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BUREAU INTERNATIONAL DES POIDS ET MESURES

REPORT TO THE COMITÉ CONSULTATIF D'ÉLECTRICITÉ FROM THE WORKING GROUP ON THE QUANTUM HALL EFFECT

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August, 1988

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1. INTRODUCTION

1.1. *Background*

So that this report may also be of use to the community of electrical metrologists at large, we begin with a brief review.

The Comité Consultatif d'Électricité (CCE) of the Comité International des Poids et Mesures (CIPM) is one of eight Consultative Committees to the CIPM which together cover most of the fields of basic metrology. These committees, which may establish temporary or permanent "Working Groups" to study special subjects, coordinate the international work carried out in their respective fields, advise the CIPM about the work of the Bureau International des Poids et Mesures (BIPM) in these fields, and propose appropriate actions to the CIPM including recommendations concerning changes in the definitions and representations of units. The CIPM may endorse, modify, or reject these recommendations, submitting as appropriate those which will have a very broad impact to the Conférence Générale des Poids et Mesures (CGPM) for further approval.

As an organ of the Convention du Mètre, one of the responsibilities of the CCE is to ensure the propagation and improvement of the Système International d'Unités or SI. The SI serves as a basis for the promotion of long-term, worldwide uniformity of electrical measurements which is of considerable technical and economic importance to commerce and industry.

As a consequence the CCE has become increasingly concerned that, because most national standards laboratories base their representation of the ohm (i.e., their national reference standard of resistance*) on the mean resistance of a particular group of precision wire-wound standard resistors, and because these artifact standards age, the various national representations of the ohm differ significantly from each other and the ohm, and some are

*The ohm means the SI unit of resistance. Occasionally it may be referred to in the literature as the absolute ohm. As-maintained ohm, representation of the ohm, laboratory representation of the ohm, "national unit of resistance," "laboratory unit of resistance," practical realization of the ohm, or other similar terms are commonly used to indicate a practical reference standard of resistance. The word *unit* should not be used in this context. The only unit of resistance in the SI is the ohm. This report uses the expression *representation of the ohm* and variations thereof.

drifting excessively. Indeed, current evidence indicates that most national representations of the ohm are from a few tenths μΩ larger to a few μΩ smaller than the ohm and that their drift rates lie in the range* -0,07 to +0,07 μΩ/year [1].

Although in principle a so called Thompson-Lampard calculable capacitor can be used to realize the ohm with an uncertainty of less than 0,1 μΩ, it is in practice a difficult experiment to carry out routinely; only one laboratory in the world has had such an apparatus in continuous operation since the method was first developed in the early 1960's [2]. Consequently, electrical metrologists enthusiastically welcomed von Klitzing's 1980 discovery of the quantum Hall effect (QHE) [3] since it promised to provide a method for basing a representation of the ohm on fundamental constants in much the same manner as the Josephson effect has provided a method for basing a representation of the volt on fundamental constants. The QHE clearly had the potential of eliminating in a relatively simple way the twin problems of nonuniformity of national representations of the ohm and their inconsistency with the SI.

Recognizing the rapid advances that have been made in understanding the QHE since its comparatively recent discovery, the CCE at its 17th meeting held in September 1986 established through Declaration E2 (1986), "Concerning the quantum Hall effect for maintaining a representation of the ohm," the Working Group on the Quantum Hall Effect [4]. The CCE charged the Working Group to (i) propose to the CCE, based upon all the relevant data that become available

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*In keeping with the preferred ISO usage, *commas* are used in this report to indicate decimal fractions. Also, in accordance with proper SI usage, symbols for units are written in Roman letters and symbols for physical quantities are italicized or underlined.

by 15th June 1988, a value of the quantized Hall resistance consistent with the SI value for use in basing an accurate representation of the ohm on the QHE; and (ii) develop detailed guidelines for the proper use of the QHE to realize reliably such a representation of the ohm.

Further, the CCE stated its intention to meet in September 1988 with a view to recommending this value of the quantized Hall resistance to come into effect on 1st January 1990 for the purpose of maintaining a highly stable and accurate representation of the ohm in all those national standards laboratories (and otherwise) that choose to base their reference standard of resistance on the QHE.

This report by the Working Group on the Quantum Hall Effect is in direct response to the charge by the CCE. It proposes the value of the quantized Hall resistance to be adopted, gives the basis for this value, and summarizes three approaches to how a representation of the ohm based on the QHE may be used in practice. Additionally, technical guidelines for the reliable measurement of the quantized Hall resistance are provided in a companion report. Since such measurements are required for the practical realization of an accurate and reproducible representation of the ohm based on the QHE, these guidelines are of exceptional importance.

1.2. *Permanence of the New Representation of the Ohm*

In its discussions leading to Declarations E1 and E2 (1986), the CCE agreed that while worldwide uniformity of electrical measurements can only be assured through the SI, in the particular areas of voltage and resistance, scientific, commercial, and industrial requirements for long-term reproducibility now exceed the accuracy with which the SI units can be readily realized. To meet these very exacting demands, the CCE believes it is necessary that representations of the volt and ohm be established that have a superior long-term reproducibility and constancy than the present direct realizations of the SI units themselves.

Although the Working Group believes that its recommended value for the quantized Hall resistance upon which the new representation of the ohm is to be based is consistent with the SI value within its assigned uncertainty, it recognizes that barring an unexpected stroke of good luck, future, more accurate measurements will no doubt show that the recommended value differs from the SI value by some small amount. In keeping with the point of view of the CCE, the Working Group envisages that should such a situation occur, the CCE could simply note the difference between the ohm and its new representation. This would be useful for those workers (mostly in the fields of realizing the SI electrical units and determining the fundamental physical constants) for whom the small difference may be significant. Since any such difference is expected to be sufficiently small that practical electrical measurements will be unaffected, the Working Group strongly believes that the recommended value will not need to be altered in the foreseeable future.

However, this last statement must not be interpreted to mean that improved realizations of the ohm are now unnecessary. Because an accurate representation of the ohm is important to science, commerce, and industry, the Working Group considers it important for laboratories to continue their efforts to realize the ohm with greater accuracy, either directly or indirectly through measurements of relevant fundamental constants. This could result in a significant reduction of the uncertainty assigned to the new representation.

1.3. *Laboratories That Do Not Use the Quantum Hall Effect*

The purpose of the new ohm representation is to improve worldwide uniformity of national representations of the ohm and their consistency with the SI. The question thus arises as to the procedure to be followed by those laboratories which will not base their representation of the ohm on the QHE. In keeping with the viewpoint expressed by the CCE during its discussions in connection with Declaration E2 (1986), the Working Group proposes that on 1st January 1990, such laboratories adjust the value of their representation of the ohm so that it is consistent with the new representation. Furthermore, this consistency should be maintained by having a transportable resistance standard periodically calibrated by a laboratory that does base its representation of the ohm on the QHE, for example BIPM.

2. DEFINITIONS, SYMBOLS, AND NOMENCLATURE

2.1. *Hall Voltage to Current Quotient or Quantized Hall Resistance*

As is now well known, the quantum Hall effects (integral and fractional) are characteristic of a two dimensional electron gas (2DEG). A 2DEG may be realized in a high mobility semiconductor device such as a silicon MOSFET (metal-oxide-semiconductor field-effect transistor) or GaAs-$Al_xGa_{1-x}As$ heterostructure, of standard Hall-bar geometry, when the applied magnetic flux density is of the order of 10 T and the device is cooled to a temperature of a few kelvin [5]. Under these conditions, the 2DEG is completely quantized and for a fixed current I through the device there are regions in the curve of Hall voltage vs. gate voltage, or of Hall voltage vs. magnetic flux density, where the Hall voltage U_H remains constant as the gate voltage or magnetic flux density is varied. These regions of constant Hall voltage are termed Hall plateaus.

In the limit of zero dissipation in the direction of current flow, the Hall resistance of the ith plateau $R_H(i)$, defined as the quotient of the Hall voltage of the ith plateau to the current I, is quantized:

$$R_H(i) = U_H(i)/I = R_K/i, \qquad (1)$$

where *i* is an integer and R_K is the *von Klitzing constant*.* (It follows from Eq. (1) that R_K is equal to the resistance of the $i = 1$ plateau, $R_H(1)$. Since $R_H(i)$ is often referred to as the quantized Hall resistance independent of plateau number *i*, to avoid confusion the Working Group proposes the use of R_K as the symbol for the Hall voltage to current quotient or resistance of the $i = 1$ plateau, and to refer to it as the von Klitzing constant after the discoverer of the quantum Hall effect.)

A significant amount of experimental evidence supports the view that the von Klitzing constant R_K is a universal quantity, provided that the particular quantum Hall effect device used meets certain criteria. While the universality of R_K has not yet been demonstrated to a level of precision approaching that of the Josephson frequency to voltage quotient or Josephson constant K_J, studies of the influence of experimental variables such as current, temperature, device type, device material, and plateau number have shown that if certain precautions are taken and tests performed, then R_K may be reproduced with a relative precision approaching one part in 10^8 or possibly even several parts in 10^9 [6-12]. Carrying out quantized Hall resistance measurements according to the companion report prepared by the Working Group entitled "Technical Guidelines for Reliable Measurements of the Quantized Hall Resistance" should allow this level of precision to be reached. Throughout the remainder of this report we assume that these guidelines are implemented.

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*We restrict ourselves to the integral quantum Hall effect for which *i* is an integer. The fractional quantum Hall effect, for which *i* is the ratio of two integers, has not yet been studied sufficiently to warrant its use as a basis for a representation of the ohm.

The current theory of the quantum Hall effects predicts, and the experimentally observed universality of the fundamental quantized Hall resistance relation [Eq. (1)] is consistent with the prediction, that R_K is equal to the invariant quotient of fundamental constants h/e^2, where h is the Planck constant and e is the elementary charge [5, 13, 14, 15]. Although the accuracy of this equality and Eq. (1) are still under active theoretical and experimental investigation, the Working Group believes that for the purpose of including data from measurements of fundamental constants, it may be assumed that $R_K = h/e^2$. (The same assumption was made by the CODATA Task Group on Fundamental Constants in obtaining their 1986 recommended values of the constants [16].)

In particular, the fine-structure constant $\underline{\alpha} \simeq 1/137$ and h/e^2 are related by defined quantities: $h/e^2 = \underline{\mu}_o c/2\underline{\alpha}$, where $\underline{\mu}_o = 4\underline{\pi} \times 10^{-7}$ N/A^2 exactly is the permeability of vacuum and $c = 299\ 792\ 458$ m/s exactly is the speed of light in vacuum. As a consequence, a measurement of $\underline{\alpha}$ having a particular relative uncertainty will yield a value of R_K in ohms with the same relative uncertainty.

2.2. *The New Representation of the Ohm and Its Practical Use*

In Appendix A of this report, we consider the currently available measurements of the von Klitzing constant R_K, deriving from them our recommended value in SI units and its associated one standard deviation assigned uncertainty*:

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*Throughout this report, we treat uncertainties following the suggestions of the BIPM Working Group on the Statement of Uncertainties as embodied in Recommendation INC-1 (1980) which has been approved by the CIPM [17]. In particular, all uncertainties are one standard deviation estimates in keeping with CIPM Recommendation 1 (CI-1986) [18].

$$R_K = 25\ 812{,}807\ \Omega \quad (2a)$$

$$\text{Standard deviation: } 0{,}005\ \Omega \quad (2b)$$

$$\text{Relative standard deviation: } 2 \times 10^{-7}. \quad (2c)$$

For the purpose of basing a representation of the ohm on the quantum Hall effect, the Working Group proposes to use Eq. (2a) to define the following conventional value for the von Klitzing constant:

$$R_{K-90} \stackrel{\text{def}}{=} 25\ 812{,}807\ \Omega \quad (3)$$

exactly, where the subscript derives from the fact that the new representation of the ohm is to come into effect starting on 1st January 1990.

The Working Group has identified three approaches to how a representation of the ohm based on the QHE and the defined physical quantity R_{K-90} may be used in practice, each having both advantages and disadvantages. These approaches are summarized below. Two are both rigorous and correct, but in one we define a new unit, Ω_{90}, and in the other we define a new physical quantity, R_{90}. The Working Group believes that the best way to avoid confusion internationally is for the national standards laboratories to adopt a uniform approach. It is imperative that the laboratories avoid giving the impression that there is more than one representation of the ohm in general use and that there may be significant differences between national realizations of the new representation of the ohm.

2.2.1. *Approach 1*

A new unit of resistance is defined via the equation

$$\Omega_{90} \stackrel{\text{def}}{=} (R_K/R_{K-90})\ \Omega \quad (4)$$

exactly. However, the *experimental* realization by a particular laboratory of the *defined* unit Ω_{90} has an associated uncertainty. Based on the quantized Hall resistance apparatus in current use, this uncertainty will generally lie in the range 0,01 $\mu\Omega$ to 0,1 $\mu\Omega$ [1]. Since Eqs. (2), (3), and (4) imply that

$$1\ \Omega_{90} = 1\ \Omega \pm 0{,}2\ \mu\Omega, \qquad (5)$$

the uncertainty with which a particular realization of Ω_{90} represents the ohm will have two components: the 0,2 $\mu\Omega$ of Eq. (5) and the experimental uncertainty associated with the realization. If to be specific we assume the latter is 0,05 $\mu\Omega$, then the resistance R' of a particular standard resistor expressed in terms of Ω_{90} would be (again to be specific)

$$R' = (1{,}000\ 003\ 59 \pm 0{,}05 \times 10^{-6})\ \Omega_{90}. \qquad (6)$$

(We also assume for simplicity a perfect resistor and no uncertainty associated with the calibration process.) It follows from Eqs. (5) and (6) that the resistance of the resistor expressed in ohms is

$$R' = (1{,}000\ 003\ 59 \pm 0{,}21 \times 10^{-6})\ \Omega. \qquad (7)$$

If it is necessary to distinguish between different experimental realizations of Ω_{90} by different laboratories the symbol $\Omega_{90\text{-LAB}}$ may be used, where LAB stands for a convenient abbreviation of the name of the laboratory carrying out the realization. Such distinction should only be necessary for work involving two or more national standards laboratories; it should not be required even in dealings with the most demanding users of calibration services.

Advantages of Approach 1

- It enables resistance measurements to be reported in a straightforward way in terms of a laboratory's realization of Ω_{90} (*i.e.*, in terms of the laboratory's representation of the ohm) with its relatively small uncertainty.
- It is consistent with current practice since most standards laboratories report the results of calibrations in terms of their representation of the ohm. Consequently, it will be readily understood by users of calibration services.

- The incorrect practice of using the physical quantity $\underline{\Omega_{LAB}}$ as a unit will be replaced by the correct practice of using the unit Ω_{90}.

Disadvantages of Approach 1

- It introduces a new unit which is likely to differ from the ohm by some small amount and which is parallel to and thus in competition with the ohm. Moreover, if Approach 1 is used to define a new unit of emf, V_{90}, based on the Josephson effect, then a complete parallel and thus competitive system of electrical units will have been introduced (*i.e.*, one would have A_{90}, W_{90}, C_{90}, F_{90}, H_{90}, T_{90}, *etc.*). This could be detrimental to the coherence of the SI in the expression of physical quantities. For example, consistency between electrical and mechanical power, assured by the SI, would no longer be guaranteed.

2.2.2. *Approach 2*

This is formally the same approach used by the Comité Consultatif de Thermométrie (CCT) to define the 1968 temperature scale and which it will likely use to define the new International Temperature Scale 1990 (ITS-90) to come into effect on 1st January 1990.

Let R be the symbol for the physical quantity resistance whose unit is the ohm. Let R_{90} be the symbol for a new physical quantity called "conventional resistance" exactly defined by

$$R_{90} \stackrel{\text{def}}{=} (R_{K-90}/R_K)R \qquad (8)$$

whose unit is also the ohm. A calibration of the same standard resistor (and under the same assumptions) discussed in Approach 1 in terms of a laboratory's experimental realization of R_{90} would be expressed as

$$R'_{90} = (1{,}000\ 003\ 59 \pm 0{,}05 \times 10^{-6})\ \Omega. \qquad (9)$$

[One way of demonstrating that Eq. (9) is correct is by combining Eqs. (4), (6), and (8).] It is important to recognize that R'_{90} is a new physical quantity; it is not the same as R' but is related to it through Eq. (8). However, the numerical value of R'_{90} expressed in ohms [Eq. (9)] is the same as the numerical value of R' expressed in terms of the unit Ω_{90} [Eq. (6)], but R'_{90} has the units of ohms. It follows from Eqs. (2), (3), (8), and (9) that R' in ohms is

$$R' = (1{,}000\ 003\ 59 \pm 0{,}21 \times 10^{-6})\ \Omega. \qquad (10)$$

As would be expected, Eq. (10) is identical to Eq. (7).

In a manner similar to that discussed under Approach 1, if it is necessary to distinguish between experimental realizations of R_{90} or measurement results such as R'_{90}, then the symbols $R_{90\text{-LAB}}$ or $R'_{90\text{-LAB}}$ may be used.

Advantages of Approach 2

It enables resistance measurements to be reported in both a straightforward and rigorous way in terms of a laboratory's representation of the ohm with its relatively small uncertainty.

- It does not introduce a new unit to compete with the ohm; measurements are reported in ohms.

Disadvantages of Approach 2

- It is not consistent with current practice in electrical metrology and is likely to cause some confusion.

- It introduces a new physical quantity for resistance which is likely to differ from resistance by some small amount. Thus the same resistor would have both a conventional resistance and a resistance. Moreover, if Approach 2 is used in a similar way to define a new physical quantity for emf, E_{90}, based on the Josephson effect, then a complete parallel set of electrical quantities will have been introduced (*i.e.*, one would have I_{90}, P_{90}, Q_{90}, C_{90}, L_{90}, B_{90}, *etc.*). However, historically, the confusion resulting from the use of concurrent systems of electrical units is well known, but experience in the area of theometry has shown that the introduction of a conventional temperature has not resulted in a comparable level of misunderstanding.

2.2.3. *Approach 3*

This approach is in reality Approach 1 but a unit such as Ω_{90} is not formally defined and used. The calibration of the above standard resistor would be reported as

$$R' = (1{,}000\ 003\ 59 \pm 0{,}05 \times 10^{-6})\ \Omega \qquad (11)$$

but with accompanying text stating in effect that the value given is not really in ohms but is actually based on the laboratory's representation of the ohm which in turn is based on the quantum Hall effect and the internationally adopted value of the von Klitzing constant as recommended by the CCE. Because the unit Ω is used in Eq. (11), equations such as (7) and (10) could not be readily given (assuming it was useful to do so). Instead, it would have to be stated in the text that the uncertainty of the resistance of the resistor in ohms is ± 0,21 $\mu\Omega$.

Advantages of Approach 3

- Because of its similarity with current practice in some laboratories, it should be readily understood.

- It avoids formally introducing a new unit of resistance or a conventional resistance.

Disadvantages of Approach 3

- It lacks rigor; Eq. (11) is incorrect since it gives the resistance in ohms but the uncertainty as if the resistance were reported in terms of the laboratory's representation of the ohm. If R' is reported in ohms, its uncertainty should be given as 0,21 $\mu\Omega$. In a variation of Approach 3, one avoids giving an incorrect equation such as Eq. (11) by deleting the unit Ω and adding further explanatory text. This increases further the amount of written material required to explain the reported value. Moreover, without such detailed information, this approach would be a continuing source of confusion.

- In contrast to Approaches 1 and 2, there is no clear indication that a new representation of the ohm is in use.

2.2.4. *Working Group Recommendation*

One member of the Working Group on the Quantum Hall Effect prefers Approach 1 because it is readily understood and consistent with current practice. Four members prefer Approach 2 because of its rigor and because it does not introduce a new unit in competition with the ohm. One member prefers Approach 3 or its variant because it is in common use and will not be a real change. He believes that the lack of rigor of this approach is of little practical consequence. (Among the members of the Working Group on the Josephson Effect, the preferences are: two members for Approach 2 and one for Approach 3.)

Because of its importance, the Working Group believes that the CCE in its entirety should consider this issue and recommend a solution.

3. CONCLUSION

- Based on direct measurements of the von Klitzing constant R_K, and indirect measurements involving fundamental physical constants, the Working Group adopts 25 812,807 Ω as its recommended value for R_K with an assigned one standard deviation uncertainty of 0,005 Ω, corresponding to a relative uncertainty of 2×10^{-7}.

- The uncertainty of the new representation of the ohm based on the quantum Hall effect and the Working Group's recommended value for R_K is 0,2 μΩ, one standard deviation estimate.

◦ The Working Group expects that its recommended value for R_K will not need to be significantly altered in the foreseeable future.

◦ Because science, commerce, and industry require an accurate and internationally uniform representation of the ohm, the Working Group strongly supports the view of the CCE that the recommended value of the von Klitzing constant be adopted simultaneously on 1st January 1990 by all those laboratories that choose to base their representation of the ohm on the quantum Hall effect, and that beginning on this date all other laboratories adjust and maintain the value of their representation of the ohm to be consistent with the recommended value.

◦ To avoid confusion internationally, the Working Group believes that the national standards laboratories should adopt a uniform approach to using the new representation of the ohm. The laboratories must avoid giving the impression that there is more than one representation of the ohm in use and that national realizations of the new representation differ significantly. This uniformity will be enhanced if laboratories refrain from using distinguishing symbols to denote their representation of the ohm.

◦ Given the importance of an accurate representation of the ohm to science, commerce, and industry, laboratories should continue their efforts to realize the ohm with improved accuracy so that the uncertainty of the new representation may be reduced.

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APPENDIX A

A.1. DERIVATION OF THE WORKING GROUP'S RECOMMENDED VALUE OF THE VON KLITZING CONSTANT R_K

A.1.1. Approach

Because the Working Group's recommended value of R_K is for use in realizing a practical representation of the ohm by means of the quantum Hall effect, we adopt the following guiding principle for its derivation: The value should be so chosen that it is unlikely to require significant change in the foreseeable future. This means that the number of digits given for the recommended value should be the minimum possible and that its uncertainty should be conservatively assigned. This principle also implies that it is unnecessary to carry out a complete least-squares adjustment of the fundamental physical constants to derive the recommended value; a straightforward treatment of the individual measurements of R_K currently available should suffice.

A.1.2 *Summary of Data*

Table A1 summarizes the measurements of R_K to be considered while Fig. 1 compares them graphically, starting from the bottom of the figure but with items 1a and 1b at the top. (To aid in the comparison, the most accurate value and its uncertainty are indicated by dashed and full lines, respectively, as well as by the usual point and error bars.) Values are included only if they were available by the 15 June 1988 date stated by the CCE in its Declaration E2 (1986) and for which some form of documentation was available to the Working Group. Although we shall assume $R_K = h/e^2 = \mu_0 c\alpha^{-1}/2$ as discussed in Sect. 2.1, only the last four entries of Table A1 (items 8 through 11) require this assumption. These values are termed *indirect*, while

Table A1. Summary of values of the von Klitzing constant R_K. For ease of comparison, the values are given in two forms: in Ω (column 2); and in parts in 10^6 relative to the convenient reference resistance 25 812,8 Ω (column 3).

Item No.	R_K (Ω)	$[(R_K/25\ 812,8\ \Omega)-1]\times 10^6$	Remarks and references
1.	25 812,809 4 ± 0,001 7	0,363 ± 0,066	CSIRO/NML quantized Hall resistance (QHR) and realization of ohm via calculable capacitor (C.C.) [A1, A2]
(a)	25 812,808 6 ± 0,001 7	0,333 ± 0,065	BIPM QHR, CSIRO/NML realization of ohm via C.C. [A3]
(b)	25 812,813 4 ± 0,002 1	0,520 ± 0,080	Gakushuin University (G.U.) QHR, CSIRO/NML realization of ohm via C.C. [A4, A5]
2.	25 812,809 2 ± 0,001 4	0,356 ± 0,054	NPL QHR and realization of ohm via C.C. [A6]
3.	25 812,801 8 ± 0,005 7	0,070 ± 0,220	LCIE QHR and realization of ohm via C.C. [A7]
4.	25 812,806 4 ± 0,006 7	0,247 ± 0,260	ETL QHR and realization of ohm via C.C. [A8]
5.	25 812,807 23 ± 0,000 61	0,280 ± 0,024	NBS QHR and realization of ohm via C.C. [A9-A11]
6.	25 812,806 5 ± 0,008 3	0,250 ± 0,320	Institute of Metrological Service (IMS) QHR, IMM realization of ohm via C.C. [A12,A13]
7.	25 812,805 5 ± 0,015 6	0,214 ± 0,606	NIM QHR and realization of ohm via C.C. [A14, A15]
8.	25 812,805 99 ± 0,000 21	0,232 1 ± 0,008 0	$\underline{\alpha}^{-1}$ from electron magnetic moment anomaly a_e [A16, A17]
9.	25 812,806 2 ± 0,004 2	0,241 ± 0,163	$\underline{\alpha}^{-1}$ from muonium ground groundstate hyperfine splitting $\underline{\nu}$(Muhfs) [A18]
10.	25 812,804 60 ± 0,000 95	0,178 ± 0,037	$\underline{\alpha}^{-1}$ from NBS $\underline{\gamma}_p'$(low), QHR, and Josephson $2e/h$ [A19, A9, A20, A11]
11.	25 812,803 3 ± 0,001 5	0,127 ± 0,056	$\underline{\alpha}^{-1}$ from NBS $\underline{\gamma}_p'$(low), realization of ohm via C.C., and Josephson $2e/h$ [A19, A10, A20, A11]

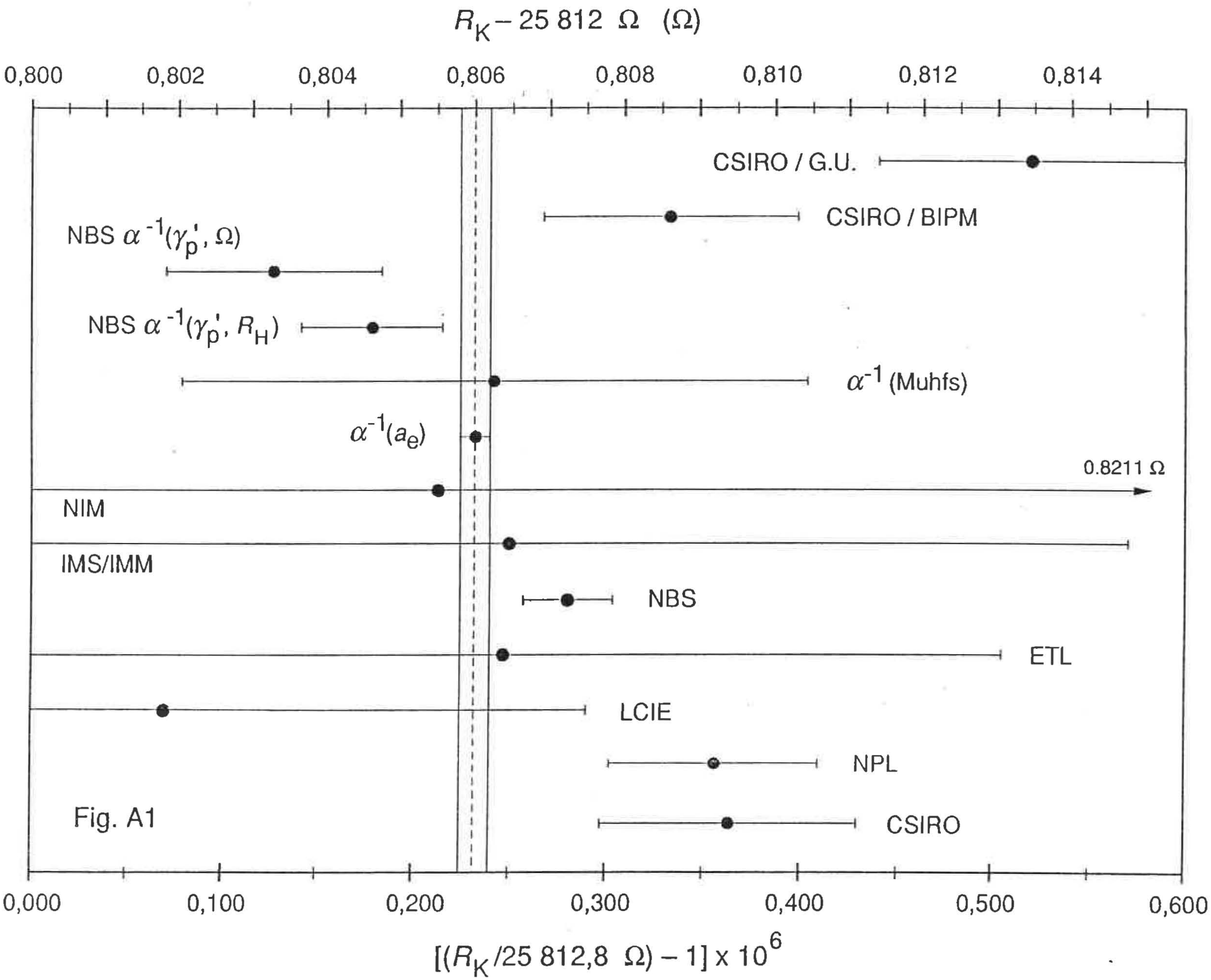

Fig. A1. Comparison of the values of R_K and their standard deviation uncertainties as given in Table A1. The vertical dashed and solid lines indicate the value and standard deviation uncertainty of the most precise result.

those which do not require this assumption (items 1 through 7) are termed *direct*. In general, we have excluded an earlier result from a particular experiment when it has been replaced by a more recent and presumably more reliable result from the same experiment.

The values given in Table A1 require further explanation.

Item 1. This result was obtained by the Commonwealth Scientific and Industrial Research Organization (CSIRO), National Measurement Laboratory (NML), Australia, from its own quantized Hall resistance (QHR) measurements and realizations of the ohm via the CSIRO/NML calculable capacitor [A1, A2]. The values labeled (a) and (b) were obtained from QHR measurements carried out at the BIPM [A3] and Gakushuin University (G.U.), Tokyo, Japan [A4,A5], respectively, and CSIRO/NML ohm realizations transferred to these laboratories by means of artifact resistance standards. Clearly, item 1 agrees well with 1a but not with 1b. The G.U. result is based on measurements using a silicon MOSFET sample. A more recent but still preliminary G.U. result using a GaAs heterostructure is about 9 parts in 10^8 smaller [A4], thereby bringing into question item 1b. (Item 6 was also obtained from silicon MOSFET measurements. All other values were obtained using heterostructures). Further, item 1 is based on QHR and ohm realization measurements both of which were carried out in CSIRO/NML. In the opinion of some members of the Working Group, it is preferable to use values of R_K based solely on data obtained in the same laboratory because of the difficulties associated with accurately transferring representations of the ohm between laboratories. For these reasons, only item 1 is included in our calculations. Item 1b does give some indication of the problems which can arise in connection with QHR measurements.

Items 2 to 7. The values of R_K from the National Physical Laboratory (NPL), U.K. (item 2) [A6]; the Laboratoire Central des Industries Électriques (LCIE), France (item 3) [A7]; the Electrotechnical Laboratory (ETL), Japan (item 4) [A8]; the National Bureau of Standards (NBS), U.S. (item 5) [A9-A11]; and the National Institute of Metrology (NIM), P.R.C. (item 7) [A14, A15], are all based on QHR and ohm realization measurements carried out in the same laboratory. For item 6, the QHR measurements were carried out in the Institute of Metrological Service (IMS), Moscow, U.S.S.R., and the ohm realization measurements in the Mendeleev Institute of Metrology (IMM), Leningrad, U.S.S.R. [A12, A13]. An artifact resistance standard was used to transfer the measurements between IMS and IMM. The variation in the uncertainty assigned the seven direct values, items 1 through 7, is mainly due to the design and construction details of the calculable capacitor and associated impedance bridges used in the ohm realization experiments.

Item 8. This indirect result is based on the value of the inverse fine-structure constant $\underline{\alpha}^{-1}$ obtained from the experimental measurement of the electron magnetic moment anomaly a_e at the University of Washington (relative uncertainty of 0,004 x 10^{-6})[A16]; and the theoretical expression for a_e given by T. Kinoshita, Cornell University (relative uncertainty of 0,007 x 10^{-6} arising from numerical integrations) [A17]. Although Kinoshita's calculations are not final, he has assigned the uncertainty conservatively and the value of R_K is not expected to change significantly. This is the most precise result currently available.

Item 9. This result is based on the value of $\underline{\alpha}^{-1}$ obtained by the Working Group from the ground-state hyperfine splitting interval of muonium (μ^+e^- atom) following the CODATA 1986 least-squares adjustment of the fundamental constants [A18]. However, the more recent and accurate value R_∞ = (10 973 731,573 ± 0,004) m^{-1} has been used for the Rydberg constant for infinite mass [A21]; and the theoretical expression for the interval used by CODATA as taken from the work of Sapirstein *et al.* has been updated to include the additional terms calculated by Eides *et al.* [A22] and Starshenka and Faustov [A23], and the exact analytic expressions obtained by Karshenboim *et al.* [A24] and by Eides *et al.* [A25] for the corresponding numerically evaluated terms given by Sapirstein *et al.*

Item 10. The relationship [A18]

$$\alpha^{-1} = [(\mu_p'/\mu_B)(2e/h)R_K/2\mu_0 R_\infty \gamma_p']^{1/3}, \quad \text{(A1)}$$

where μ_p'/μ_B is the magnetic moment of the proton in units of the Bohr magneton and γ_p' is the gyromagnetic ratio of the proton (the prime indicates a spherical, pure H_2O nuclear magnetic resonance or NMR sample at 25 °C), has the unique property that it remains valid if $2e/h$ is measured by the Josephson effect and is expressed in terms of V_{LAB}; R_K is expressed in terms of Ω_{LAB}; and γ_p' is measured by the so-called low-field method and is expressed in terms of $T_{LAB} \propto A_{LAB} = V_{LAB}/\Omega_{LAB}$, where T_{LAB}, V_{LAB}, and Ω_{LAB} are the laboratory representations of the tesla, volt, and ohm, respectively. Item 10 was obtained by NBS from this equation and a new NBS determination of γ_p'(low) [A19], maintenance of V_{NBS} using Josephson arrays [A20], measurements of R_K in terms of Ω_{NBS} [A9], the 1986 CODATA value of μ_p'/μ_B, and the value of R_∞ given under item 9. Because items 5 and 10 are based on the same QHR measurements, they are not totally independent; their correlation coefficient is 0,04.

Item 11. If the equation $R_K = \mu_0 c\alpha^{-1}/2$ is used to eliminate R_K from Eq. (A1), the resulting expression is

$$\alpha^{-1} = [c(\mu_p'/\mu_B)(2e/h)/4R_\infty \gamma_p']^{1/2}. \quad \text{(A2)}$$

If as before $2e/h$ is measured by the Josephson effect and is expressed in V_{LAB}, and γ_p' is measured by the low-field method and is expressed in $T_{LAB} \propto A_{LAB} = V_{LAB}/\Omega_{LAB}$, the quantity Ω_{LAB} is introduced in the denominator. That is, γ_p' is replaced with $\Omega_{LAB}\gamma_p'(\text{low})$, where Ω_{LAB} is to be expressed in ohms. Item 11 was obtained by NBS from this equation using the result of its recent experiment to realize the ohm via the NBS calculable capacitor [A10]. Because the same ohm realization result was used by NBS to obtain item 5, the two are not independent; their correlation coefficient is -0,17. Similarly, the correlation coefficient of items 10 and 11 is 0,98, mainly because both are based on the same value of $\gamma_p'(\text{low})$. NBS items 5, 10, and 11 are related in such a way that, assuming their correlations are properly considered, the weighted mean of any two of them gives the same result. This is taken into account as appropriate in the calculations carried out in the following section.

A.1.3. *Analysis of Data*

The simple mean and standard deviation of the mean of the ten measurements 1 through 9 plus 11 are

$$R_K = (25\ 812{,}806\ 26 \pm 0{,}000\ 80)\ \Omega \qquad \text{(A3a)}$$

$$= R_o[1 + (0{,}242 \pm 0{,}031) \times 10^{-6}], \qquad \text{(A3b)}$$

where the convenient reference resistance 25 812,8 Ω is denoted by the symbol R_o. (Including item 10 instead of item 11 yields a similar result.)

However, the simple mean and its standard deviation have little significance in the present case because of the large differences in precision of the measurements. The more appropriate weighted mean, taking as the weight of each measurement the reciprocal of the square of its assigned one standard deviation uncertainty, $w_i = 1/s_i^2$, yields

$$R_K = (25\ 812{,}806\ 15 \pm 0{,}000\ 25)\ \Omega \qquad \text{(A4a)}$$

$$= R_0[1 + (0{,}238\ 4 \pm 0{,}009\ 6)], \qquad \text{(A4b)}$$

where the uncertainty has been calculated on the basis of external consistency. That is, the usual standard deviation of the weighted mean calculated on the basis of internal consistency, $s_I = [\sum_{i=1}^{N} w_i]^{-1/2}$, has been multiplied by the scale factor or Birge ratio $R_B = [\chi^2/\nu]^{1/2}$, where χ^2 is the statistic "chi square" and ν is the number of degrees of freedom ($\nu = 9$ in the present case). The reason is that the data are only marginally in agreement; $R_B = 1{,}31$ and $\chi^2 = 15{,}6$ compared with its expected value of $\nu = 9$. The probability that this value of χ^2 has occurred by chance is about 8%, i.e., $P(15{,}6|9) \approx 0{,}08$. (We assume as usual that $P > 0{,}05$ indicates an acceptable level of agreement.)

It is clear that the value of R_K from $\alpha^{-1}(a_e)$, item 8, will dominate any weighted mean in which it is included because its assigned uncertainty is significantly smaller than that of any other value. If it is deleted, one obtains

$$R_K = (25\ 812{,}806\ 97 \pm 0{,}000\ 56)\ \Omega \qquad \text{(A5a)}$$

$$= R_0[1 + (0{,}270 \pm 0{,}022) \times 10^{-6}], \qquad \text{(A5b)}$$

where the uncertainty is calculated on the basis of external consistency; $\chi^2 = 11{,}8$ for $\nu = 8$, $R_B = 1{,}22$, and $P(11{,}8|8) \approx 0{,}16$. The agreement is reasonable.

It is of interest to calculate a value of R_K based solely on the seven direct measurements, items 1 through 7. The result is

$$R_K = (25\ 812{,}807\ 65 \pm 0{,}000\ 52)\ \Omega \qquad \text{(A6a)}$$

$$= R_0[1 + (0{,}296 \pm 0{,}020) \times 10^{-6}], \qquad \text{(A6b)}$$

where the uncertainty has been calculated on the basis of internal consistency; $\chi^2 = 3{,}85$ for $\underline{\nu} = 6$, $R_B = 0{,}80$, and $P(3{,}85|6) \simeq 0{,}70$. The agreement is excellent.

Because items 1, 2, and 5 are significantly more precise than items 3, 4, 6, and 7, they essentially determine the weighted mean of the seven direct measurements. Indeed, the weighted mean of just these three more precise values is

$$R_K = (25812{,}807\ 72 \pm 0{,}000\ 61)\ \Omega \qquad \text{(A7a)}$$

$$= R_o\ [1 + (0{,}299 \pm 0{,}024) \times 10^{-6}], \qquad \text{(A7b)}$$

where the uncertainty has been calculated on the basis of external consistency; $\chi^2 = 2{,}70$ for $\underline{\nu} = 2$, $R_B = 1{,}16$, and $P(2{,}70|2) \simeq 0{,}26$. The agreement is quite reasonable.

An indirect value based on the weighted mean of items 8, 9, and 11 may be obtained for comparison:

$$R_K = (25812{,}805\ 94 \pm 0{,}000\ 27)\ \Omega \qquad \text{(A8a)}$$

$$= R_o[1 + (0{,}230 \pm 0{,}010) \times 10^{-6}], \qquad \text{(A8b)}$$

where the uncertainty has been calculated on the basis of external consistency; $\chi^2 = 3{,}40$ for $\underline{\nu} = 2$, $R_B = 1{,}30$, and $P(3{,}40|2) \simeq 0{,}18$. The agreement is reasonable. The difference between Eqs. (A7) and (A8) is $(0{,}001\ 78 \pm 0{,}000\ 67)\ \Omega$, which corresponds to a relative difference of $(0{,}069 \pm 0{,}026) \times 10^{-6}$. Compared in this way, the direct and indirect values are not in particularly good agreement. (Using item 10 in place of item 11 yields a similar result.) Indeed, the uncertainty of item 8 is sufficiently small compared with the uncertainties of the other indirect values that it essentially determines the indirect value. The difference between Eq. (A7) and item 8 is (0,001 73 ± 0,000 65) Ω, which corresponds to a fractional difference of (0,067 ± 0,025).

Finally, the result of the above comparison of Eqs. (A7) and (A8) leads us to calculate the weighted mean of just items 1, 2, 5, 8, and 10. These five more precise values of R_K have relative uncertainties of less than 7 x 10^{-8}, which is less than one half that of the next most precise value. They yield

$$R_K = (25\ 812{,}806\ 16 \pm 0{,}000\ 37)\ \Omega \qquad \text{(A9a)}$$

$$= R_o[1 + (0{,}239 \pm 0{,}014) \times 10^{-6}, \qquad \text{(A9b)}$$

where the uncertainty has been calculated on the basis of external consistency; $\chi^2 = 15{,}0$ for $\nu = 4$, $R_B = 1{,}93$, and $P(15{,}0|4) \simeq 0{,}005$. As could be anticipated from the above comparison, the agreement is poor. (Including item 11 in place of item 10 yields the identical result because of the relationship between items 5, 10, and 11 discussed previously.)

A.1.4. *Selection of Recommended Value*

Based on the weighted mean of all the data as given in Eq. (A4), and the Working Group's adopted guiding principle discussed in the first section of this appendix, an obvious choice for the recommended value is 25 812,806 Ω. That this is identical to the value of R_K obtained from $\alpha^{-1}(a_e)$, item 8, is in large part due to the latter's small uncertainty in comparison with the uncertainties of the other values. On the other hand, based on the weighted mean of the direct measurements only rather than all the measurements, an obvious choice is 25 812,808 Ω as given in either Eqs. (A6) or (A7).

The Working Group believes that its recommended value of R_K should not be dominated by a single indirect value which has not been verified by independent experiments and calculations, and that it should reflect the results of the direct measurements. On this basis, the recommended value is taken as the simple mean of the above two values, namely, 25 812,807 Ω.

The question remains as to the one standard deviation uncertainty to be assigned this value which will also be consistent with the Working Group's guiding principle. Considering that the peak-to-peak scatter among the values of R_K given in Table A1 (including item 1b) is about 0,01 Ω, and that there are still lingering questions about the equality $R_K = \mu_0 c \alpha^{-1}/2$, the Working Group believes that adopting 0,005 Ω as the one standard deviation uncertainty, which corresponds to a relative uncertainty of 2 x 10^{-7}, is consistent with both its guiding principle and the current situation. Thus the Working Group's recommended value and assigned uncertainty are

$$R_K = 25\ 812{,}807\ \Omega \qquad \text{(A10a)}$$

Standard deviation: 0,005 Ω (A10b)

Relative standard deviation: 2 x 10^{-7}. (A10c)

Figure A2 graphically compares this value with the data of Table A.1. (The dashed line is the recommended value and the shading delimits its uncertainty.) Equation (A10) is consistent with the 1986 CODATA value (25 812,805 6 ± 0,001 2) Ω [A18].

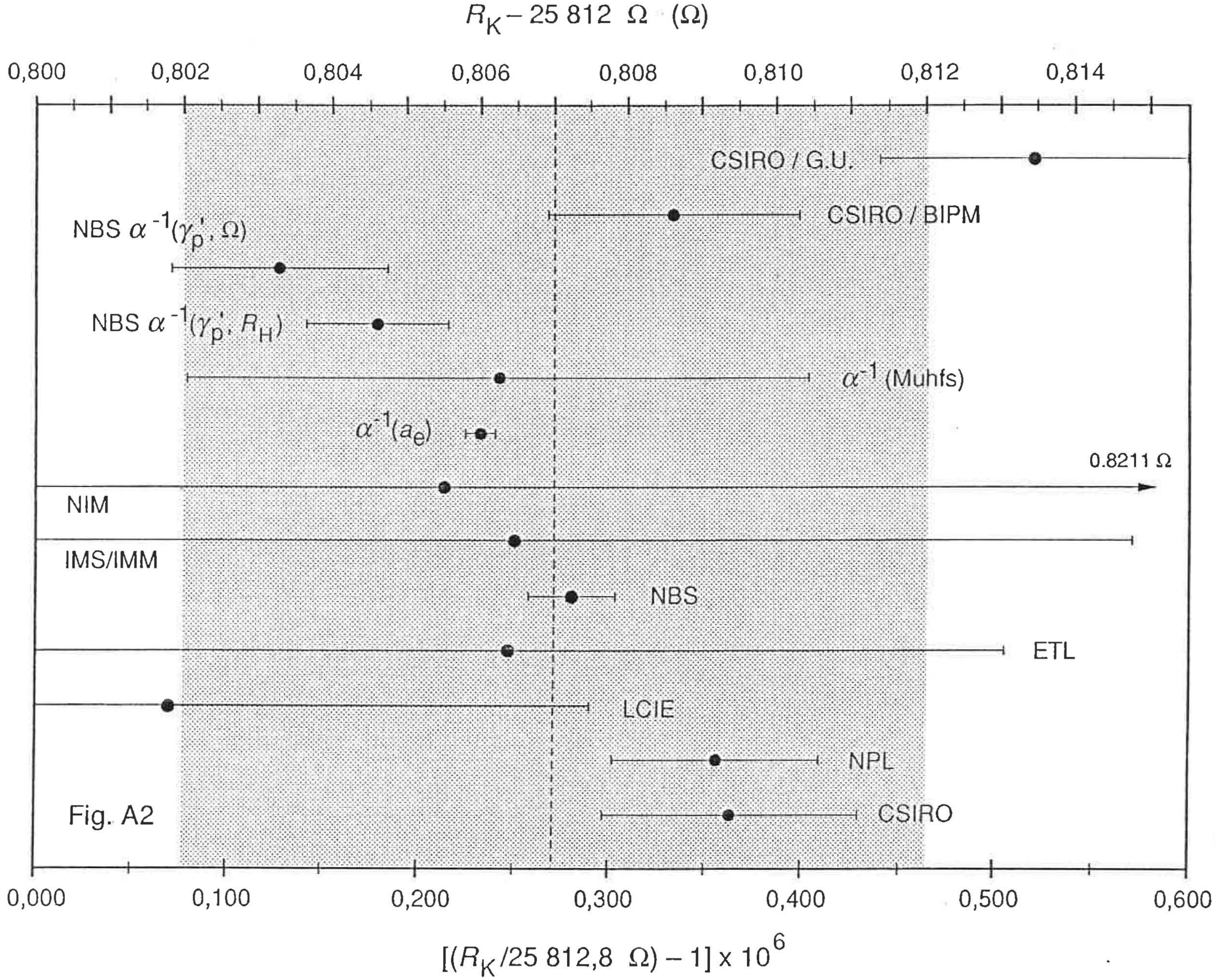

Fig. A2. Comparison of the recommended value or R_K (vertical dashed line) and its standard deviation uncertainty (delimited by the shading) with the values of R_K and their standard deviation uncertainties given in Table A1.

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