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On the effect of two extended dead times in series

Jörg W. Müller

The purpose of this note is to determine in an exact way the influence of two consecutive dead times of the extended type on the count rate if they are inserted into a sequence of pulses. The original process, which may be due to the decays of a radioactive source, is assumed to be of the Poisson type (Fig. 1). It will be shown that insertion of the first dead time τ', contrary to naive expectation, results in increasing the count rate R at the output rather than diminishing it, a result which - at first sight at least - seems to be at variance with common sense which considers a dead time as a passive element causing always losses of counts.

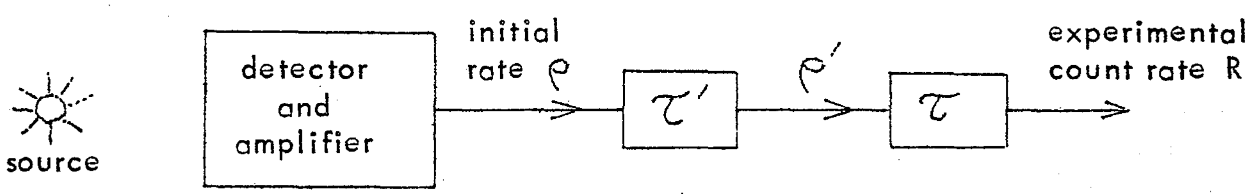

Fig. 1: Schematic arrangement of the experiment

As has been seen earlier in the treatment of the analogous problem for non-extended dead times [1], the only case of real interest is for $\tau > \tau'$ since otherwise the second dead time would have no influence at all. We can therefore put

$$\tau' = \alpha \cdot \tau \qquad \text{with } 0 \leq \alpha \leq 1 . \tag{1}$$

In order to evaluate the effect of the dead times, it is practical to consider first the time distribution of the events. Thereby we assume that detection and amplification of the pulses from the source (with sufficiently long half-life to neglect decay during the measurement) result in a random selection among all the particles emitted by the source, without any additional time distortion. The sequence of pulses reaching the first dead time circuit is thus still Poissonian with a mean count rate ρ , including finite solid angle and detection efficiency. The corresponding density for the time differences between subsequent pulses is therefore a simple exponential

$$f(t) = \rho \cdot e^{-\rho t} , \qquad \text{for} \quad t > 0 . \tag{2}$$

The effect of an extended dead time on the interval density has been studied earlier in detail [2] . It follows from formula (21) given there that the time distribution of the pulses which have passed the first dead time τ' is given by

$$F(t) = \sum_{i=1}^{\infty} A_i(t) ,$$

$$\text{where } A_i(t) = U(t - i\tau') \cdot \frac{(-1)^{i-1}}{(i-1)!} \cdot \rho^i (t - i\tau')^{i-1} \cdot e^{-i\rho\tau'} \tag{3}$$

and U is the unit step function.

The rate of the process at this point in the circuit (between the two dead times) is easily found to be given by

$$\rho' = \rho \cdot e^{-\rho\tau'} . \tag{4}$$

It is well known that an extended dead time of length τ has the effect of eliminating any pulse the distance of which with respect to its predecessor is smaller than τ . But instead of determining the new interval density - a task which would be quite cumbersome, we note that for evaluating the resulting count rate it is sufficient to know the relative percentage P of the pulses which are eliminated by τ .

Since $F(t)$ is normalized to unity, this fraction is given by

$$P = \int_0^{\tau} F(t)\,dt = \sum_{i=1}^{\infty} \int_0^{\tau} A_i(t)\,dt = \sum_{i=1}^{J} \int_{i\tau'}^{\tau} A_i(t)\,dt \equiv \sum_{i=1}^{J} p_i , \tag{5}$$

where $J \equiv \left[\left[1/\alpha\right]\right]$ is the largest integer below $1/\alpha = \tau/\tau'$.

Once P is known, the final experimental count rate R after both dead times is simply determined by

$$R = \rho' \cdot (1 - P) . \tag{6}$$

The only remaining problem therefore consists of evaluating (5). For this purpose let us consider the integral

$$p_i = \int_{i\tau'}^{\tau} A_i(t)\,dt = \frac{(-1)^{i-1}}{(i-1)!} \cdot \rho^i \cdot e^{-i\rho\alpha\tau} \cdot \int_{i\alpha\tau}^{\tau} (t - i\alpha\tau)^{i-1}\,dt$$

$$= \frac{(-1)^{i-1}}{(i-1)!} \cdot \rho^i \cdot e^{-i\rho\alpha\tau} \cdot \int_0^{\tau(1-i\alpha)} y^{i-1}\,dy$$

$$= \frac{\rho\tau(-\rho\tau)^{i-1}}{i!} \cdot e^{-i\rho\alpha\tau} \cdot (1 - i\alpha)^i \quad , \qquad (7)$$

for $i < 1/\alpha$.

With the abbreviation $\rho\tau \equiv x$, this results, according to (5), in a relative loss P due to the second dead time τ of

$$P = \sum_i p_i = x \cdot \sum_{i=1}^{J} \frac{(-x)^{i-1}}{i!} \cdot e^{-i\alpha x} \cdot (1 - i\alpha)^i$$

$$= \sum_{i=1}^{J} \frac{(-1)^{i-1}}{i!} \cdot \left[x\,(1 - i\alpha)\right]^i \cdot e^{-i\alpha x} \quad . \qquad (8)$$

The rate R of particles at the end of both dead times ($\alpha\tau$ and τ , in this order) is thus given by (6) as

$$R = \rho e^{-\rho\tau'} \cdot (1 - P) = \rho e^{-\alpha x} \cdot \left\{ 1 + \sum_{i=1}^{J} \frac{\left[-x(1 - i\alpha)\right]^i}{i\,!} \cdot e^{-i\alpha x} \right\} \quad . \qquad (9)$$

In particular let us denote by R_o the count rate at the output in the absence of a first dead time, i.e. for $\alpha = 0$.

We can then put

$$R = R_o \cdot T \; , \qquad (10)$$

where T is a transmission factor which indicates the relative change in the output count rate due to the presence of τ'.

Since by analogy with (4) we have

$$R_o = \rho \cdot e^{-\rho\tau} , \qquad (11)$$

the transmission turns out to be

$$T = \frac{R}{R_o} = e^{x(1-\alpha)} \cdot \left\{ 1 + \sum_{j=1}^{J} \frac{[-x(1-j\alpha)]^j}{j!} \cdot e^{-j\alpha x} \right\}$$

$$= e^{-\alpha x} \cdot \sum_{j=0}^{J} \frac{[-x(1-j\alpha)]^j}{j!} \cdot e^{x(1-j\alpha)} \qquad (12)$$

with

$$x = \rho \cdot \tau \, ,$$

$$\alpha = \tau'/\tau \leq 1 \qquad \text{and}$$

$$J = \left\lceil [1/\alpha] \right\rceil .$$

Numerical values of T as a function of α , calculated on the IBM 1130 computer of the BIPM, are given for various parameters x by the graphs in Fig. 2. A closer inspection of the transmission $T(\alpha)$ shows that it has in the range $0 \leq \alpha \leq 1$ a single maximum – again in contrast to the more complicated structure found earlier for the non-extended case [1] – and becomes unity at the ends. Whereas this maximum lies at $\alpha = 3/2$ for very low values of the parameter $x = \rho\tau$, its position is steadily shifted towards smaller ratios α if x increases, reaching $\alpha = 1/2$ for $x \cong 2.5$. We note in passing that for all values of α which are below the position of the maximum, an increase of the first dead time τ' augments the output rate R. If $x \ll 1$, the behaviour of T can be approximated by

$$T \cong 1 + \frac{1}{2}(\alpha x)^2 \qquad \text{for} \quad \alpha \longrightarrow 0$$

and

$$T \cong 1 + x^2(1-\alpha) - 2x^2(1-\alpha)^2 \qquad \text{for} \quad \alpha \longrightarrow 1.$$

The fact that T always exceeds unity shows that τ' , instead of producing additional losses (as we probably anticipated), has just the opposite effect. But since a dead time clearly cannot produce additional pulses, the cause for the surprising effect needs an indirect explanation. In fact, the first dead time τ' deforms the initial exponential interval density in such a way that the subsequent elimination of pulses by τ is much less effective than it would have been if τ' were absent.

Thus in contrast to the result for two non-extended dead times [1] where τ' always produced additional losses, in the case of the extended type discussed here the first dead time has the unexpected opposite effect of reducing the losses, and augmenting thereby the experimental count rates at the output. The first tentative experiments tend to confirm very well the theoretical expectation described above with regard to sign as well as to the magnitude of the effect.

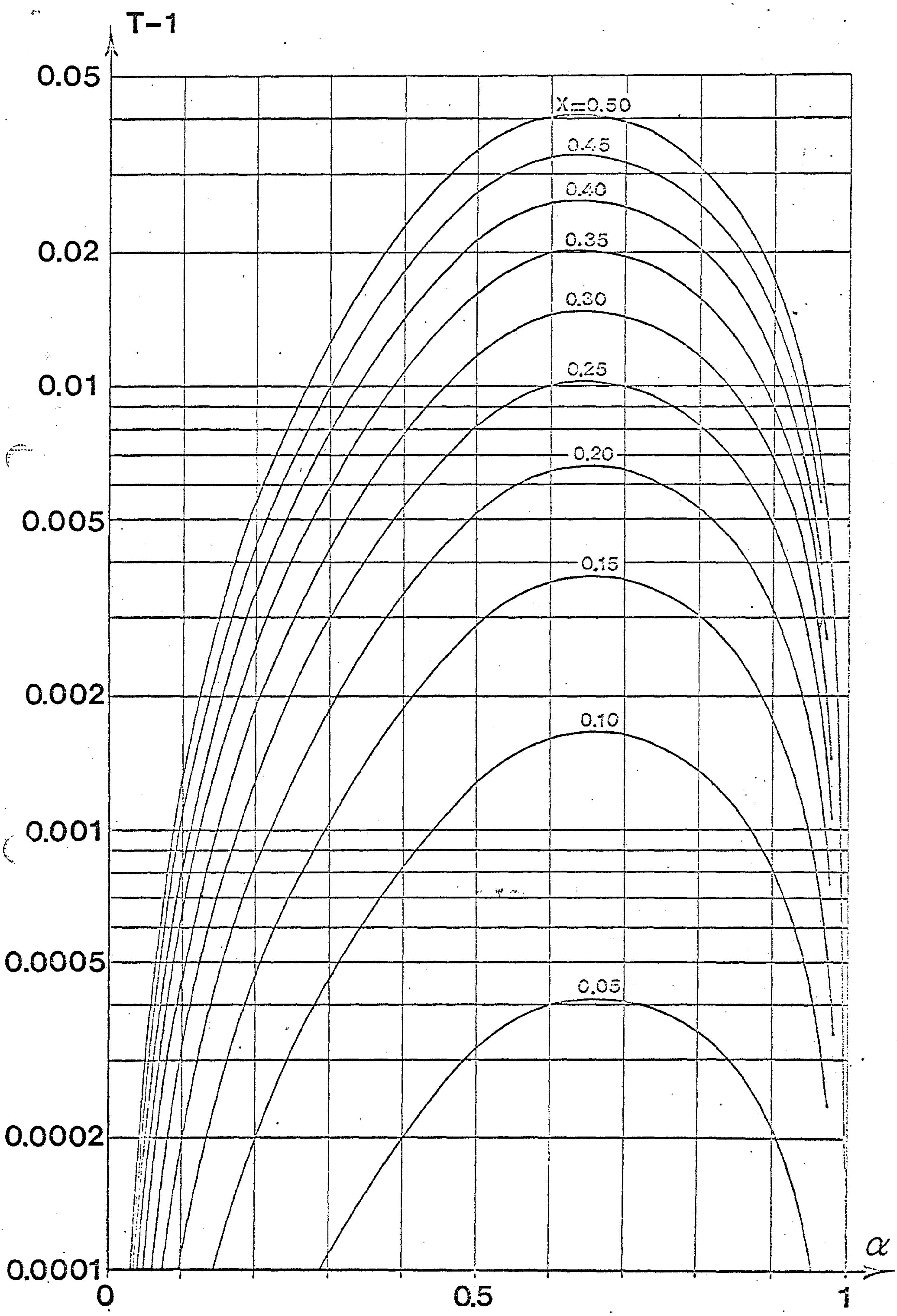

Fig. 2a. - Transmission factor T for x from 0.05 to 0.5

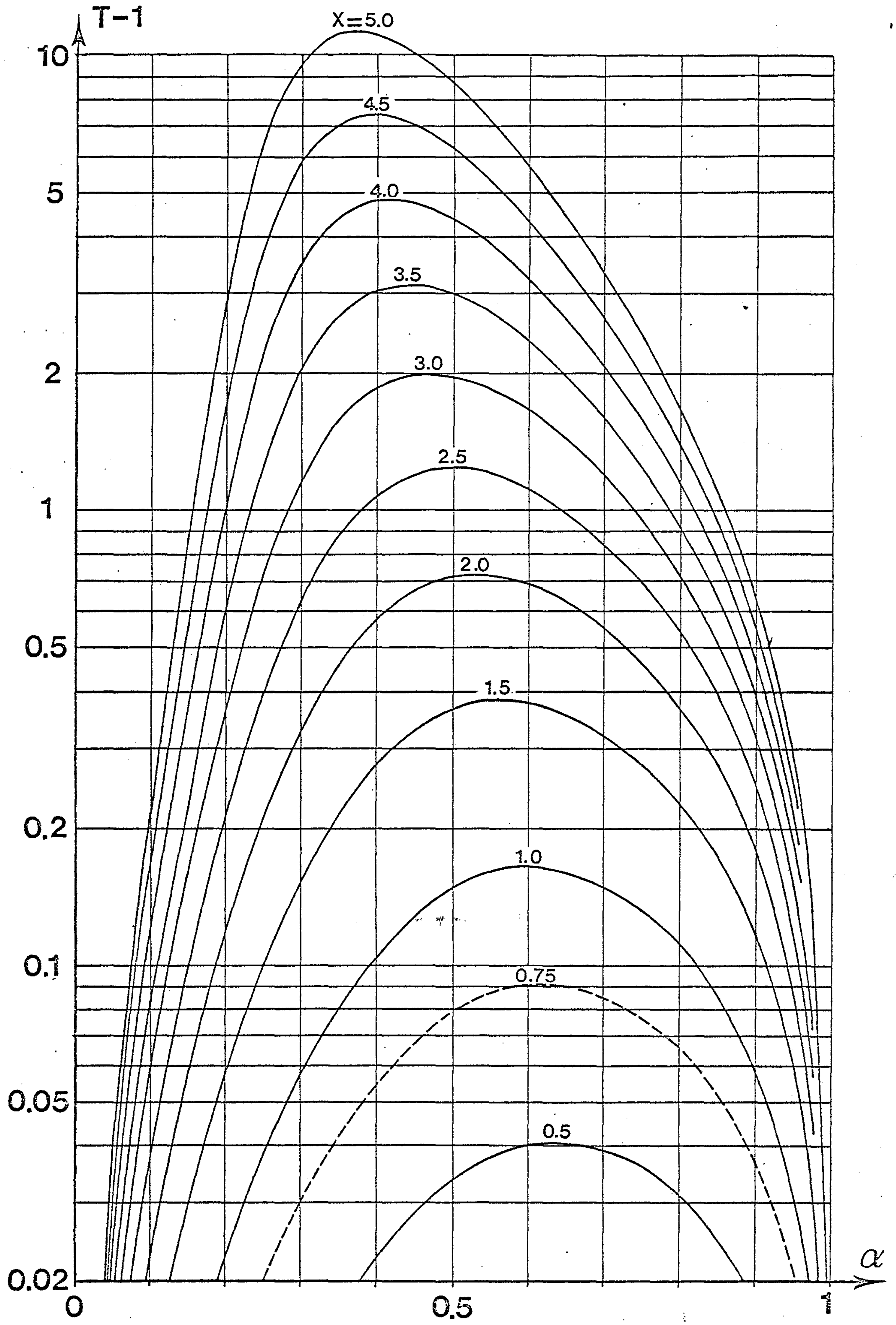

Fig. 2b. - Transmission factor T for x from 0.5 to 5.0

References

[1] J.W. Müller: "On the influence of two consecutive dead times", Report BIPM-106 (March 1968). For numerical values see "Procès-Verbaux du Comité International des Poids et Mesures", 36 (1968) p. 70.

[2] J.W. Müller: "Interval densities for extended dead times", Report BIPM-112 (March 1971).

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