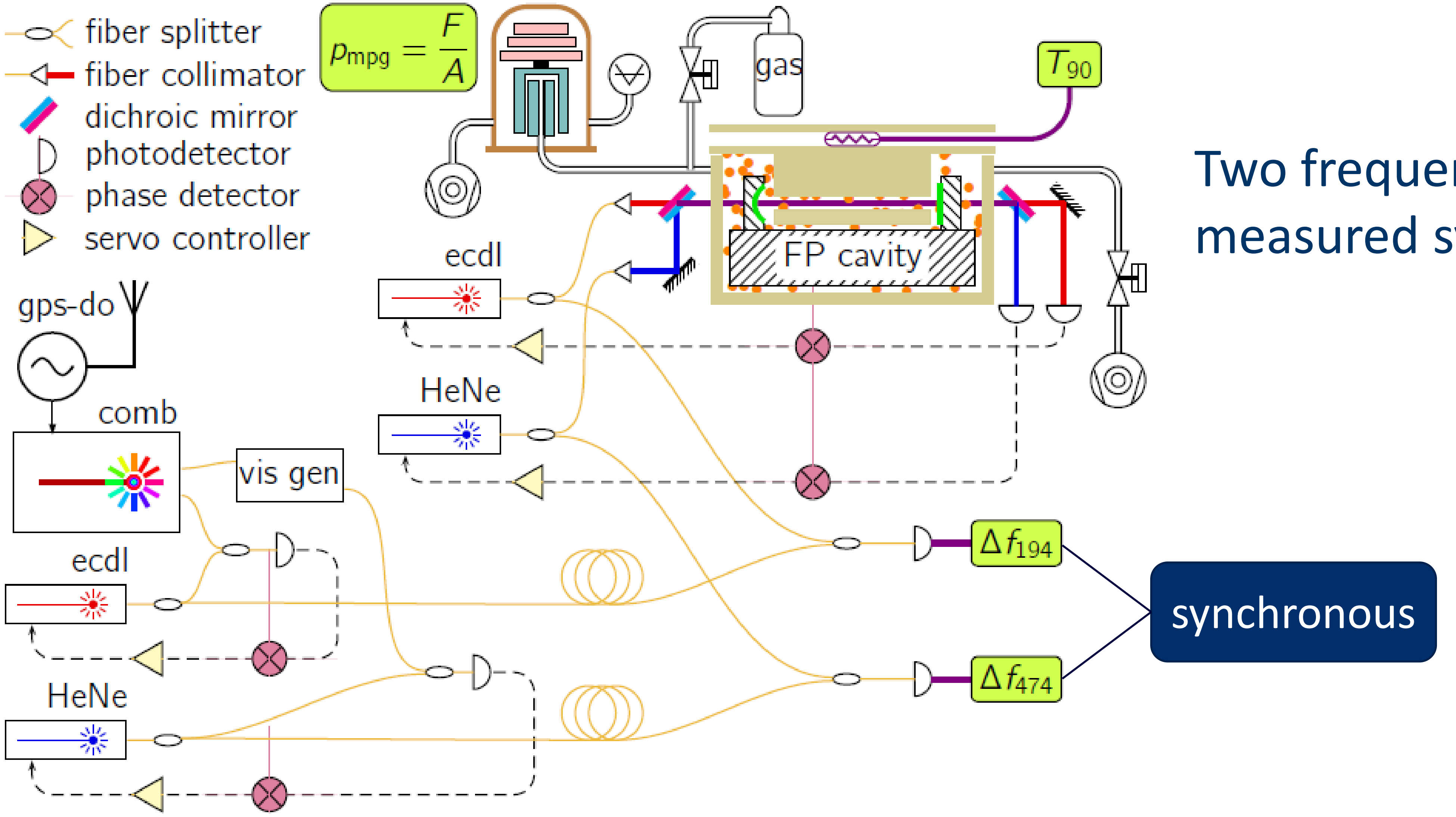
Two color method – correlated measurement

Two laser beams sense the same pressure, temperature, and cavity length simultaneously.



Mirrors are dual-coated at 633 nm and 1542 nm.

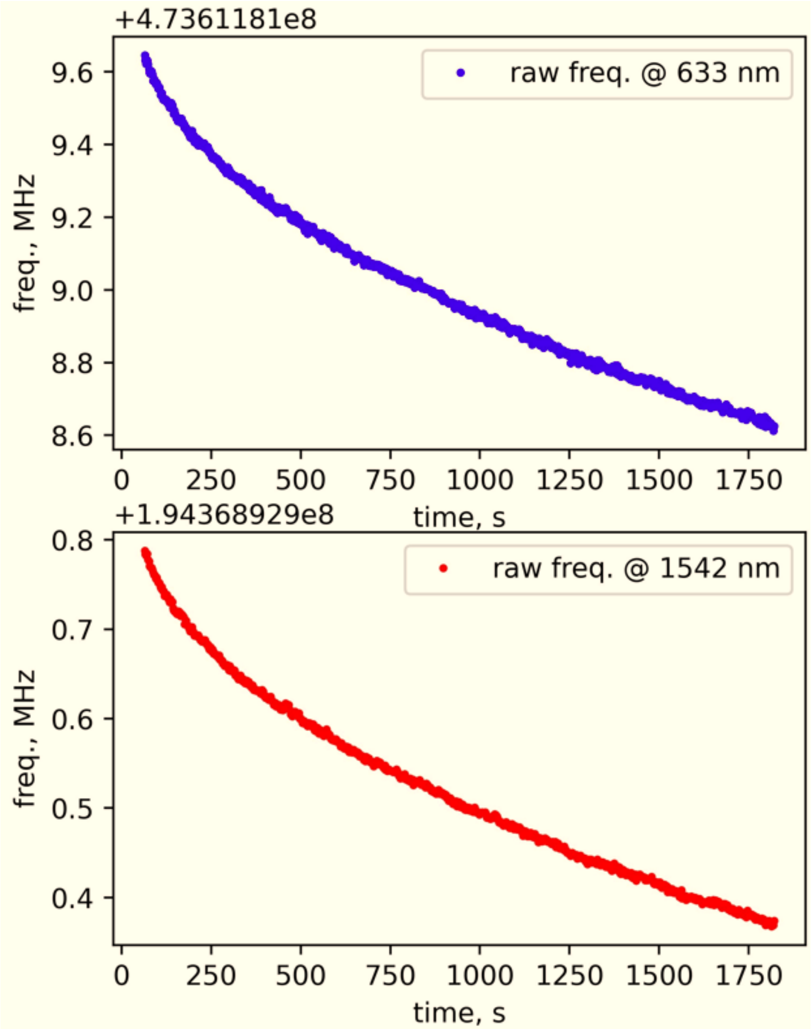
Two color method – correlated measurement



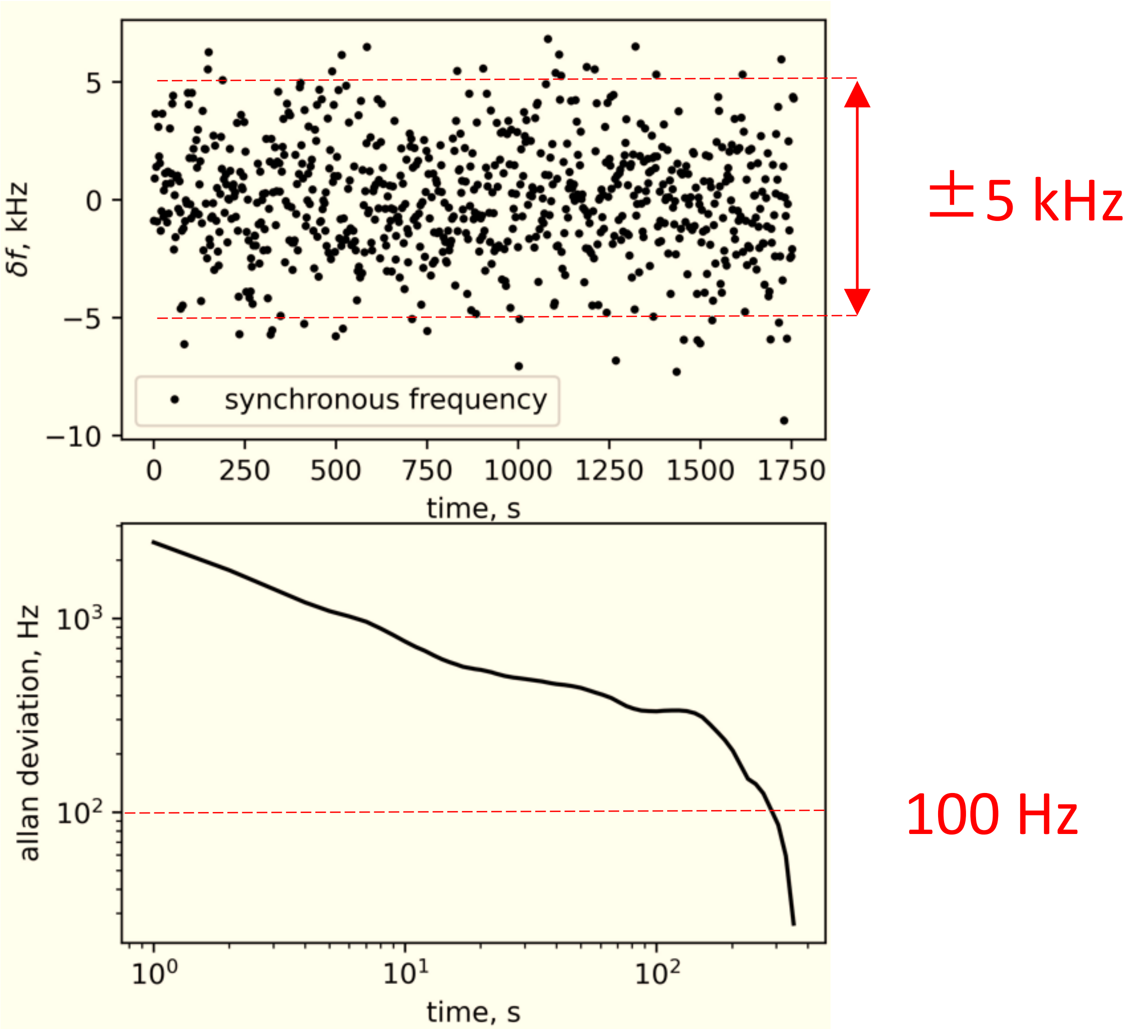
Two frequencies should be measured synchronously!

Noise of synchronous frequency

Frequencies drift due to helium diffusion into the ULE glass



Synchronous frequency doesn’t drift



Uncertainty analysis

$$p_{\text{ops}} = \frac{2RT}{3A_2\left(\overline{\nu}_{\text{blue}}^2 - \overline{\nu}_{\text{red}}^2\right)}\left[\left(\frac{\Delta f}{\nu}\right)_{\text{red}} - \left(\frac{\Delta f}{\nu}\right)_{\text{blue}}\right]$$

Synchronous frequency

$$p_{\text{ops}} = \frac{\left(\frac{\Delta f}{\nu}\right)_{\text{red}} - \left(\frac{\Delta f}{\nu}\right)_{\text{blue}}}{1.5 \times 10^{-12}\ \text{Pa}^{-1}} = \frac{\Delta f_{\text{red}} - \frac{\nu_{\text{red}}}{\nu_{\text{blue}}}\Delta f_{\text{blue}}}{711}\ \text{Pa}$$

100 Hz is equivalent to 0.14 Pa, i.e. 1.4 ppm @ 100 kPa

Uncertainty analysis

CCM

$$p_{\text{ops}} = \frac{2RT}{3A_2\left(\bar{\nu}_{\text{blue}}^2 - \bar{\nu}_{\text{red}}^2\right)}\left[\left(\frac{\Delta f}{\nu}\right)_{\text{red}} - \left(\frac{\Delta f}{\nu}\right)_{\text{blue}}\right]$$

$$A_R(\bar{\nu}) = A_\varepsilon + A_\mu + A_2\bar{\nu}^2 + A_4\bar{\nu}^4 + \cdots$$

$$\bar{\nu} = \frac{\nu}{10^{14}\ \text{Hz}}$$

4 ppm

Puchalski et al., PRA 93, 032515, 2016

Retardation correction: 40 ppm

Puchalski et al., PRA 99, 041803, 2019

Uncertainty analysis

$$p_{\mathrm{ops}} = \frac{2R{\color{red}T}}{3A_2\left(\bar{\nu}_{\mathrm{blue}}^2 - \bar{\nu}_{\mathrm{red}}^2\right)}\left[\left(\frac{\Delta f}{\nu}\right)_{\mathrm{red}} - \left(\frac{\Delta f}{\nu}\right)_{\mathrm{blue}}\right]$$

T

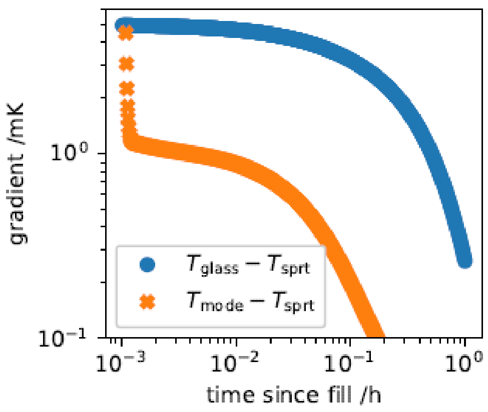
2.5 ppm

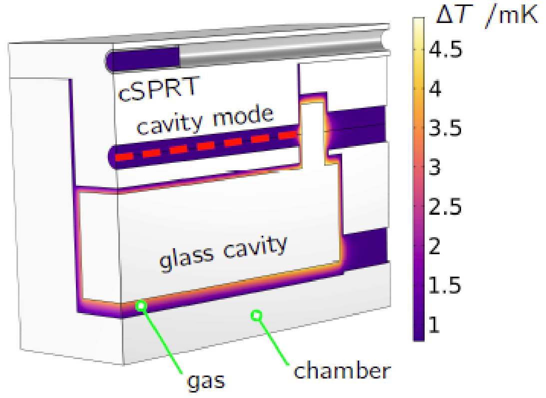
0.3 mK

- Calibration of cSPRT against ITS-90
- Temperature gradient

0.7 mK

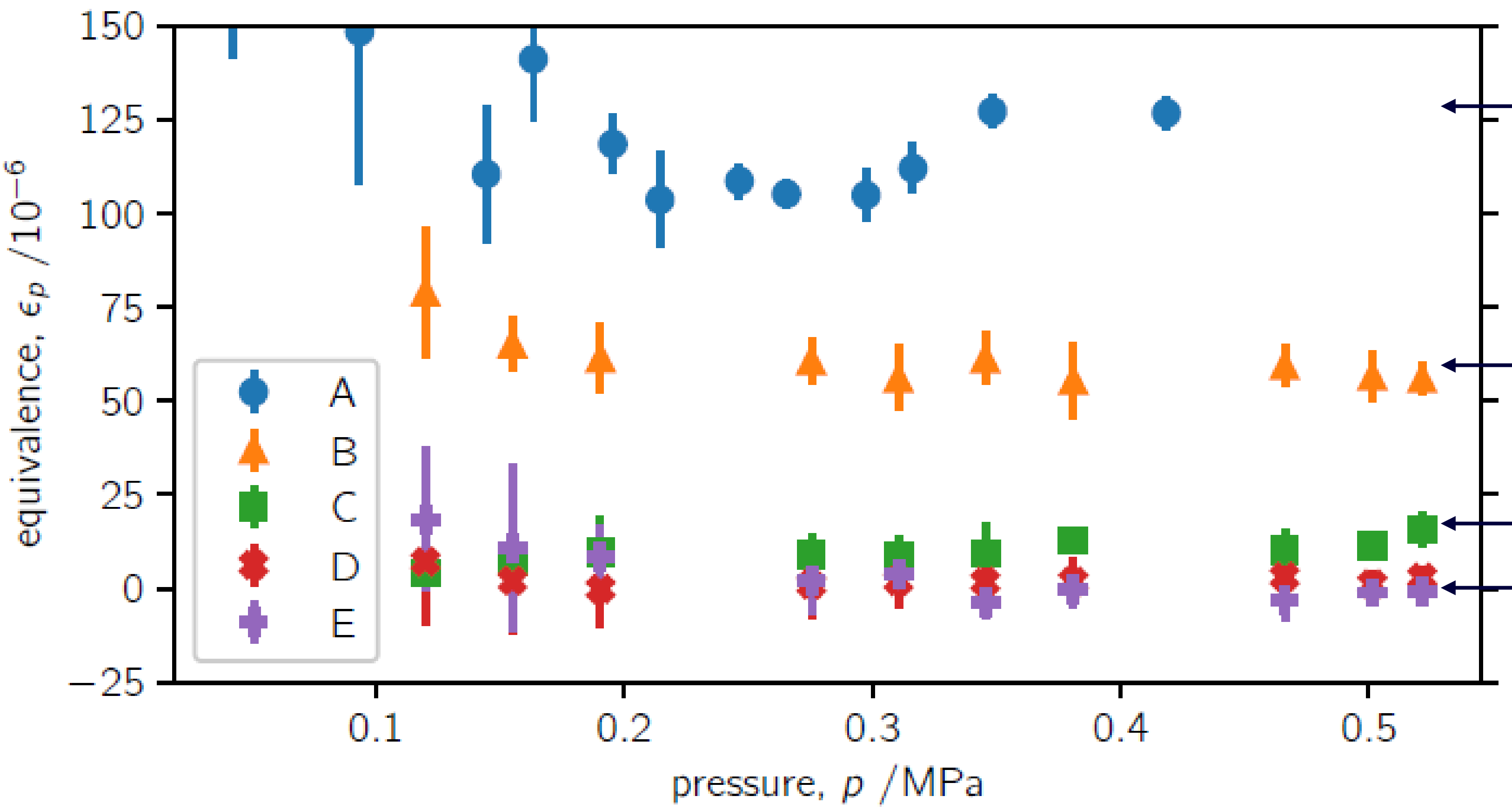
- Converting ITS-90 to thermodynamic temperature T





Gaiser et al., 2022 Update for the Differences Between Thermodynamic Temperature and ITS-90 Below 335 K, Journal of Physical and Chemical Reference Data, 51, 043105 (2022).

Helium impurity



Helium from an old depleted cylinder

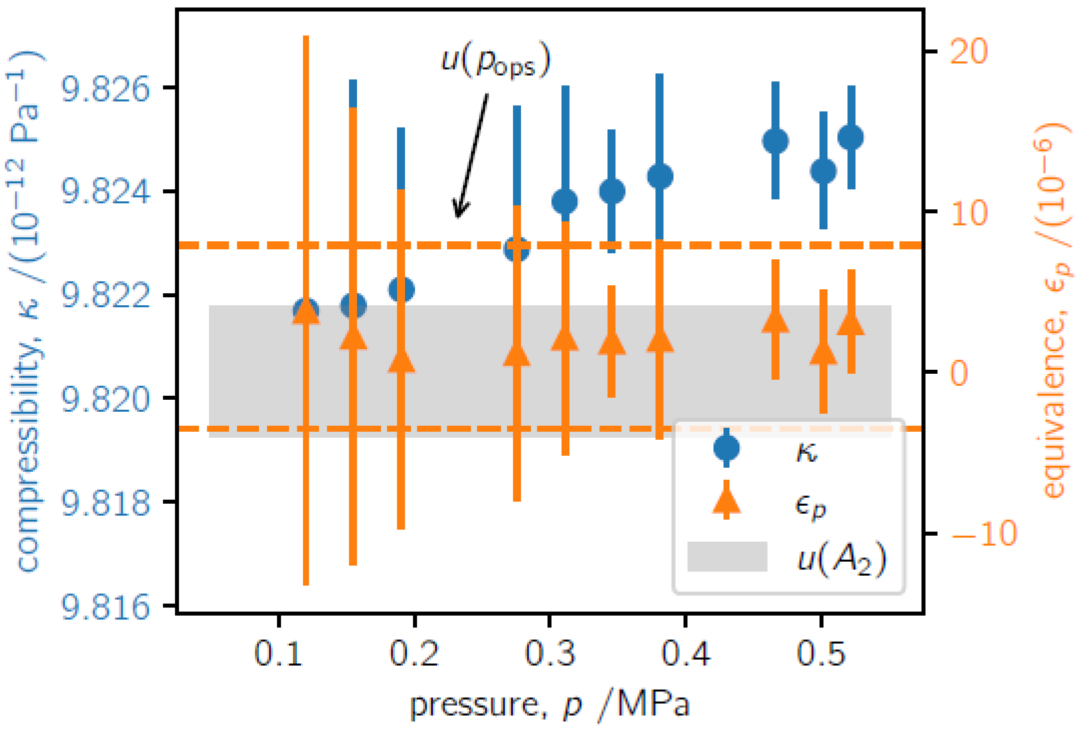
Helium cylinder with claimed 99.9999% purity

Purified by a getter

Purified by a getter and liquid-helium coldtrap

Fully-primary optical pressure scale

CCM



component	$u_r(p_{ops})$ $/10^{-6}$	$u_r(\kappa)$ $/10^{-4}$
He theory $A_R(\nu), B_\rho, \ldots$	4.0	< 0.1
temperature T	2.5	< 0.1
pressure p	—	1.0
frequency $\Delta f/\Delta\nu_{fsr}$	0.2	< 0.1
diffusion β	0.6	0.2
impurity	0.7	0.2
statistical	3.0	1.7
combined $k = 1$	5.7	2.0

Comparison with NIST 2 ppm piston gauge: 2.2 ppm ± 5.7 ppm

Neon, Argon, and nitrogen

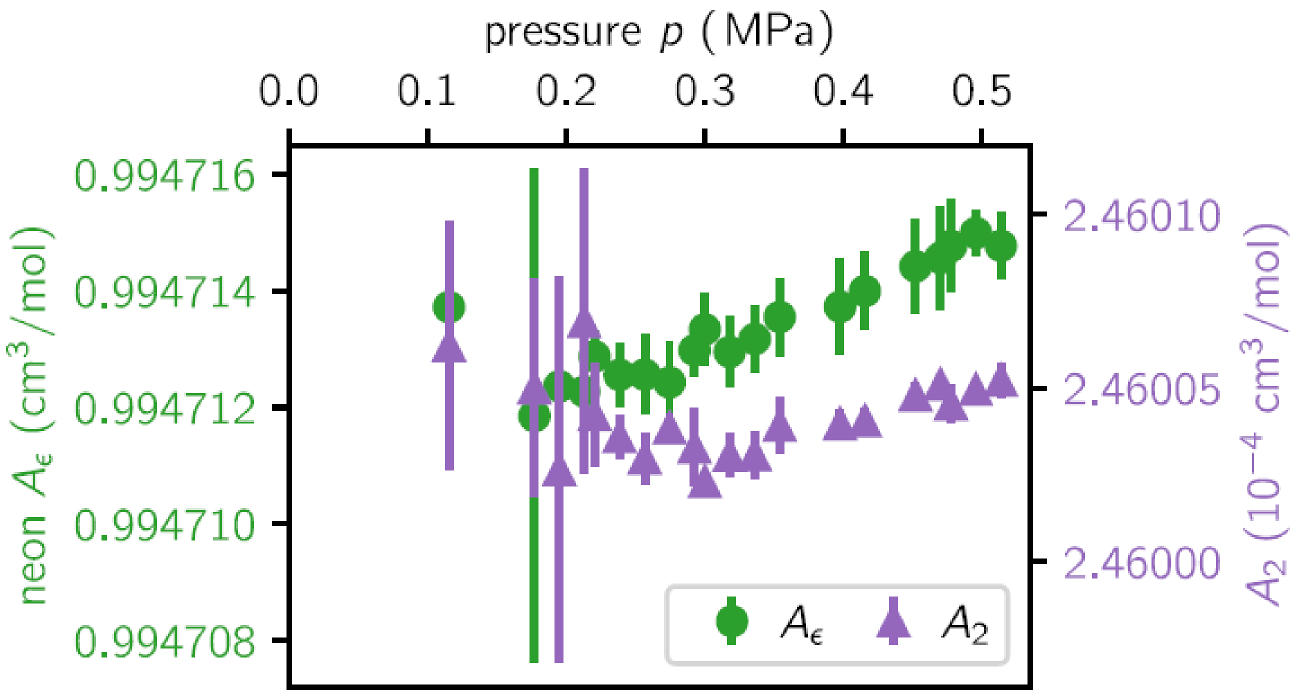
Using κ determined from helium measurement, and p from piston gauge calibrated by the optical pressure scale.

633nm $$p\frac{3\left(A_{\varepsilon}+A_{\mu}+A_{2}\bar{\nu}_{633}{}^{2}+A_{4}\bar{\nu}_{633}{}^{4}+\cdots\right)}{2RT}=\left(\frac{\Delta f}{\nu}\right)_{633}+n_{\mathrm{gas}}\kappa p$$

1542nm $$p\frac{3\left(A_{\varepsilon}+A_{\mu}+A_{2}\bar{\nu}_{1542}{}^{2}+A_{4}\bar{\nu}_{1542}{}^{4}+\cdots\right)}{2RT}=\left(\frac{\Delta f}{\nu}\right)_{1542}+n_{\mathrm{gas}}\kappa p$$

Two equations for two unknowns

Neon

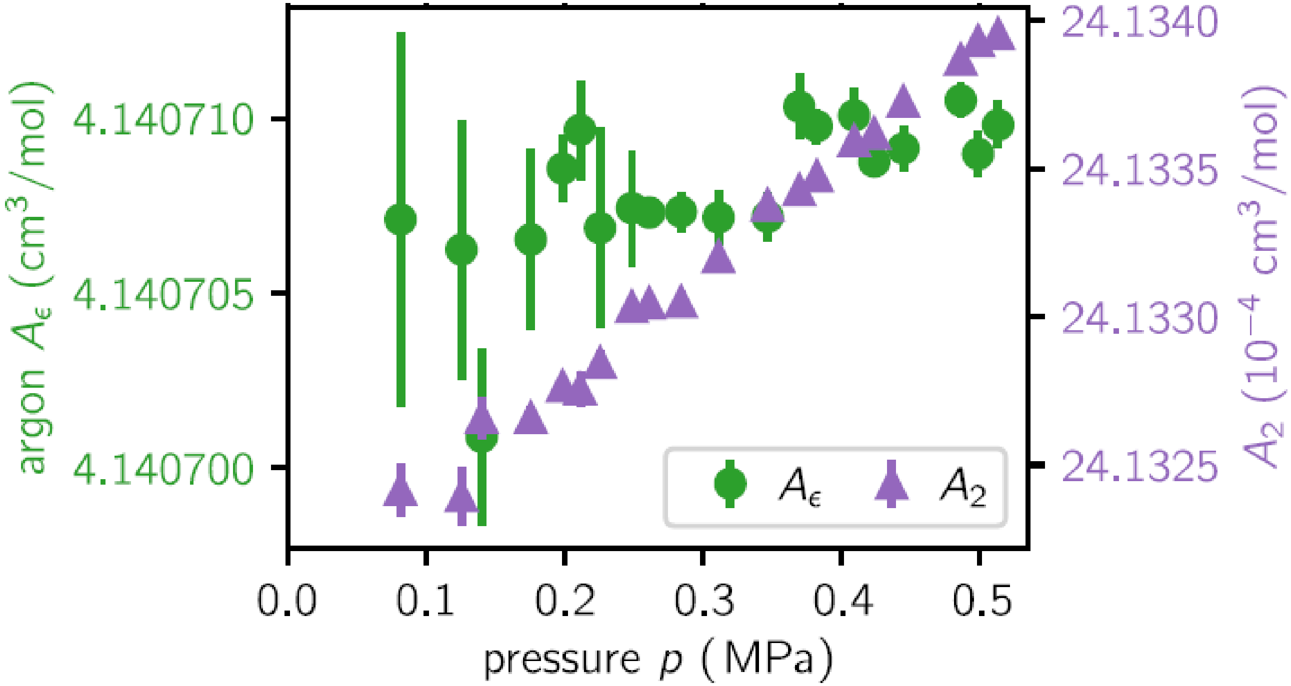


A_ε /(cm^3/mol)	A_2 /(10^{-4}cm^3/mol)	Ref.
0.9946(1)	2.461(5)	Theo. [1]
0.994711(2)		Exp. [2]
0.994709(8)	2.46000(8)	This work

Theo. [1]: Lesiuk et al., PRA 102, 052816 (2020).

Exp. [2]: Gaiser and Fellmuth, PRL 120, 123203 (2018).

Argon

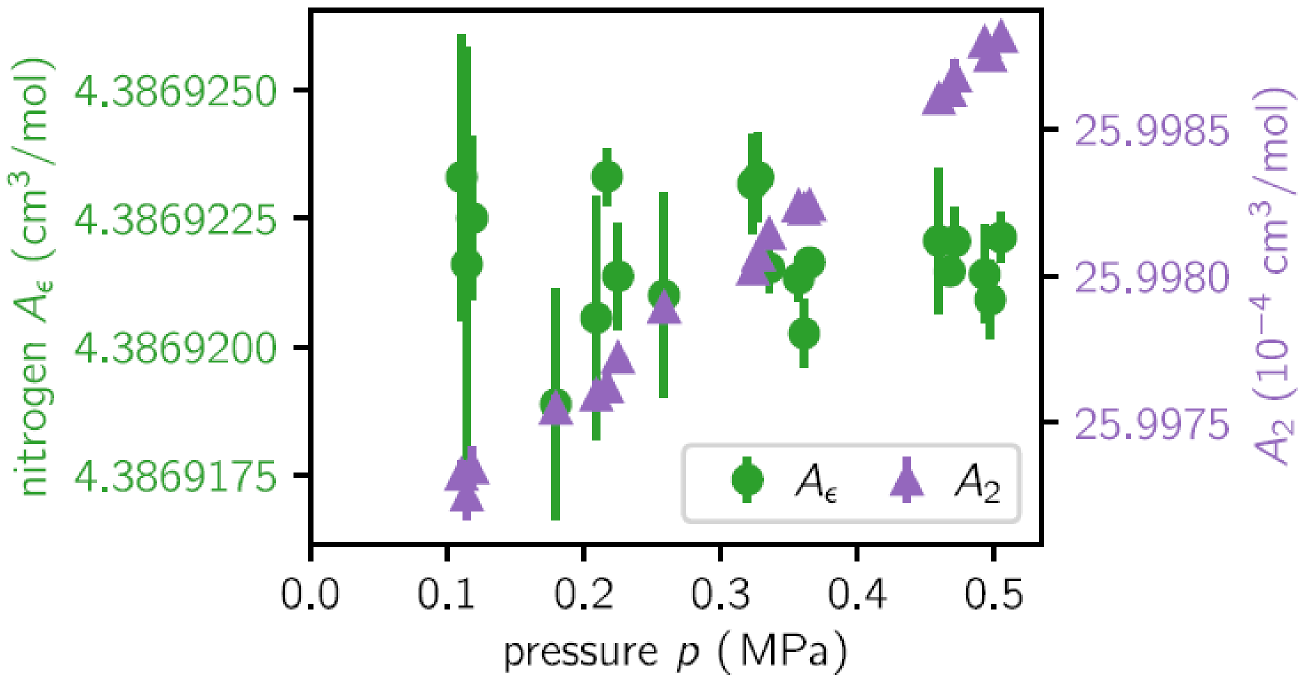


A_ε /(cm^3/mol)	A_2 /(10^{-4}cm^3/mol)	Ref.
4.1408(7)	24.16(1)	Theo. [1]
4.14069(1)		Exp. [2]
4.14070(3)	24.1320(8)	This work

Theo. [1]: Lesiuk et al., PRA 107, 042805 (2023).

Exp. [2]: Gaiser and Fellmuth, PRL 120, 123203 (2018).

Nitrogen



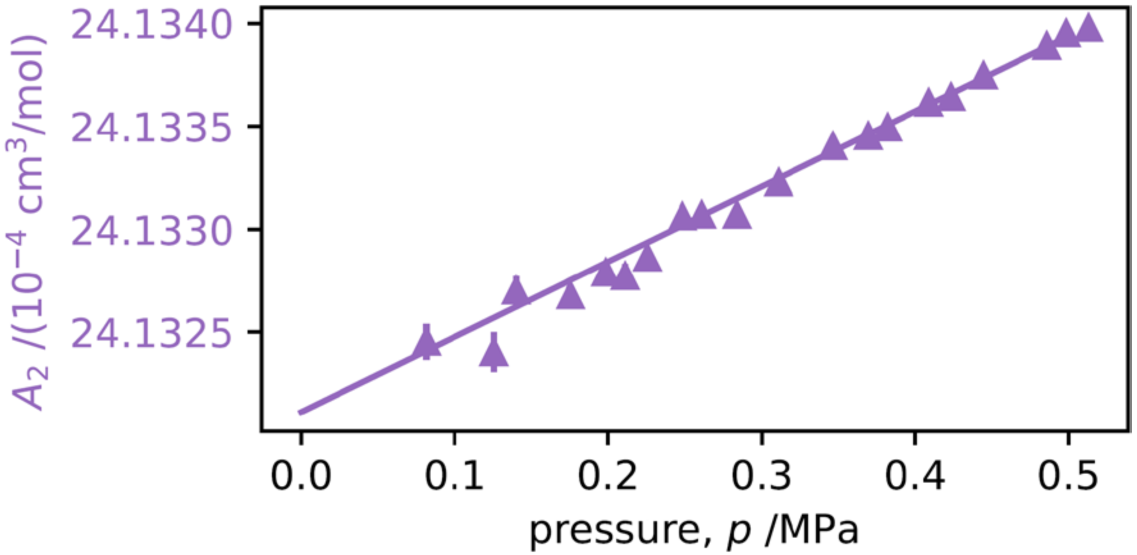
A_ε /(cm^3/mol)	A_2 /(10^{-4}cm^3/mol)	Ref.
4.388	25.70	Exp. [1]
4.3877(2)		Exp. [2]
4.38692(3)	25.997(5)	This work

Exp. [1]: Kumar et al., Theor. Chim. Acta 82, 131 (1992).

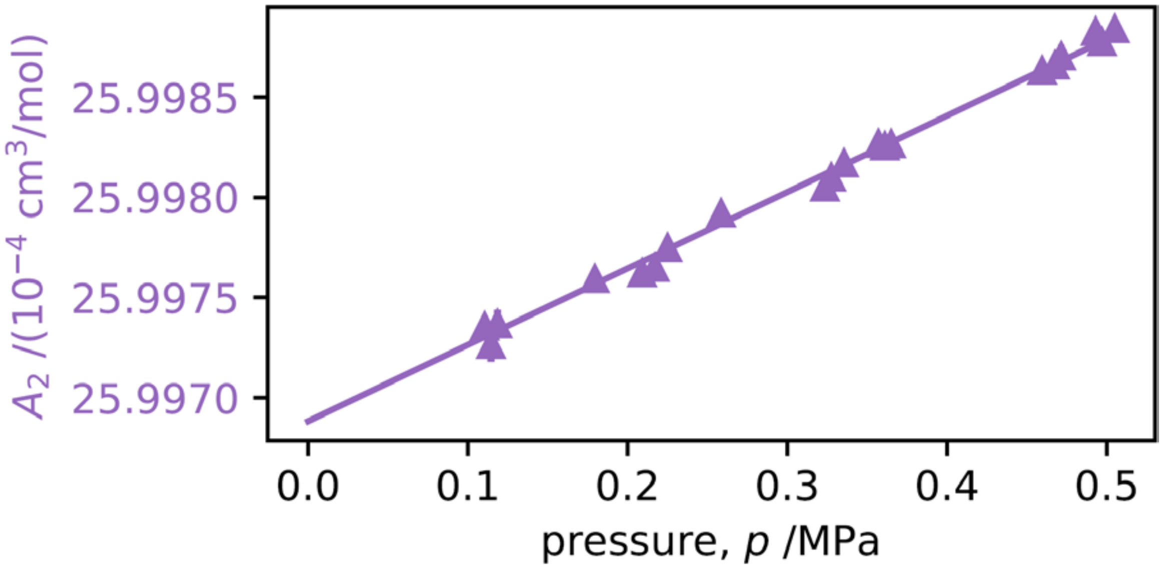
Exp. [2]: Schmidt and Moldover, Int. J. Thermophys. 24, 375 (2003).

Argon and nitrogen: trend in $A_2(p)$

argon



nitrogen

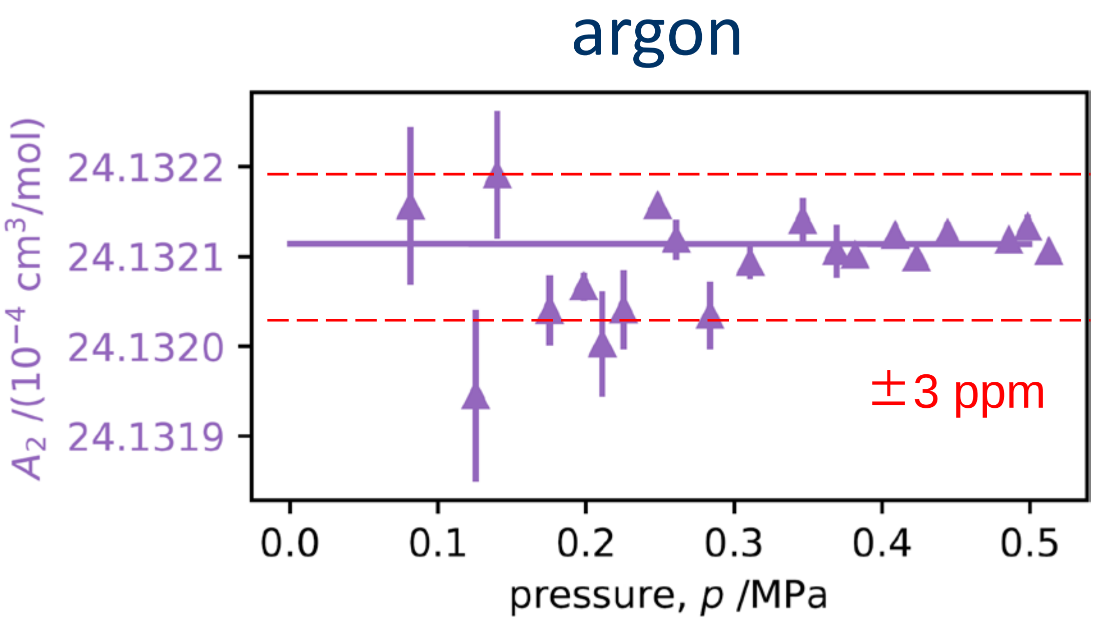


Argon and nitrogen: trend in $A_2(p)$

$$n - 1 = p\frac{3A_{\mathrm{R}}}{2RT} + p^2\frac{3}{8(RT)^2}\left(A_{\mathrm{R}}^2 - 4A_{\mathrm{R}}B_\rho + 4B_{\mathrm{R}}\right) + o(p^3)$$

$$B_{\mathrm{R}}(\nu) = B_\varepsilon + B_{\mathrm{R}}^{(2)}\bar{\nu}^2$$

Argon: adjusted $B_{\mathrm{R}}^{(2)}$ for no trend in $A_2(p)$



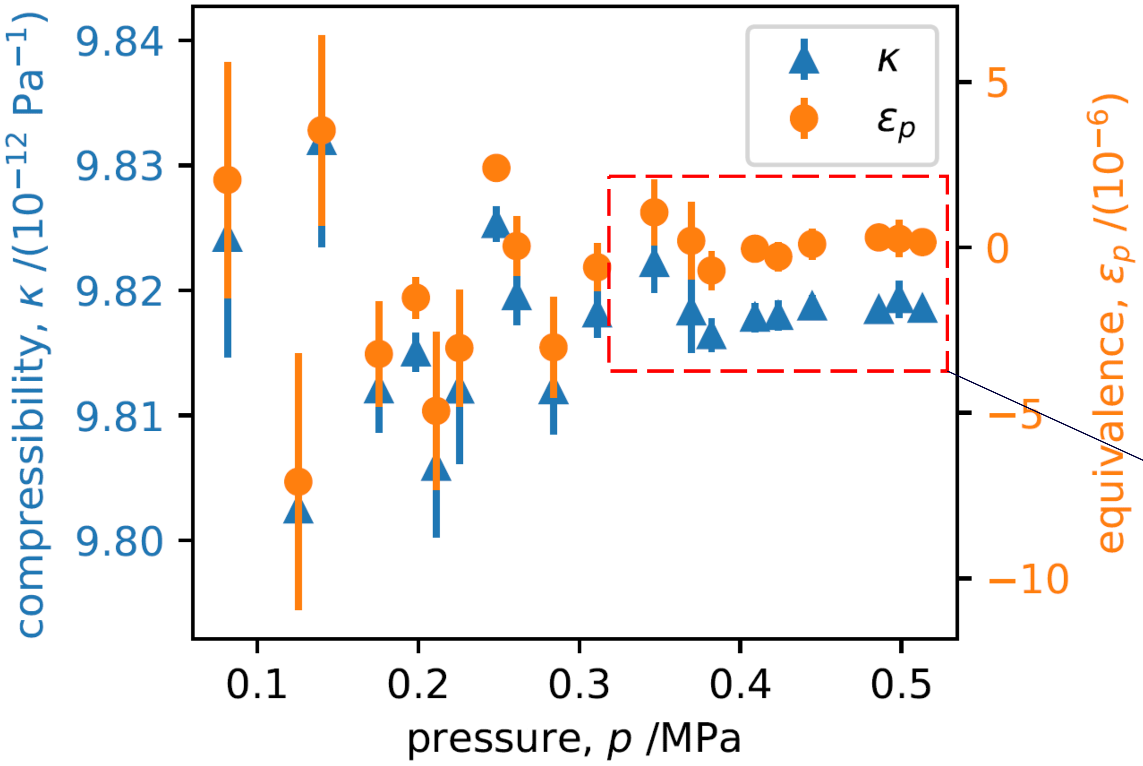
Unit: cm^6/mol^2

T/(K)	B_ε	$U(B_\varepsilon)$	$B_{\mathrm{R}}^{(2)}$	B_{R} 633 nm	B_{R} 1542 nm	Ref.
300	1.71	0.11	4.79	1.73	1.71	Theor.
300	1.71	0.11	8.76	1.75	1.71	This work

Theor. : Garberoglio and Harvey, J. Res. Natl. Inst. Stand. Technol. 125, 125022 (2020).

Practical optical pressure scale using argon

Using A_ε, A_2, and $B_\mathrm{R}^{(2)}$ obtained in this work, solving p and κ from the two-color equations.

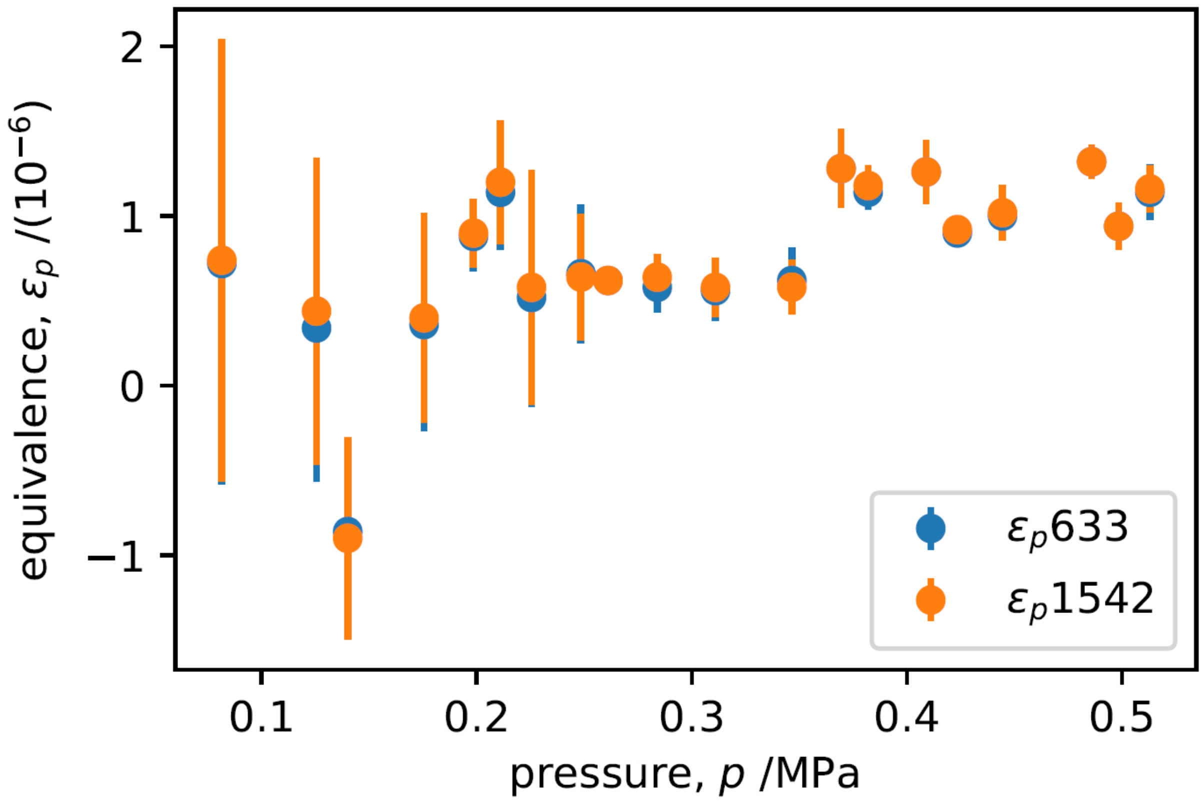


- Primary
- No diffusion problem
- Less sensitive to impurity than helium

Sub-ppm noise at high pressures

Practical optical pressure scale using argon

Using κ determined at high pressures, then apply single wavelength calculations.



Statistic uncertainty below 1 ppm!

Summary

- First realization of a fully-primary optical pressure scale based on the dispersion property of helium.
- Most accurate dispersion data for neon, argon, and nitrogen.
- Pressure trend in $A_2(p)$ with current literature data of $B_R^{(2)}$.
- Practical optical pressure scale using argon can achieve statistic uncertainty below 1 ppm.

Thank you!