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BUREAU INTERNATIONAL DES POIDS ET MESURES

REPORT TO THE COMITÉ CONSULTATIF D'ÉLECTRICITÉ FROM THE WORKING GROUP ON THE JOSEPHSON EFFECT

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August, 1988

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REPORT

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1. INTRODUCTION

1.1. *Background*

So that this report may also be of use to the community of electrical metrologists at large, we begin with a brief review.

The Comité Consultatif d'Électricité (CCE) of the Comité International des Poids et Mesures (CIPM) is one of eight Consultative Committees to the CIPM which together cover most of the areas of basic metrology. These committees, which may establish temporary or permanent "Working Groups" to study special subjects, coordinate the international work carried out in their respective fields, advise the CIPM about the work of the Bureau International des Poids et Mesures (BIPM) in these fields, and propose appropriate actions to the CIPM including recommendations concerning changes in the definitions and representations of units. The CIPM may endorse, modify, or reject these recommendations, submitting as appropriate those which will have a very broad impact to the Conférence Générale des Poids et Mesures (CGPM) for final approval.

As an organ of the Convention du Mètre, one of the responsibilities of the CCE is to ensure the propagation and improvement of the Système International d'Unités or SI. The SI serves as a basis for the promotion of long-term, worldwide uniformity of electrical measurements which is of considerable technical and economic importance to commerce and industry.

Consequently, at its 13th meeting held in 1972, the CCE suggested that the national standards laboratories adopt* 483 594,0 GHz/V as the conventional value of the Josephson frequency to voltage quotient for use in realizing and maintaining laboratory representations of the volt† by means of the Josephson effect [1]. While most national laboratories did in fact

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*In keeping with the preferred ISO usage, *commas* are used in this report to indicate decimal fractions. Also, in accordance with proper SI usage, symbols for units are written in Roman letters and symbols for physical quantities are italicized or underlined.

†The volt means the SI unit of electric potential difference or electromotive force. Occasionally it may be referred to in the literature as the absolute volt. As-maintained volt, representation of the volt, laboratory representation of the volt, "national unit of voltage," "laboratory unit of voltage," practical realization of the volt, or other similar terms are commonly used to indicate a practical reference standard of voltage. The word *unit* should not be used in this context. The only unit of electric potential difference in the SI is the volt. This report uses the expression *representation of the volt* and variations thereof.

adopt this value, three did not. The U.S., France, and the U.S.S.R. adopted values of the quotient which are, respectively, $(1 - 1,20 \times 10^{-6})$, $(1 + 1,32 \times 10^{-6})$, and $(1 + 4,50 \times 10^{-6})$ times the CCE 1972 stated value [2]. As a consequence, the national representations of the volt of these countries differ by -1,20 μV, 1,32 μV, and 4,50 μV, respectively, from the national representations of those countries which use the 1972 value. Moreover, it has recently become evident that the 1972 value is about $(1 - 8 \times 10^{-6})$ times the SI value and thus the national representations of the volt of those countries that have adopted it are about 8 μV smaller than the SI unit [2]. For the U.S., France, and the U.S.S.R., the differences from SI are about -9,2 μV, -6,7 μV, and -3,5 μV, respectively.

To address these two problems, nonuniformity among countries and inconsistency with the SI, the CCE at its 17th meeting held in September 1986 established through Declaration E1 (1986), "Concerning the Josephson effect for maintaining the representation of the volt," the CCE Working Group on the Josephson Effect [3]. The CCE charged the Working Group with making a proposal for a new value of the Josephson frequency to voltage quotient consistent with the SI value based upon all relevant data that become available by 15th June 1988.

Further, the CCE stated its intention to meet in September 1988 with a view to recommending a new value of this quotient to come into effect on 1st January 1990 for the purpose of maintaining a highly stable and accurate representation of the volt in all those national standards laboratories (and otherwise) that base their representation of the volt on the Josephson effect.

This report by the Working Group on the Josephson Effect is in direct response to the charge by the CCE. It proposes a new value of the Josephson frequency to voltage quotient, gives the basis for this new value, and summarizes three approaches to how a representation of the volt based on the Josephson effect may be used in practice.

1.2. *Permanence of the New Representation of the Volt*

In its discussions leading to Declarations E1 and E2 (1986), the CCE agreed that while worldwide uniformity of electrical measurements can only be assured through the SI, in the particular areas of voltage and resistance, scientific, commercial, and industrial requirements for long-term reproducibility now exceed the accuracy with which the SI units can be readily realized. To meet these very exacting demands, the CCE believes it is necessary that representations of the volt and the ohm be established that have a superior long term reproducibility and constancy than the present direct realizations of the SI units themselves.

Although the Working Group believes that its recommended value for the Josephson frequency to voltage quotient upon which the new representation of the volt is to be based is consistent with the SI value within its assigned uncertainty, it recognizes that barring an unexpected stroke of good luck, future, more accurate measurements will no doubt show that the new recommended value differs from the SI value by some small amount. In keeping with the point of view of the CCE, the Working Group envisages that should such a situation occur, the CCE could simply note the difference between the volt and its new representation. This would be useful for those workers (mostly in the fields of realizing the electrical units and determining the fundamental physical constants) for whom the small difference may be significant. Since any such difference is expected to be sufficiently small that practical electrical measurements will be unaffected, the Working Group strongly believes that the new recommended value will not need to be significantly altered in the foreseeable future.

However, this last statement must not be interpreted to mean that improved realizations of the volt are now unnecessary. Because an accurate representation of the volt is important to science, commerce, and industry, the Working Group considers it important for laboratories to continue their efforts to realize the volt with greater accuracy, either directly or indirectly through measurements of relevant fundamental constants. This could result in a significant reduction of the uncertainty assigned to the new representation.

1.3. *Laboratories That Do Not Use the Josephson Effect*

The purpose of the new volt representation is to improve worldwide uniformity of national representations of the volt and their consistency with the SI. The question thus arises as to the procedure to be followed by those laboratories which do not base their representation of the volt on the Josephson effect. In keeping with the viewpoint expressed by the CCE during its discussions in connection with Declaration E1 (1986), the Working Group proposes that on 1st January 1990, such laboratories adjust the value of their representation of the volt so that it is consistent with the new representation. Furthermore, this consistency should be maintained by having a transportable voltage standard periodically calibrated by a laboratory that does base its representation of the volt on the Josephson effect, for example, BIPM.

2. DEFINITIONS, SYMBOLS, AND NOMENCLATURE

2.1. *Josephson Frequency to Voltage Quotient*

As is now well known, the Josephson effects (AC and DC) are characteristic of weakly coupled superconductors when cooled below their transition temperatures [4]. An example is two thin films of superconducting lead separated by an approximately 1 nm-thick thermally-grown oxide layer.

When, under the proper experimental conditions, such a "Josephson device" is irradiated with electromagnetic radiation of frequency ν, its current-voltage curve exhibits current steps at highly precise quantized Josephson voltages U_J. The voltage of the nth step $U_J(n)$, n an integer, is related to the frequency of the radiation by

$$U_J(n) = n\nu/K_J, \tag{1}$$

where K_J is the *Josephson frequency to voltage quotient* which we term the *Josephson constant*. (Since no symbol has yet been adopted for this quotient, the Working Group proposes the use of K_J. It follows from Eq. (1) that the Josephson constant is equal to the frequency to voltage quotient of the $n = 1$ step.)

A significant amount of experimental evidence supports the view that the Josephson constant K_J is a universal quantity independent of experimental variables, for example, type of superconductor, temperature, and irradiation frequency and power, to very high precision [5-15]. Indeed, in one experiment it was shown that K_J was the same for two Josephson devices made of different superconducting materials to within a relative difference of 2×10^{-16} [11]. A Josephson device may thus be viewed as a nearly perfect frequency to voltage transducer.

The theory of the Josephson effects predicts, and the experimentally observed universality of the Josephson frequency-voltage relation [Eq. (1)] is consistent with the prediction, that K_J is equal to the invariant quotient of fundamental constants $2e/h$, where e is the elementary charge and h is the Planck constant [4, 16, 17, 18]. The Working Group thus assumes for the purpose of including data from measurements of fundamental constants that $K_J = 2e/h$. (The same assumption was made by the CODATA Task Group on Fundamental Constants in obtaining their 1986 set of recommended values of the constants [19].)

2.2. *The New Representation of the Volt and Its Practical Use*

In Appendix A of this report, we consider the currently available measurements of the Josephson constant K_J, deriving from them our recommended value in SI units and its associated one standard deviation assigned uncertainty*:

$$K_J = 483\ 597{,}9\ \text{GHz/V} \tag{2a}$$

$$\text{Standard deviation: } 0{,}2\ \text{GHz/V} \tag{2b}$$

$$\text{Relative standard deviation: } 4 \times 10^{-7}. \tag{2c}$$

For the purpose of basing a representation of the volt on the Josephson effect, the Working Group proposes to use Eq. (2a) to define the following conventional value for the Josephson constant:

$$K_{J-90} \overset{\text{def}}{=} 483\ 597{,}9\ \text{GHz/V} \tag{3}$$

exactly, where the subscript 90 derives from the fact that the new representation of the volt is to come into effect starting on 1st January 1990.

The Working Group has identified three approaches to how a representation of the volt based on the Josephson effect and the defined physical quantity K_{J-90} may be used in practice, each having both advantages and disadvantages. These approaches are summarized below. Two are both rigorous and correct, but in one we define a new unit, V_{90}, and in the other we define a new physical quantity, E_{90}. The Working Group believes that the best way to avoid

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*Throughout this report, we treat uncertainties following the suggestions of the BIPM Working Group on the Statement of Uncertainties as embodied in Recommendation INC-1 (1980) which has been approved by the CIPM [20]. In particular, all uncertainties are given as one standard deviation estimates in keeping with CIPM Recommendation 1 (CI-1986) [21].

confusion internationally is for the national standards laboratories to adopt a uniform approach. It is imperative that the laboratories avoid giving the impression that there is more than one representation of the volt in general use and that there may be significant differences between national realizations of the new representation of the volt.

2.2.1. *Approach 1*

A new unit of electromotive force (or electric potential difference) is defined via the equation

$$\mathrm{V}_{90} \stackrel{\mathrm{def}}{=} (K_{\mathrm{J}\text{-}90}/K_{\mathrm{J}})\ \mathrm{V} \tag{4}$$

exactly.* However, the *experimental* realization by a particular laboratory of the *defined* unit V_{90} has an associated uncertainty. Based on the Josephson effect apparatus in current use, this uncertainty will generally lie in the range 0,01 μV to 0,1 μV [2]. Since Eqs. (2), (3), and (4) imply that

$$1\ \mathrm{V}_{90} = 1\ \mathrm{V} \pm 0{,}4\ \mu\mathrm{V}, \tag{5}$$

the uncertainty with which a particular realization of V_{90} represents the volt will have two components: the 0,4 μV of Eq. (5) and the experimental uncertainty associated with the realization. If to be specific we assume the latter is 0,07 μV, then the emf E' of a particular standard cell expressed in terms of V_{90} would be (again to be specific)

$$E' = (1{,}018\ 603\ 59 \pm 0{,}07 \times 10^{-6})\ \mathrm{V}_{90}. \tag{6}$$

(We also assume for simplicity a perfect standard cell and no uncertainty associated with the calibration process.) It follows from Eqs. (5) and (6) that the emf of the cell expressed in volts is

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*Equation (4) is reminiscent of the familiar relation $V_{76\text{-BI}} = 483\ 594\ \mathrm{GHz}/K_{\mathrm{J}}$, where the physical quantity $V_{76\text{-BI}}$ is the BIPM representation of the volt based on the Josephson effect starting on 1st January 1976.

$$E' = (1{,}018\ 603\ 59 \pm 0{,}41 \times 10^{-6})\ \text{V}. \tag{7}$$

If it is necessary to distinguish between different experimental realizations of V_{90}, the symbol $V_{90\text{-LAB}}$ may be used, where LAB stands for a convenient abbreviation of the name of the laboratory carrying out the realization. Such distinction should only be necessary for work involving two or more national standards laboratories; it should not be required even in dealings with the most demanding users of calibration services.

Advantages of Approach 1

- It enables voltage measurements to be reported in a straightforward way in terms of a laboratory's realization of V_{90} (*i.e.*, in terms of the laboratory's representation of the volt) with its relatively small uncertainty.

- It is consistent with current practice since most standards laboratories report the results of calibrations in terms of their representation of the volt. Consequently, it will be readily understood by users of calibration services.

- The incorrect practice of using the physical quantity V_{LAB} as a unit will be replaced by the correct practice of using the unit V_{90}.

Disadvantages of Approach 1

- It introduces a new unit which is likely to differ from the volt by some small amount and which is parallel to and thus in competition with the volt. Moreover, if Approach 1 is used in a similar way to define a new unit of resistance, Ω_{90}, based on the quantum Hall effect, then a complete parallel and thus competitive system of electrical units will have been introduced (*i.e.*, one would have A_{90}, W_{90}, C_{90}, F_{90}, H_{90}, T_{90}, *etc.*). This could be detrimental to coherence in the expression of physical quantities. For example, consistency between electrical and mechanical power, assured by the SI, would no longer be guaranteed.

2.2.2. *Approach 2*

This is formally the same approach used by the Comité Consultatif de Thermométrie (CCT) to define the 1968 temperature scale and which it will likely use to define the new International Temperature Scale 1990 (ITS-90) to come into effect on 1st January 1990.

Let E be the symbol for the physical quantity electromotive force whose unit is the volt. Let E_{90} be the symbol for a new physical quantity called "conventional electromotive force" exactly defined by

$$E_{90} \stackrel{\text{def}}{=} (K_J/K_{J\text{-}90})E \tag{8}$$

whose unit is also the volt. (Clearly electric potential difference U may be treated in a similar way.) A calibration of the same standard cell (and under the same assumptions) discussed in Approach 1 in terms of a laboratory's experimental realization of E_{90} would be expressed as

$$E'_{90} = (1{,}018\ 603\ 59 \pm 0{,}07 \times 10^{-6})\ \text{V}. \tag{9}$$

[One way of demonstrating that Eq. (9) is correct is by combining Eqs. (4), (6), and (8).] It is important to recognize that E'_{90} is a new physical quantity; it is not the same as E' but is related to it through Eq. (8). However, the numerical value of E'_{90} expressed in volts [Eq. (9)] is the same as the numerical value of E' expressed in terms of the unit V_{90} [Eq. (6)], but E'_{90} has the units of volts. It follows from Eqs. (2), (3), (8), and (9) that E' in volts is

$$E' = (1{,}018\ 603\ 59 \pm 0{,}41 \times 10^{-6})\ \text{V}. \tag{10}$$

As would be expected, Eq. (10) is identical to Eq. (7).

In a manner similar to that discussed under Approach 1, if it is necessary to distinguish between experimental realizations of E_{90} or measurement results such as E'_{90}, then the symbols $E_{90\text{-LAB}}$ or $E'_{90\text{-LAB}}$ may be used.

Advantages of Approach 2

It enables voltage measurements to be reported in both a straightforward and rigorous way in terms of a laboratory's representation of the volt with its relatively small uncertainty.

- It does not introduce a new unit to compete with the volt; measurements are reported in volts.

Disadvantages of Approach 2

- It is not consistent with current practice in electrical metrology and is likely to cause some confusion.

- It introduces a new physical quantity for emf which is likely to differ from emf by some small amount. Thus the same standard cell would have both a conventional emf and an emf. Moreover, if Approach 2 is used in a similar way to define a new physical quantity for resistance, R_{90}, based on the quantum Hall effect, then a complete parallel set of electrical quantities will have been introduced (*i.e.*, one would have I_{90}, P_{90}, Q_{90}, C_{90}, L_{90}, B_{90}, *etc.*). However, historically, the confusion resulting from the use of concurrent systems of electrical units is well known, but experience in the area of thermometry has shown that the introduction of a conventional temperature has not resulted in a comparable level of misunderstanding.

2.2.3. *Approach 3*

This approach is in reality Approach 1 but a unit such as V_{90} is not formally defined and used. The calibration of the above standard cell would be reported as

$$E' = (1{,}018\ 603\ 59 \pm 0{,}07 \times 10^{-6})\ \text{V} \qquad (11)$$

but with accompanying text stating in effect that the value given is not really in volts but is actually based on the laboratory's representation of the volt which in turn is based on the Josephson effect and the internationally adopted value of the Josephson constant as recommended by the CCE. Because the unit V is used in Eq. (11), equations such as (7) and (10) could not be readily given (assuming it was useful to do so). Instead, it would have to be stated in the text that the uncertainty of the emf of the cell in volts is ± 0,41 μV.

Advantages of Approach 3

- Because of its similarity with current practice in some laboratories, it should be readily understood.
- It avoids formally introducing a new unit of emf or a conventional emf.

Disadvantages of Approach 3.

- It lacks rigor; Eq. (11) is incorrect since it gives the emf in volts but the uncertainty as if the emf were reported in terms of the laboratory's representation of the volt. If E' is reported in volts, its uncertainty should be given as 0,41 μV. In a variation of Approach 3, one avoids giving an incorrect equation such as Eq. (11) by deleting the unit V and adding further explanatory text. This increases further the amount of written material required to explain the reported value. Moreover, without such detailed information, this approach would be a continuing source of confusion.
- In contrast to Approaches 1 and 2, there is no clear indication that a new representation of the volt is in use.

2.2.4. *Working Group Recommendation*

Two members of the Working Group on the Josephson effect prefer Approach 2 because of its rigor and because it does not introduce a new unit in competition with the volt. One member prefers Approach 3 or its variant because it is in common use and will not be a real change. He believes that the lack of rigor of this approach is of little practical consequence. (Among the members of the Working Group on the Quantum Hall Effect, the preferences are: one member for Approach 1, four for Approach 2, and one for Approach 3.)

Because of its importance, the Working Group believes that the CCE in its entirety should consider this issue and recommend a solution.

3. CONCLUSION

∘ Based on direct measurements of the Josephson constant K_J, and indirect measurements involving fundamental physical constants, the Working Group adopts 483 597,9 GHz/V as its recommended value for K_J with an assigned one standard deviation uncertainty of 0,2 GHz/V, corresponding to a relative uncertainty of 4×10^{-7}.

∘ The uncertainty of the new representation of the volt based on the Josephson effect and the Working Group's recommended value for K_J is 0.4 μV, one standard deviation estimate.

∘ The Working Group expects that its new recommended value for K_J will not need to be significantly altered in the foreseeable future.

∘ Because science, commerce, and industry require an accurate and internationally uniform representation of the volt, the Working Group strongly supports the view of the CCE that the new value of the Josephson constant be adopted simultaneously on 1st January 1990 by all those laboratories that base their representation of the volt on the Josephson effect, and that beginning on this date all other laboratories adjust and maintain the value of their representation of the volt to be consistent with the new value.

∘ The new recommended value for the Josephson constant is approximately $(1 + 8{,}06 \times 10^{-6})$ times the value 483 594 GHz/V stated by the CCE in 1972. This implies that the new representation of the volt will exceed a representation of the volt based on the 1972 value by about 8,06 μV.

◦ To avoid confusion internationally, the Working Group believes that the national standards laboratories should adopt a uniform approach to using the new representation of the volt. The laboratories must avoid giving the impression that there is more than one representation of the volt in use and that national realizations of the new representation differ significantly. This uniformity will be enhanced if laboratories refrain from using distinguishing symbols to denote their representation of the volt.

◦ Given the importance of an accurate representation of the volt to science, commerce, and industry, laboratories should continue their efforts to realize the volt with improved accuracy so that the uncertainty of the new representation may be reduced.

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APPENDIX A

A.1. DERIVATION OF THE WORKING GROUP'S RECOMMENDED VALUE OF THE JOSEPHSON CONSTANT K_J

A.1.1. *Approach*

Because the Working Group's recommended value of K_J is for use in realizing a practical representation of the volt by means of the Josephson effect, we adopt the following guiding principle for its derivation: The value should be so chosen that it is unlikely to require significant change in the foreseeable future. This means that the number of digits given for the recommended value should be the minimum possible and that its uncertainty should be conservatively assigned. This principle also implies that it is unnecessary to carry out a complete least-squares adjustment of the fundamental physical constants to derive the recommended value; a straightforward treatment of the individual measurements of K_J currently available should suffice.

A.1.2. *Summary of Data*

Table A1 summarizes the measurements of K_J to be considered while Fig. A1 compares them graphically, starting from the bottom of the figure. (To aid in the comparison, the most accurate value and its uncertainty are indicated by dashed and full lines, respectively, as well as by the usual point and error bars.) Values are included only if they were available by the 15 June 1988 date stated by the CCE in its Declaration E1 (1986) and for which some form of documentation was available to the Working Group. Although we shall assume $K_J = 2e/h$ as discussed in Sect. 2.1, only the last three entries of Table A1 (items 8, 9, 10) require this assumption. Such values are usually termed

Table A.1. Summary of values of the Josephson constant K_J. For ease of comparison, the values are given in two forms: in GHz/V (column 2); and in parts in 10^6 relative to the value for the Josephson constant stated by the CCE in 1972, namely, 483 594 GHz/V (column 3).

Item No.	K_J (GHz/V)	$[(K_J/483\ 594\ \mathrm{GHz/V})-1]\times10^6$	Remarks and references
1.	483 597,91 ± 0,13	8,09 ± 0,27	CSIRO/NML Hg electrometer [A1, A2]
2.	483 598,77 ± 0,17	9,86 ± 0,35	U. Zagreb capacitor volt balance, realization of farad via calculable capacitor and voltage calibrations in terms of K_J from other laboratories [A3]
3.	483 597,903 ± 0,037	8,070 ± 0,077	NPL realization of watt via moving-coil balance, realization of ohm via calculable capacitor [A4]
4.	483 597,84 ± 0,32	7,94 ± 0,67	NBS realization of watt via moving-coil balance, realization of ohm via calculable capacitor [A5-A7]
5.	483 597,94 ± 0,33	8,15 ± 0,67	NBS γ'_p(high) from F, γ'_p(low), realization of ohm via calculable capacitor [A8]
6.	483 597,88 ± 0,48	8,02 ± 0,99	NIM γ'_p(high), γ'_p(low), realization of ohm via calculable capacitor [A9-A12]
7.	483 597,54 ± 0,25	7,33 ± 0,52	NPL γ'_p(high), NBS γ'_p(low), NBS and NML realizations of ohm via calculable capacitor [A13]
8.	483 597,40 ± 0,29	7,03 ± 0,60	$2e/h$ from NBS N_A, $\underline{\alpha}(a_e)$ [A14]
9.	483 597,70 ± 0,32	7,65 ± 0,66	$2e/h$ from PTB N_A using NBS silicon reference sample of known molar mass, $\underline{\alpha}(a_e)$ [A15]
10.	483 595,90 ± 0,39	3,92 ± 0,80	$2e/h$ from ASMW γ'_p via γ'_p(low) and γ'_p(high), $\underline{\alpha}(a_e)$ [A16]

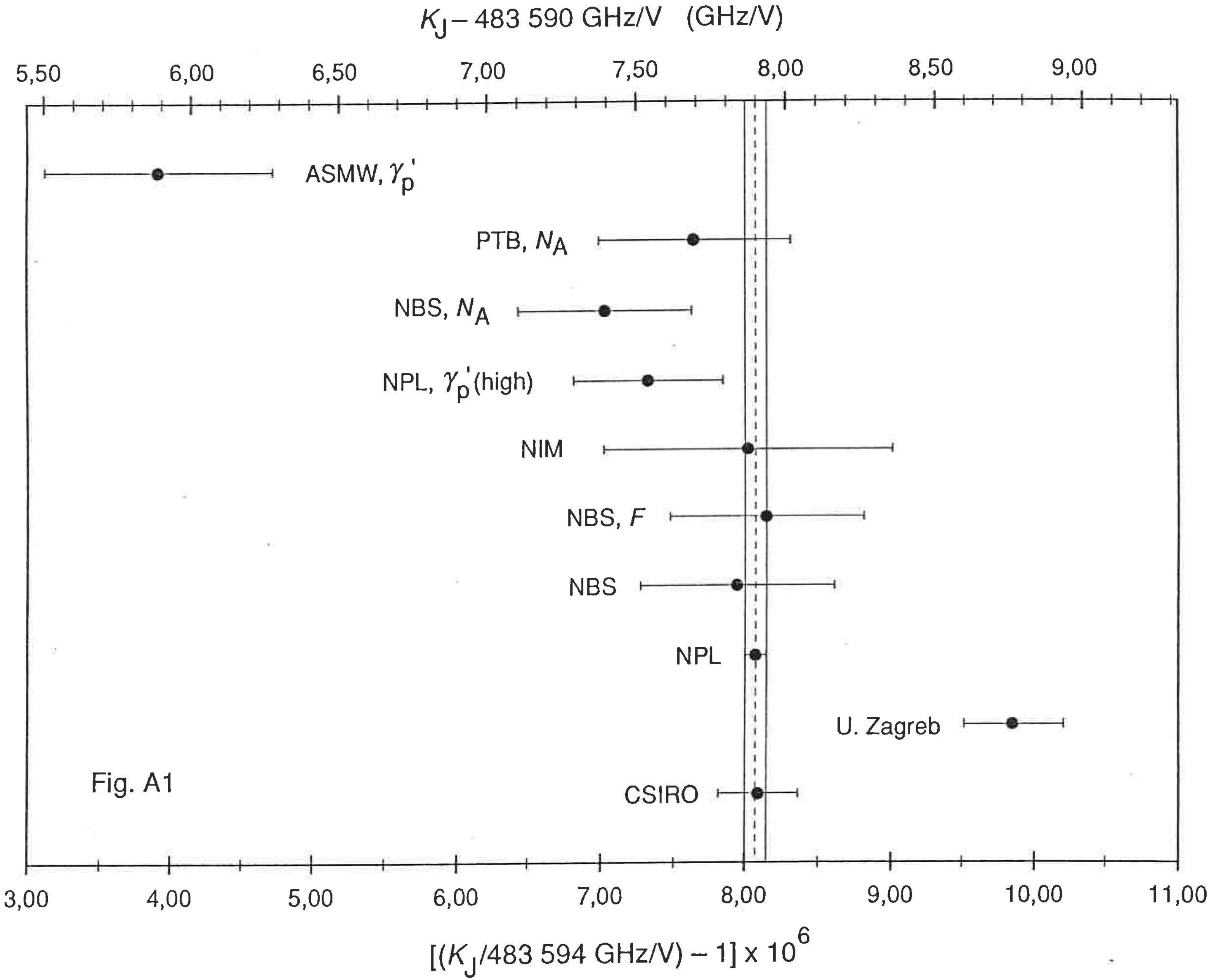

Fig. A1. Comparison of the values of K_J and their standard deviation uncertainties as given in Table A1. The vertical dashed and solid lines indicate the value and standard deviation uncertainty of the most precise result.

indirect, while those which do not require this assumption (items 1 through 7) are termed *direct*.

In general, we have excluded an earlier result from a particular experiment when it has been replaced by a more recent and presumably more reliable result from the same experiment. Also excluded are measurements having a relative uncertainty larger than about 1 x 10^{-6} because they cannot contribute in a significant way to the derivation of our recommended value.

The values given in Table 1 require further explanation.

Item 1. The relative uncertainty of the Commonwealth Scientific and Industrial Research Organization (CSIRO), National Measurement Laboratory (NML), Australia, elevated mercury column or so-called mercury electrometer result has been reduced from 0,31 x 10^{-6} to 0,27 x 10^{-6} based on further measurements relating to the density of the reference mercury used in the NML experiment and to the stability of the density of mercury during long-term storage [A1, A2].

Item 2. The result from the Faculty of Electrical Engineering, University of Zagreb, Yugoslavia, given in the table is from their latest and most precise measurements [A3]. It was obtained using volt balance ETF-84 during late 1987 and the first half of 1988. However, it differs significantly from the results obtained from 1981 to 1985 using volt balances ETF-80 and ETF-82. Possible sources of systematic error in the present balance and associated equipment are being vigorously investigated. This experiment requires knowledge of the value of a reference capacitor in farads, but since it enters into the calculation of K_J to the one-half power, its contribution to the uncertainty is reduced by a factor of two.

Item 3. To obtain a value of K_J from a watt realization experiment of the moving-coil type developed at the National Physical Laboratory (NPL), U.K., requires knowledge of a reference resistance in ohms. This resistance can either be an artifact-based resistance standard or the quantized Hall resistance. (The contribution of the uncertainty of this reference resistance to K_J is reduced by a factor of two since it enters to the one-half power.) The NPL value of K_J given in the table [A4] is based on an NPL realization of the ohm using a calculable capacitor. If it were based on the value of the von Klitzing constant recommended by the CCE Working Group on the quantum Hall effect (QHE), it would be 0,042 parts in 10^6 larger, a little over one half the standard deviation of the NPL value. Because this is a comparatively small shift and the relative uncertainty assigned by the QHE Working Group to the von Klitzing constant is 2×10^{-7}, we take the NPL result as given.

Item 4. The experiment to realize the watt by the moving-coil method at the National Bureau of Standards (NBS), U.S.A., is similar to that at NPL but it has not yet reached the same level of precision because a much weaker magnetic field is currently being used. The NBS result [A5-A7] is based on a realization of the ohm at NBS via a calculable capacitor. Using instead the value of the von Klitzing constant recommended by the QHE Working Group would increase the NBS result by less than one part in 10^8.

Item 5. A value of K_J can be obtained from so-called low and high field measurements of the gyromagnetic ratio of the proton, $\gamma_p'(\text{low})$ and $\gamma_p'(\text{high})$, and a realization of the ohm [A17] (the prime indicates a spherical, pure H_2O nuclear magnetic resonance or NMR sample at 25 °C). For this result, $\gamma_p'(\text{high})$ was derived from an NBS measurement of the faraday constant F and the accepted values of well-known constants [A8]. The experiments to realize the ohm via the NBS calculable capacitor and to measure F and γ'_p(low) were carried out at NBS during the period 1973 to 1978.

Item 6. This result from the National Institute of Metrology (NIM), P.R.C., was obtained in the same way as item 5 except γ'_p(high) was measured directly using NMR and a force balance [A9-A12]. The experiments to realize the ohm and to measure γ'_p(low) and γ'_p(high) upon which it is based were carried out from Oct. 1987 to May 1988 and supercede those of the 1970's.

Item 7. Like data items 5 and 6, data item 7 is based on a realization of the ohm and measurements of γ'_p(low) and γ'_p(high). It was obtained by the Working Group from the 1974 NPL measurement of γ'_p(high) [A13], the 1978 NBS measurement of γ'_p(low) [A18], a 1973 NBS calculable capacitor realization of the ohm [A19], NML calculable capacitor realizations of the ohm carried out over the period 1964 to 1987 [A20, A21], and the results of the international comparisons of national representations of the ohm organized by BIPM over the same period. Because the same value of γ'_p(low) was used to obtain item 5, items 5 and 7 are not completely independent; their correlation coefficient is 0.03. This correlation is taken into account in the calculations carried out in the next section. It is relatively small because the uncertainties of the two values of γ'_p(high) upon which items 5 and 7 are based are about six and five times larger, respectively, than the uncertainty of the NBS 1978 γ'_p(low) value.

Item 8. A value of $2e/h = K_J$ can be obtained from a measurement of the Avogadro constant N_A via the relation

$$K_J = [16R_\infty(m_p/m_e)N_A/\mu_0 c^2 M_p \alpha]^{1/2}, \qquad \text{(A1)}$$

where R_∞ is the Rydberg constant for infinite mass, m_p/m_e is the proton to electron mass ratio, $\mu_0 = 4\pi \times 10^{-7}$ N/A^2 exactly is the permeability of vacuum, c = 299 792 458 m/s exactly is the speed of light in vacuum, M_p is the molar mass of the proton, and α is the fine-structure constant. Item 8 was derived by the Working Group from this equation using (i) N_A = 6,022 129 7(72) x 10^{23} mol^{-1} based on the most recent NBS silicon lattice spacing measurements [A14] and the NBS value for the molar volume of silicon used in the 1986 CODATA adjustment of the fundamental constants [A17] but updated to account for a new mass adjustment [A22] (the change is inconsequential); (ii) the CODATA value for m_p/m_e [A17]; (iii) R_∞ = 10 973 731,573(4) m^{-1} [A23], a more recent and accurate value than that of CODATA; (iv) M_p = 1,007 276 468(7) x 10^{-3} kg/mol based on the new value for the nuclidic mass of hydrogen from the new mass adjustment [A22]; and (v), the most recent value of the fine-structure constant from the electron magnetic moment anomaly a_e [A24], $1/\alpha(a_e)$ = 137,035 991 4(11). While the NBS silicon lattice spacing result is not final, the value is unlikely to change by an amount of any significance in comparison with the 1.15 x 10^{-6} relative uncertainty of the silicon molar volume.

Item 9. This result for $2e/h = K_J$ was derived by the Working Group from Eq. (A1) using the value N_A = 6,022 137 3(79) x 10^{23} mol^{-1} as obtained from the Physikalisch-Technische Bundesanstalt, F.R.G., measurements of the lattice spacing and molar volume of silicon [A15]. Data items 8 and 9 are not entirely independent because they are based on the molar mass of the same silicon reference material. The 0,42 x 10^{-6} relative uncertainty of the molar mass of this material leads to a correlation coefficient between the two values of 0,11. This correlation is taken into account as appropriate in the calculations carried out in the next section.

Item 10. This result for $2e/h = K_J$ was derived by the Working Group from the Amt für Standardisierung, Messwesen, und Warenprüfung (ASMW), D.D.R., low and high field measurements of the proton gyromagnetic ratio completed in 1985 [A16]. It is based on the relations

$$\gamma_p' = [\{\gamma_p'(\text{low})\}\{\gamma_p'(\text{high})\}]^{1/2}\ \text{s}^{-1}\text{T}^{-1} \tag{A2}$$

$$2e/h = 4R_\infty\gamma_p'/c\alpha^2(\mu_p'/\mu_B), \tag{A3}$$

where { } indicates numerical value only and it is assumed that $\gamma_p'(\text{low})$ and $\gamma_p'(\text{high})$ are measured in terms of the same laboratory representations of the volt and ohm; μ_p'/μ_B is the magnetic moment of the proton in units of the Bohr magneton. Using the CODATA value of μ_p'/μ_B, $\gamma_p' = 2{,}675\ 142\ 7(21) \times 10^8\ \text{s}^{-1}\text{T}^{-1}$ from the ASMW measurements, and the values for the other constants indicated previously, yields the result in the table.

An alternate approach would have been to reexpress the two ASMW measurements in terms of the BIPM representations of the volt and ohm and to use the NML calculable capacitor realization of the ohm to obtain a value of K_J rather than $2e/h$ in a manner similar to that used to obtain items 5, 6, and 7. However, the use of Eq. (A3) minimizes the considerable problems associated with the transfer to the BIPM representations without introducing any significant additional uncertainty since the constants entering Eq. (A3) are well known in comparison with the $0{,}80 \times 10^{-6}$ relative uncertainty of the ASMW value of γ_p'.

A.1.3. *Analysis of Data*

The simple mean and standard deviation of the mean of the ten measurements given in Table A1 are

$$K_J = (483\ 597{,}68 \pm 0{,}23)\ \text{GHz/V} \tag{A4a}$$

$$= K_{J\text{-}72}[1 + (7{,}61 \pm 0{,}47) \times 10^{-6}], \tag{A4b}$$

where for convenience the value of the Josephson constant stated by the CCE in 1972 is denoted by the symbol K_{J-72}; that is, K_{J-72} = 483 594 GHz/V exactly.

The simple mean and its standard deviation have little significance in the present case because of the large differences in precision among the measurements. The more appropriate weighted mean, taking as the weight of each measurement the reciprocal of the square of its assigned one standard deviation uncertainty, $w_i = 1/s_i^2$, yields

$$K_J = (483\ 597{,}907 \pm 0{,}086)\ \text{GHz/V} \tag{A5a}$$

$$= K_{J-72}[1 + (8{,}08 \pm 0{,}18) \times 10^{-6}], \tag{A5b}$$

where the uncertainty has been calculated on the basis of external consistency. That is, the usual standard deviation of the weighted mean calculated on the basis of internal consistency, $s_I = [\sum_{i=1}^{N} w_i]^{-1/2}$, has been multiplied by the scale factor or Birge ratio $R_B = [\chi^2/\nu]^{1/2}$, where χ^2 is the statistic "chi square" and ν is the number of degrees of freedom ($\nu = N - 1 =$ 9 in the present case). The reason is that the data are in disagreement; R_B = 2,55 and χ^2 = 58,7 compared with its expected value of ν = 9. The probability that such a large value of χ^2 has occurred by chance is essentially zero [*i.e.*, $P(58{,}7|9) \simeq 0$].

The problem, of course, is that the University of Zagreb result, item 2, and the ASMW result, item 10, strongly disagree with each other as well as with most of the remaining data. This is readily apparent from an examination of Table A1 and Fig. A1. Indeed, item 2 accounts for 44% and item 10 for 46% of the above value of χ^2, respectively. If these two clearly discrepant items are deleted, one finds for the weighted mean

$$K_J = (483\ 597{,}887 \pm 0{,}035)\ \text{GHz/V} \tag{A6a}$$

$$= K_{J-72}[1 + (8{,}039 \pm 0{,}071) \times 10^{-6}], \qquad (A6b)$$

where the uncertainty is now calculated on the basis of internal consistency (as will be the case for the remainder of this section). The eight values are in excellent agreement; $\chi^2 = 5{,}22$ for $\underline{\nu} = 7$, $R_B = 0{,}86$, and $P(5{,}22|7) \approx 0{,}63$. (We assume as usual that $P > 0{,}05$ indicates an acceptable level of agreement.)

It is clear that the NPL result, item 3, will dominate any weighted mean in which it is included because its assigned uncertainty is significantly smaller than that of any other value. If it is deleted along with the discrepant items 2 and 10, the weighted mean of the remaining seven items is

$$K_J = (483\ 597{,}794 \pm 0{,}092)\ \mathrm{GHz/V} \qquad (A7a)$$

$$= K_{J-72}[1 + (7{,}84 \pm 0{,}19) \times 10^{-6}], \qquad (A7b)$$

with $\chi^2 = 4{,}03$ for $\underline{\nu} = 6$, $R_B = 0{,}82$, and $P(4{,}03|6) = 0{,}67$. Again, these values are in excellent agreement among themselves. Moreover, their weighted mean is consistent with the highly accurate NPL result, item 3. The relative difference is $(0{,}23 \pm 0{,}21) \times 10^{-6}$.

If the next most precise value, the CSIRO/NML result (item 1), is deleted along with the two discrepant items 2 and 10 and the NPL result (item 3), one finds

$$K_J = (483\ 597{,}67 \pm 0{,}13)\ \mathrm{GHz/V} \qquad (A8a)$$

$$= K_{J-72}[1 + (7{,}60 \pm 0{,}27) \times 10^{-6}], \qquad (A8b)$$

with $\chi^2 = 2.38$ for $\underline{\nu} = 5$, $R_B = 0{,}69$, and $P(2{,}38|5) = 0{,}79$. The relative difference between this value and the NPL value is $(0{,}47 \pm 0{,}28) \times 10^{-6}$, which is acceptable agreement.

Because item 10 is discrepant and the two remaining indirect values, items 8 and 9, are of low precision relative to the two most precise direct values, items 1 and 3, little can be learned from a detailed comparison of the means of the direct and indirect values. However, we do note that the relative difference between the weighted mean of the six consistent direct measurements, items 1, 3, 4, 5, 6, and 7, and that of items 8 and 9 is $(0{,}75 \pm 0{,}47) \times 10^{-6}$. The agreement is acceptable.

A.1.4. *Selection of Recommended Value*

It is clear from the above analysis that taking as the recommended value $K_J = 483\ 597{,}9$ GHz/V is highly consistent with any reasonable treatment of the data and the Working Group's adopted guiding principle discussed in the first section of this appendix. The question remains as to the one standard deviation uncertainty to be assigned this value which will also be consistent with the principle.

Considering (i) that the peak-to-peak scatter among the individual measurements upon which the highly accurate NPL result is based is about 0,34 GHz/V, which corresponds to a relative peak-to-peak scatter of $0{,}7 \times 10^{-6}$; (ii) that the difference between Eqs. (A6) and (A8) is $(0{,}21 \pm 0{,}14)$ GHz/V, which corresponds to a relative difference of $(0{,}44 \pm 0{,}28) \times 10^{-6}$; and (iii) the existence of the discrepant items 2 and 10, the Working Group believes that adopting 0,2 GHz/V as the one standard deviation uncertainty, which corresponds to a relative uncertainty of 4×10^{-7}, is consistent with both its guiding principle and the data. Thus the Working Group's recommended value and assigned uncertainty are

$$K_J = 483\ 597{,}9 \text{ GHz/V} \qquad \text{(A9a)}$$

$$\text{Standard deviation: } 0{,}2 \text{ GHz/V} \qquad \text{(A9b)}$$

$$\text{Relative standard deviation: } 4 \times 10^{-7}. \qquad \text{(A9c)}$$

Figure A2 graphically compares this value with the data of Table A1. (The dashed line is the recommended value and the shading delimits its uncertainty.) Equation (A9) is consistent with the 1986 CODATA value $(483\ 597{,}67 \pm 0{,}14)$ GHz/V [A17].

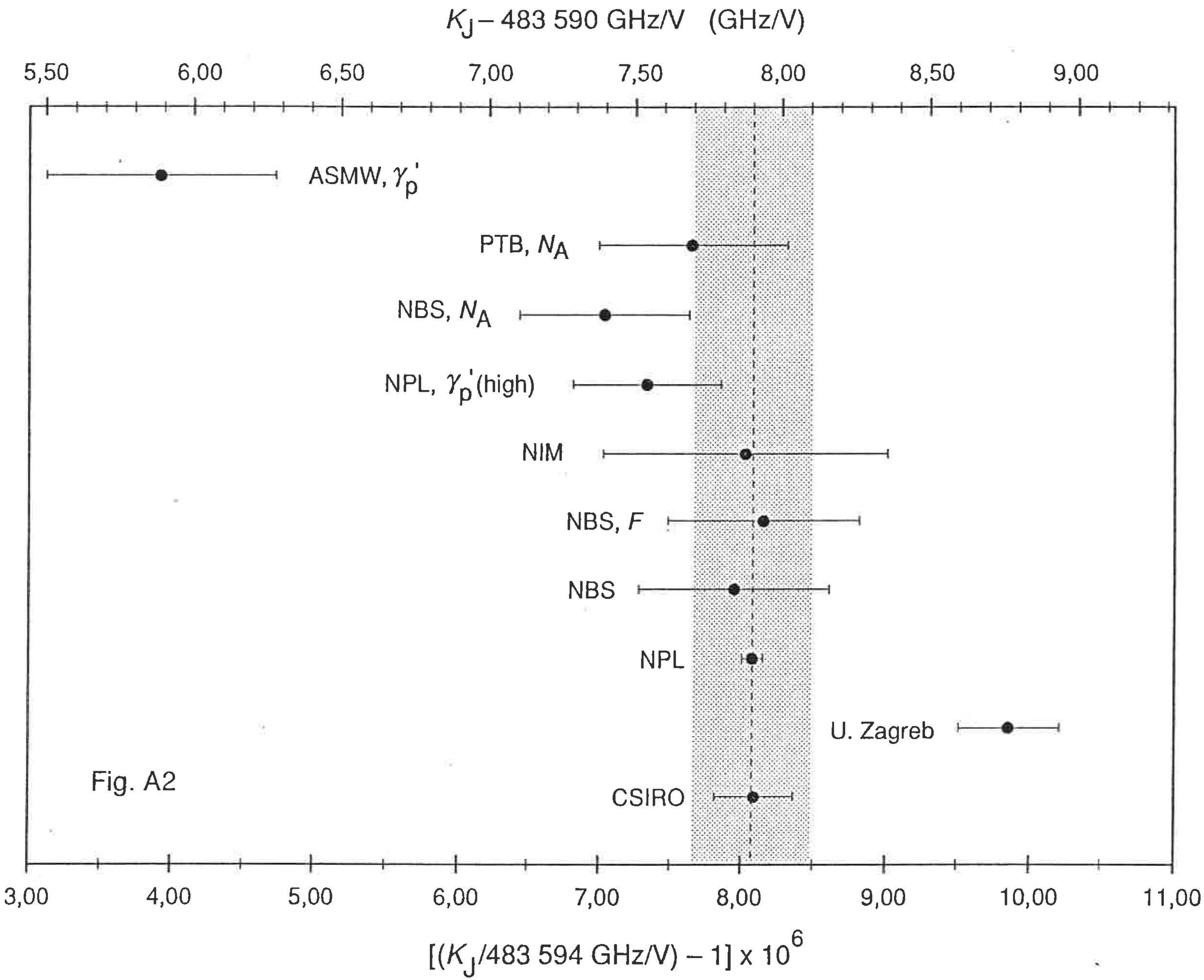

Fig. A2. Comparison of the recommended value of K_J (vertical dashed line) and its standard deviation uncertainty (delimited by the shading) with the values of K_J and their standard deviation uncertainties given in Table A1.

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