Report BIPM-67/3

On a statistical method for detecting the influence of dilution errors

We assume that the values for the specific activities and their (random) standard deviation

$$R_{di} \pm \sigma_{di}$$

are experimentally known for all "strong" sources.

Then we easily get the corresponding values per dilution d, i.e.

$$R_d = \frac{\sum_i R_{di} \, \sigma_{di}^{-2}}{\sum_i \sigma_{di}^{-2}} ,$$

$$\sigma_d^2 = \frac{\sum_i (R_{di} - R_d)^2 \cdot \sigma_{di}^{-2}}{(n_d - 1) \sum_i \sigma_{di}^{-2}} ,$$

where n_d is the number of sources prepared from d.

Let us call now "reduced" specific activity of a dilution $d = 1, 2, 3, 4$ the quantity

$$Q_d = (DF)_d \cdot R_d ,$$

$(DF)_d$ being the dilution factor for d.

Now, the general idea is the following: eventual random errors in the dilution factor have the effect, that the expectation value $E(Q_d)$ is no longer the same for any dilution. If these deviations become large enough, we can detect this contribution to the error by means of a simple analysis of the variance.

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The total mean value for the reduced specific activity is easily determined to be

$$Q = \frac{\sum_d p_d \cdot Q_d}{\sum p_d} ,$$

where $p_d = \left[(DF)_d \cdot \sigma_d \right]^{-2}$ is the statistical weight of Q_d.

As for the variance of Q, we use two different methods. Whereas the first is based on the deviations among the partial means Q_d, the second only takes into account the individual standard deviations σ_d. We then get for these two quantities

$$S^2 = \frac{\sum_d p_d (Q_d - Q)^2}{3 \cdot \sum_d p_d} \quad \text{and}$$

$$s^2 = \left[\sum p_d \right]^{-1} ,$$

respectively, from which we form the ratio

$$F = S^2 / s^2 .$$

This quantity can be shown to follow a F-distribution with $f_1 = 3$ and $f_2 = \sum_d n_d - 4$ degrees of freedom.

In the case that F exceeds the upper limit corresponding to a probability chosen in advance, the assumption $E(Q_d) = \text{const.}$ has to be abandonned, proving with this the influence of random errors due to the dilution techniques applied.

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