

**Bilateral Comparison of 1.018 V Standards between the VNIIM and the BIPM,
November 1998 through January 1999**

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A comparison of the voltage reference standards of the BIPM and the D.I. Mendeleev Institute for Metrology (VNIIM) was carried out from November 1998 to January 1999. Two BIPM 732B Zener diode-based travelling standards, BIPM4 and BIPM5, were used as the travelling standards. At the BIPM the 1.018 V outputs were measured with respect to a reference standard cell whose value is known with respect to the BIPM Josephson voltage standard with a combined standard uncertainty of 10 nV. The travelling standards were hand-carried by airplane as carry-on luggage and measured at the VNIIM directly with its Josephson array voltage standard over the period from 14 to 18 December 1998. Results of all measurements were corrected for the dependence of the output voltage on ambient temperature and pressure.

Figure 1 shows the results of the 1.018 V measurements of BIPM4 in both laboratories. The BIPM measurements were analyzed using a linear least-squares fit to the voltages measured at the BIPM as a function of time. A dashed line in the figure indicates this fit. The slope of the fit was used to estimate the drift of the Zener over the entire time period from November through January. Using this slope, the predicted behavior of the Zener voltage at the VNIIM was modeled by a linear equation that yields the mean value of the VNIIM measurements on the mean date of the VNIIM measurements. Graphically, this corresponds to representing the VNIIM measurement results by a straight line having the same slope as the line describing the BIPM results but passing through the mean value of the VNIIM results on the mean date of the VNIIM measurements, 16 December 1998. This is the reference date of the comparison. The BIPM value and uncertainty on the reference date are calculated from the least-squares fit to the BIPM data. Similarly, the uncertainties for the VNIIM measurements are calculated from the differences between the values that were measured by the VNIIM and the values that are predicted by the model. The analysis of the measurements of BIPM5 was done in the same way and the results are shown in Figure 2.

Table 1 lists the results of the 1.018 V comparison and the component uncertainty contributions. In estimating the uncertainty we have calculated the *a priori* uncertainty based on all

known sources except that associated with the stability of the standards when transported. We compare this with the *a posteriori* uncertainty estimated by the standard deviation of the mean of the results from the two travelling standards. With only two travelling standards, the uncertainty of the latter is comparable to the uncertainty itself. If the *a posteriori* uncertainty is significantly greater than the *a priori* uncertainty, we assume that a standard has changed in an unusual way and that we must use the *a posteriori* uncertainty in calculating the final uncertainty. In this comparison the estimated *a priori* uncertainty is in excellent agreement with the *a posteriori* uncertainty.

Uncertainties in the BIPM measurements of the temperature and pressure coefficients would lead to a type B uncertainty if only one travelling standard was used. In the case of more than one, we do not expect significant correlation among the corrections for different standards and in our uncertainty table they are treated as type A uncertainties.

The final results of the comparison are presented as the difference between the value assigned to a 1.018 V standard by the VNIIM, U_{VNIIM} , and that assigned by the BIPM, U_{BIPM} , on the reference date. The result is

$$U_{\text{VNIIM}} - U_{\text{BIPM}} = 0.00 \mu\text{V}; \quad u_c = 0.03 \mu\text{V} \text{ on } 1998/12/16,$$

where u_c is the combined type A and type B standard uncertainty from both laboratories. This is a remarkably good result. Our experience normally leads us to expect typical values of the order of 0.050 μV for the transfer uncertainty.

Z4 à 1,018 V

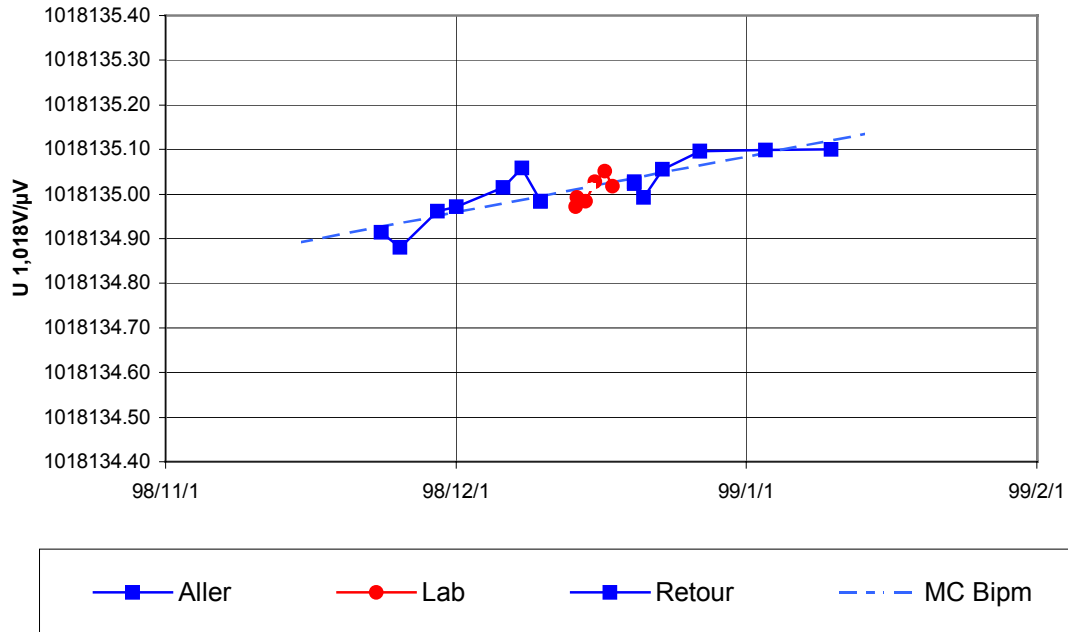


Figure 1 Values of the 1.018 V output of BIPM4 during the comparison: Corrections for temperature and pressure have been applied. The dashed straight line indicates the result of a linear least-squares fit to the BIPM results.

Z5 à 1,018 V

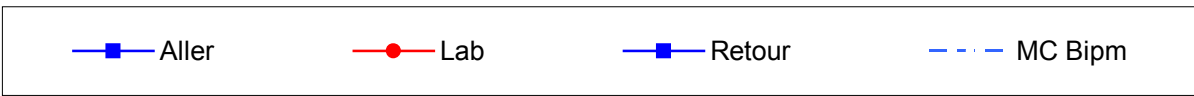
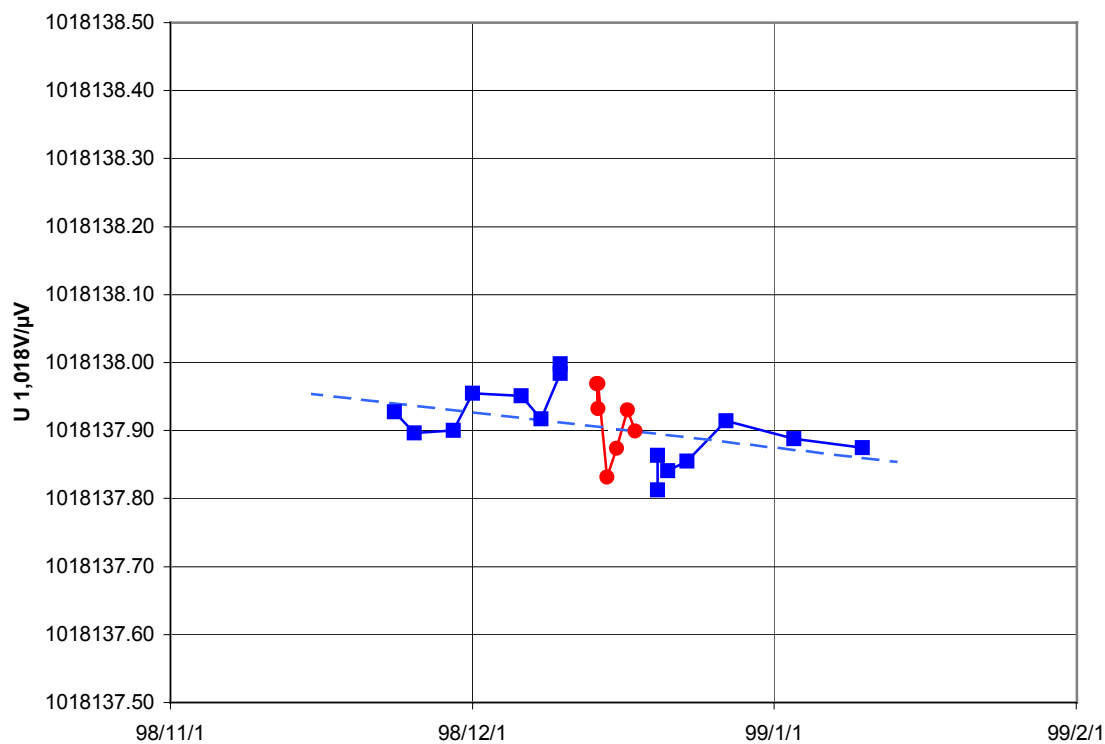


Figure 2. Values of the 1.018 V output of BIPM5 during the comparison: Corrections for temperature and pressure have been applied. The dashed straight line indicates the result of a linear least-squares fit to the BIPM results.

Table 1. Results of the VNIIM/BIPM bilateral comparison of 1.018 V standards using Zener travelling standards: Mean Date 16 December 1998. Uncertainties are 1- σ estimates.

VNIIM/BIPM Bilateral voltage comparison using travelling Zener standards BIPM4 and BIPM5.				
Units are μV				
		BIPM4@1.018 V	BIPM5@1.018 V	
1	VNIIM value, U_{VNIIM}	1018135.007	1018137.915	
2	VNIIM unc (A)	0.010	0.019	r
3	VNIIM unc (B)	0.022	0.022	s
4	VNIIM unc (total)	0.024	0.029	
5	BIPM value, U_{Bi}	1018135.017	1018137.903	
6	BIPM unc (A)	0.009	0.013	t
7	BIPM unc (B)	0.010	0.010	u
8	BIPM unc (tot)	0.013	0.016	
9	pc & tc unc, uncorrelated	0.006	0.006	v
10	tot rss uncorr for each Zener	0.013	0.024	$w = \text{rss}(r, t, v)$
11	$U_{\text{VNIIM}} - U_{\text{BIPM}}$	-0.010	0.012	
12	mean $U_{\text{VNIIM}} - U_{\text{BIPM}}$	0.001		
13	unc of transfer	0.011		x
14	<i>a priori</i> type A unc for 2 Zeners	0.011		$y = 1/\{\text{sqrt}[w_4^{-2} + w_5^{-2}]\}$
15	Total unc of comparison	0.027		$z = \text{rss}(x, s_4, u_4)$ $= \text{rss}(x, s_5, u_5)$
	mean date yy/mm/dd	98/12/16	98/12/16	

References to Table 2.

- 1, 2, 3 and 4 are the VNIIM value, type A, type B and combined uncertainties;
- 2, The stability of the Zeners can be described by flicker noise ($1/f$ noise) with a floor value of about 1 to 2 parts in 10^8 . If the VNIIM results for BIPM 4 from each day are used in a linear least-squares fit, the standard deviation of the residuals is 0.020. With respect to the model that assumes a constant drift rate of the travelling standard, the standard deviation of the value assigned by the VNIIM on the mean date of the VNIIM measurements, is the standard deviation of the residuals divided by the square root of the number of degrees of freedom (number of daily measurement results minus two) or about 0.010, if the daily measurement values are uncorrelated. But, in fact, they are correlated in a way that is difficult to estimate from the data. We therefore estimate a value between the two limits of 0.020 and 0.010 for the type A uncertainty and set it at 0.010. A similar argument was applied for the estimated type A uncertainty for BIPM 5.
- 5, 6, 7 and 8 are the BIPM value, type A, type B and combined uncertainties.
- 9 is the root-sum-square (rss) total uncertainty associated with the corrections for temperature and pressure; uncertainties in the pressure and thermistor measurements are negligible and that the total uncertainty is dominated by that of the coefficients.
- 10 is the total *a priori* combined type A uncertainty for each Zener. This is the rss of 2, 6 and 9.
- 11 is the comparison result from each Zener.
- 12 is the mean difference for all n ($=2$) Zeners.

13 is the *a posteriori* type A uncertainty and includes effects due to transport. It is the standard deviation of the results from both travelling standards. There were only two travelling standards and so this uncertainty has a rather large uncertainty.

14 is the *a priori* type A standard uncertainty of the comparison and is the uncertainty of the mean or the results from the two Zeners. This should be compared with 13 which contains the same uncertainty components and transport effects. In this case the two estimates are equal so we assume that transport-related effects are almost negligible.

15 is the total uncertainty of the comparison calculated from the rss of 3, 7 and 13.