Bilateral Comparison of 10 V Standards between the NML, Ireland and the BIPM,

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A comparison of the 10 V voltage reference standards of the BIPM and the National Metrology Laboratory, (NML), Dublin, Ireland, was carried out in June 1999. Two BIPM 732B Zener diode-based travelling standards, BIPM4 and BIPM5, were transported by freight. The BIPM measurements of the travelling standards were carried out by direct comparison to the Josephson effect standard or by dividing the 10 V outputs to 1.018 V by means of a resistive divider and comparing these values to the electromotive force of a standard cell. Results obtained from this indirect method were checked against those obtained by measuring the 10 V outputs directly with the BIPM Josephson array and the two were found to differ by less than 0.1 μV. Results of all measurements were corrected for the dependence of the output voltage on ambient temperature and pressure.

Figures 1 and 2 show the measured values obtained for the two standards by the two laboratories. The BIPM values and uncertainties, and those of the NML are calculated for the reference date from the least-squares fits.

Table 1 lists the results of the 10 V comparison and the component uncertainty contributions for the comparison NML/BIPM. Experience has shown that flicker or 1/*f* noise dominates the stability characteristics of Zener-diode standards and it is not appropriate to use the standard deviation of the mean to characterize the dispersion of measured values. For the present standards, the flicker floor voltage is about 0.1 μV.

In estimating the uncertainty we have calculated the *a priori* uncertainty based on all known sources except that associated with the stability of the standards when transported. We compare this with the *a posteriori* uncertainty estimated by the standard deviation of the mean of the results from the two travelling standards. With only two travelling standards, the uncertainty of the standard deviation of the mean is comparable to the value of the standard deviation of the mean itself. If the *a posteriori* uncertainty is significantly different from the *a priori* uncertainty, we assume that a standard has changed in an unusual way and we use the larger of these two estimates in calculating the final uncertainty.

Remarks on the Items in Table 1

Items 1, 2, 3 and 4 are the NML values, the type-A, type-B and the combined standard uncertainties, respectively, for each Zener, expressed in μV. Item 2 is estimated by noting that the stability of the Zeners can be described by flicker noise (1/*f* noise) with a floor value of about 1 part in 10^8. This means that successive measured values are correlated and that the standard deviation of the mean is greater than the standard deviation divided by the square root of the number of measurements. As estimates of the standard deviation of the mean, taking correlation into account, we take 0.1 μV for BIPM4 and 0.1 μV for BIPM5.

Table 2 summarises the estimated type-B standard uncertainty for the BIPM measurements using the resistive divider. These estimates are deduced by comparing values obtained for Zener calibrations by the resistive divider method with those obtained by direct measurements with a BIPM 10 V array. Table 3 summarises the estimated type-B standard uncertainty for the BIPM measurements using the Josephson array voltage standard. Table 4 lists the type-B uncertainties of the NML.

Items 5, 6, 7 and 8 of Table 1 are the BIPM values, the type-A, type-B and the combined standard uncertainties, respectively, for each Zener.

Apart from the uncertainties due to the measuring instruments which are already included in the type-B uncertainty for each laboratory, uncertainties for the voltage corrections are estimated from the uncertainties in the BIPM measurements of the temperature and pressure coefficients. In each case the voltage correction uncertainty is split into two parts, one associated with fluctuations of the pressure (or temperature) about the local mean value and the other associated with the difference between the local mean pressures (or temperatures) of the laboratories. The first part is assumed to be already included in the type-A uncertainty. The second part is listed separately in Item 9, not as a contribution associated with one laboratory but as one associated with the two laboratories. This value, due to uncertainties in the pressure and temperature coefficients, is the root-sum-square (rss) of the temperature and pressure uncertainties related to the differences between the mean temperatures and pressures of the two laboratories.

Item 10 is the total *a priori* combined *type-A* uncertainty for each Zener. This is the rss of Items 2, 6 and 9.

Item 11 is the comparison result obtained from each Zener and Item 12 is the mean, $\overline{R}$, of the comparison results for all ($n = 2$) Zeners, R_1 and R_2. Then, from elementary statistics, if the R's are independent, $\mathrm{var}(\overline{R}) = \mathrm{var}\left[(\sum_{i=1}^{2} R_i)/2\right] = 2^{-2}\sum_{i=1}^{2}\mathrm{var}(R_i)$ or $\sigma(\overline{R}) = 2^{-1}\left[\sum_{i=1}^{2}\mathrm{var}(R_i)\right]^{1/2}$. This is given in Item 13 and it is the expected or *a priori* type-A uncertainty of the mean (Item 12).

In contrast, Item 14 is the *a posteriori* type-A standard uncertainty of the comparison and is the standard deviation of the mean, s_M, of the results obtained from the two Zeners. With only two travelling standards, the uncertainty of the standard deviation of the mean is comparable to the value of the standard deviation of the mean itself. Item 14 should be compared with Item 13, which we would expect to contain the same uncertainty components except for transport effects. In cases where the two are not consistent, we use the *larger* of the two estimates as the type-A uncertainty.

Item 15 is the total combined uncertainty of the comparison calculated from the rss of Item 3, Item 7, and either Item 13 or Item 14.

The final result of the comparison is presented as the difference between the value assigned to a 10 V standard by each laboratory. The difference between the value assigned to a 10 V standard by the NML, at the NML, U_NML, and that assigned by the BIPM, at the BIPM, U_BIPM, for the reference date is

$$U_\mathrm{NML} - U_\mathrm{BIPM} = +\ 0.29\ \mu\mathrm{V};\ u_\mathrm{c} = 2.3\ \mu\mathrm{V}\ \text{on 1999/06/20,}$$

where u_c is the combined type-A and type-B standard uncertainty from both laboratories.

This is an excellent result. The difference between the values assigned to the travelling standards by the two laboratories is well within the estimated total standard uncertainty.

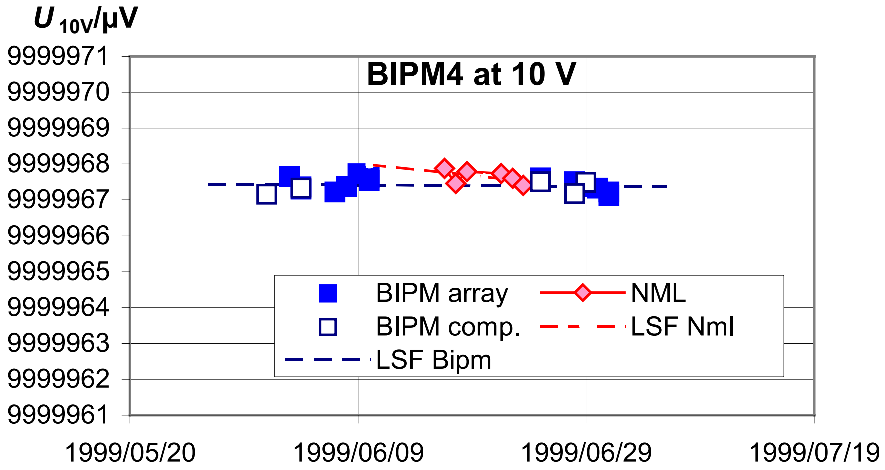

Figure 1. Voltage of BIPM4 vs time with linear least-squares fits (LSF) to the measurements of each laboratory

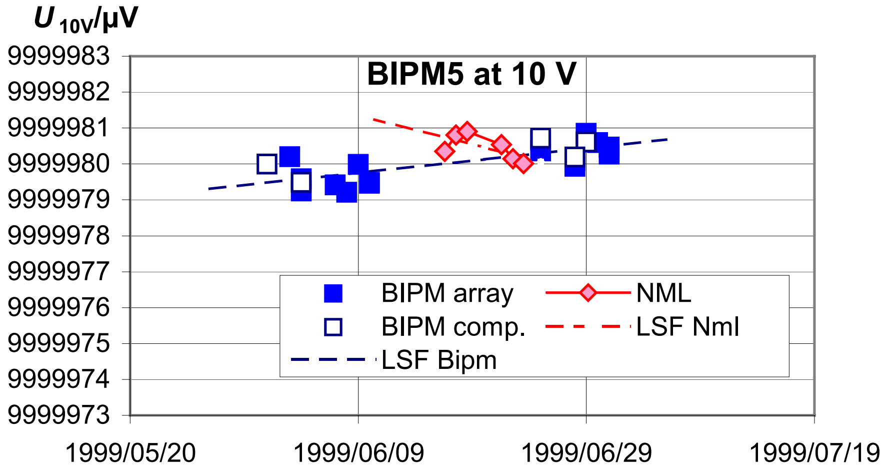

Figure 2. Voltage of BIPM5 vs time with linear least-squares fits (LSF) to the measurements of each laboratory

Table 1. Results of the NML/BIPM bilateral comparison of 10 V standards using Zener travelling standards: Mean Date 20 June 1999. Uncertainties are 1-σ estimates.

NML/BIPM Bilateral voltage comparison using travelling
Zener standards BIPM4 and BIPM5
Units are µV

		BIPM4	BIPM5	
1	NML value,U_{NML}	9999967.64	9999980.46	
2	NML unc (A)	0.10	0.12	r
3	NML unc (B)	2.30	2.30	s
4	NML unc (total)	2.30	2.30	
5	BIPM value,U_{BI}	9999967.39	9999980.13	
6	BIPM unc (A)	0.10	0.10	t
7	BIPM unc (B)	0.10	0.10	u
8	BIPM unc (tot)	0.14	0.14	
9	pc & tc unc.	0.11	0.12	v
10	tot rss uncorr for each Zener	0.18	0.20	$w=[r^2+t^2+v^2]^{1/2}$
11	U_{NML} -U_{BIPM}	0.25	0.33	
12	mean U_{NML} -U_{BIPM}	0.29		
13	Expected unc of transfer	0.13		$y=[w_4^2+w_5^2]^{1/2}/2$
14	s_M of difference for 2 Zeners	0.04		
15	Total unc of comparison	2.31		
	mean date yy/mm/dd	99/06/20	99/06/20	

Table 2. Estimated type-B standard uncertainties for Zener calibrations with the BIPM resistive divider system. The uncertainty evaluations follow from comparisons of results of Zener calibrations obtained with the divider system with those obtained by direct measurements with a BIPM 10 V array.

	Value/nV
Uncertainty in the difference in calibration results by the two methods	50
Temporal stability of divider calibration	90
Total	100

Table 3. Estimated type-B standard uncertainties for Zener calibrations with the BIPM Josephson array voltage standard.

	BIPM/nV
Thermal emfs	1.0
Frequency	0.3
Leakage resistance	0.2
Detector	0.5
rss total	1.2

Table 4. Estimated type-B standard uncertainties for the NML from information supplied by the participant. Units are μV.

	NML/μV
Reference group stability	2.2
Uncorrected parasitic voltages	0.3
Detector	0.5
rss total	2.3