

Some measurements of equivalent dead times

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Abstract

It can be shown that a series arrangement of two dead times, as far as the count rates are concerned, may be replaced by a single dead time of generalized type. Experimental measurements of such "equivalent" dead times have been performed for the case where the first element is of the extended and the second of the non-extended type. The results are in good agreement with the values expected by theory.

1. Introduction

Nuclear particles, as a result of the interaction with their environment, can be detected individually and appropriate detectors allow to identify and count them. This type of particle registration is a basic and most important operation in nuclear physics as well as in all related fields, such as nuclear medicine.

Among several factors which are of importance in the nuclear medical procedures used for dynamic studies and diagnosis of tumor lesions, two in particular attract our attention since they are largely responsible for the quality of the images obtained by present-day scanning techniques.

The first factor is the availability of accurate radioactive standards issued by or traceable to the national laboratories. These standards allow the calibration of ionization chamber systems used in performing quantitative assays of the radiopharmaceuticals delivered to patients. These standards also serve to calibrate the detection systems used for in-vivo measurements [1]. The second factor concerns the precision of results of measurements based on the registration of nuclear events. These are inevitably affected by counting losses in the measuring system due to its limited resolution in time [2]. Therefore, both factors ultimately rely on the possibility of performing accurate counting.

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Any detection system, after having accepted an electronic pulse, is unable to handle possible further arrivals immediately afterwards. The minimum time interval necessary to separate individual counts is called the dead time of the electronic circuit. This leads inevitably to counting losses for the measurement, which have no counterpart in the corresponding biological effects. The resulting distortion in the temporal sequence of the measured electronic pulses can be so large that simply neglecting them would lead to errors which may become quite unacceptable for medical applications.

In order to have a clear-cut experimental situation for the counted pulses, one generally introduces artificially a dead time of length τ . This will modify the stream of incoming events in a well-defined way, allowing us to evaluate the original count rate from the measured one.

2. Some general remarks on the effect of dead times

Traditionally, two types of dead times are considered [3]. They differ in the way of responding to pulses which arrive during the dead-time period:

- a) For a non-extended dead time, these pulses are just ignored,
- b) For an extended (also called cumulative or paralyzable) dead time, they extend the dead time by τ , measured from the moment of the last arrival.

Thus, while in a) dead times are only imposed by registered events, they are caused in b) by all detected pulses.

Normally, such a dead time is inserted as early as possible into an electronic measuring chain in order to minimize pile-up effects. Its value is usually chosen to be well beyond the one estimated for the "inherent" dead time of the system. This allows to reduce, and often practically to eliminate, the effect of the latter.

The precise value of the unavoidable inherent dead time, which is mainly due to the electronic chain, is difficult, but not impossible, to measure. The effect of this "first" dead time can be expressed by a transmission factor T_1 which may be used for the determination of the original count rate. How this can actually be done has been described elsewhere [4].

The recent availability of dead times of the so-called generalized type has changed the situation and widened the possibilities. It can be shown that any series arrangement of two dead times may be replaced by a single generalized dead time, as far as the count rates are concerned. When the value of this dead time is chosen equal to that of the second element in the chain, there only remains its parameter τ to be determined. Hence, if a method can be found which permits to predict τ ,

the advantage of the new approach is obvious, as it then allows us to bypass the use of the complicated mathematical expressions which occur in the description of a series arrangement of two dead times. All we then need is the inversion formula applicable for a generalized dead time, and this can be readily programmed on a pocket computer [5, 6].

Whenever we have two dead times in series, four possible combinations can be formed depending on whether they are of the extended "E" or non-extended "N" type. These combinations are "E-E", "N-E", "E-N" and "N-N".

For a given set of parameters (count rate ρ , dead times τ_1 and τ_2 , each of specified type), the corresponding generalized "type parameter" \mathcal{P} can be determined. Let the output count rate be denoted by R for the series arrangement of dead times and by R' for the generalized dead time.

For our measurements we have chosen the arrangement where τ_1 and τ_2 are of type "E" and "N", respectively (Fig. 1). For such an arrangement "E-N" we have [7] the output count rate

$$R = \frac{\rho}{(1-\alpha)x + e^{\alpha x}} \cong \frac{\rho}{1 + x + \frac{1}{2} \alpha^2 x^2 + \dots}, \quad (1)$$

whereas, according to the Takács formula [8], it is given for a generalized dead time by

$$R' = \frac{\rho}{e^{x\tau_2} + 1} \cong \frac{\rho}{1 + x + \frac{1}{2} x^2 + \dots}, \quad (2)$$

where $x = \rho\tau_2$ and $\alpha \cong \tau_1/\tau_2$, with values of α between 0 and 1.

The requirement that R' should be equal to R imposes a condition on \mathcal{P} . If \mathcal{P} can be chosen such that $R' = R$, the dead time is said to be equivalent to the corresponding series arrangement. A comparison of the two series developments given in (1) and (2) shows that if one stops at the terms of order x^2 , the correspondence is $\mathcal{P} = \alpha^2$. However, developments involving higher-order terms of x will in general lead to better approximations of \mathcal{P} . It has been derived in [7] that, up to fourth order in x ,

$$\begin{aligned} \mathcal{P} = & \alpha^2 + \frac{1}{3} \alpha^3(1-\alpha)x + \frac{1}{36} \alpha^4(1-\alpha) (3-5\alpha)x^2 + \frac{1}{540} \alpha^5(1-\alpha) (9-41\alpha + 34\alpha^2)x^3 \\ & + \frac{1}{6480} \alpha^6(1-\alpha) (18 - 174\alpha + 351\alpha^2 - 193\alpha^3)x^4 + \dots \end{aligned} \quad (3)$$

Numerical checks have shown that, for our experimental conditions, this theoretical development is more than adequate.

The aim of our experiment is to check whether the theoretical expressions for \mathcal{V} recently given in [7] can be confirmed by precise measurements. This experimental part takes advantage of some of the advanced techniques presently available in this field.

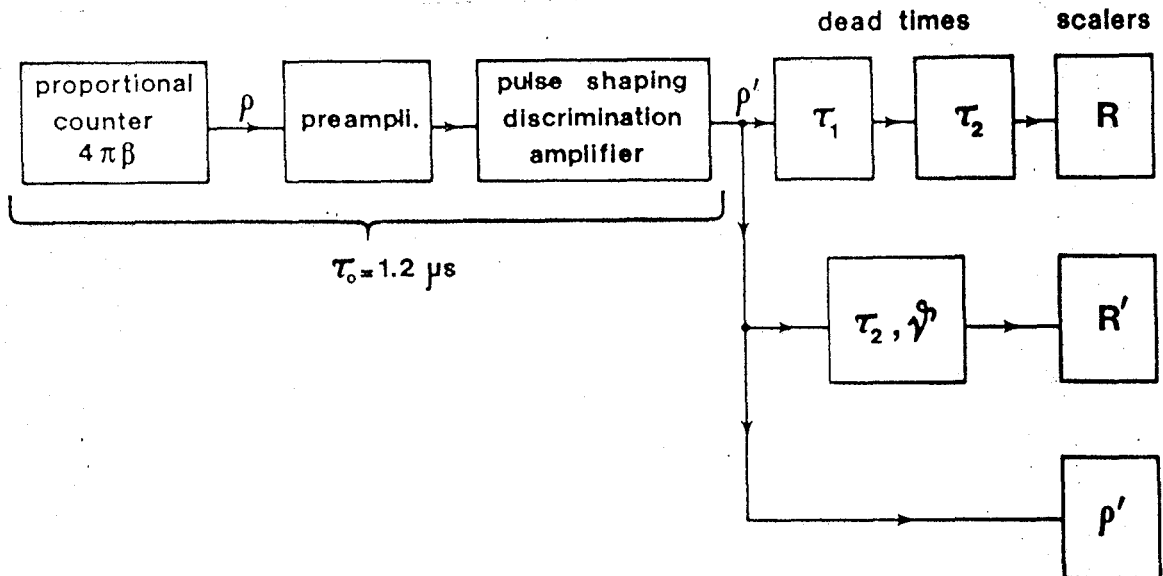


Fig. 1 - Schematic diagram of the measuring electronic chain

3. Experimental procedure

A series of ^{60}Co sources with count rates in the range of about 2 500 to 15 000 s^{-1} , deposited on VYNS films, was specially prepared for this experiment following the routine procedure of source preparation used at BIPM for absolute activity measurements.

For the sake of simplicity we can say that the electronic measuring chain is composed of two parts (Fig. 1): first the $4\pi\beta$ proportional counter and associated electronics (preamplifier, pulse-shape discriminator and amplifier) with an intrinsic dead time τ_0 of about 1.2 μs ; second, it uses, in parallel, both a series arrangement of two dead times (with values τ_1 and τ_2) and a single generalized dead time (of value τ_2 and with parameter \mathcal{V}). In addition, a monitor measures continuously the count rate ρ' which permits the evaluation of the original count rate ρ once the dead time τ_0 is known.

All the measurements are performed simultaneously in order to average out statistical fluctuations of the count rate which are due to the source. The dead times τ_1 and τ_2 as well as the generalized "type parameter" \mathcal{V} are electronically inserted and a description of the system is planned for publication [9]. In the adopted model of a generalized

dead time it is assumed that the type of dead time associated with an incoming pulse is randomly determined, but in such a way that its probability of being extended is given by the numerical value of \mathcal{P} . Thus, the special cases $\mathcal{P} = 0$ and $\mathcal{P} = 1$ correspond to a purely non-extending and extending dead time, respectively.

Apart from these limiting cases all intermediate situations are covered. In the general situation, an extending type is chosen randomly with probability \mathcal{P} , and a non-extending type with probability $1 - \mathcal{P}$. The experimental realization of an electronic circuit capable of producing such a dead time is based on an approximate simulation [10]. An electronic switch which determines whether an extended or non-extended behaviour is applied, is activated by a periodic signal with two stages, the ratio of which is \mathcal{P} to $1 - \mathcal{P}$. A schematic diagram of this periodical signal is given in figure 2; its frequency $\nu = 1/T$ should be well in excess of the experimental count rate under study.

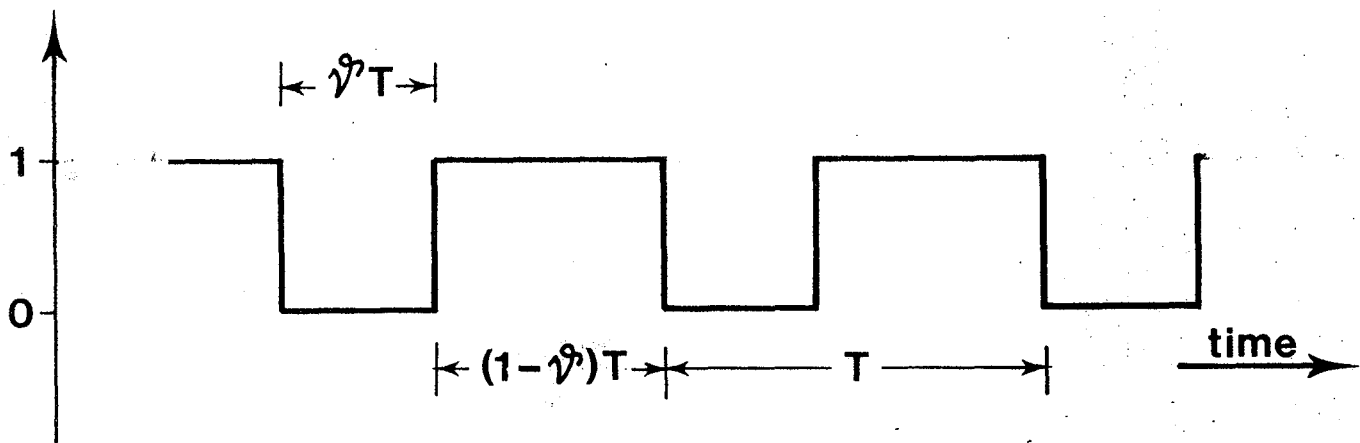


Fig. 2 - Schematic diagram of the periodical signal

An approximate value of \mathcal{P} can then be obtained directly from a look at the periodical signal, for instance by means of an oscilloscope. However, practical measurements have revealed that this simple method of determining \mathcal{P} is not accurate enough. It has therefore been replaced by a more elaborate method which takes advantage of a special sequence of triple pulses produced by a signal generator. This method [9] allows to reduce the determination of \mathcal{P} to a simple counting of pulses and it can be easily shown that this approach permits its measurement within 100 s to about 0.2 %, which is sufficient for our purposes.

In order to assure our confidence in the stability of the whole measuring system, five readings have been taken for each nominal value of \mathcal{P} and a sufficient number of readings for R and R' , providing thereby a reasonably good statistical basis for the evaluation of the results.

As a normal procedure prior to the start of a new series of measurements, a new radioactive source was left for a sufficient time (e.g. about 30 min) in the proportional counter, allowing thereby the measurement conditions to reach a state of equilibrium. The numerical values of τ_1 and τ_2 were measured for every new choice of x by using an automatic measuring device described in [11].

4. Results

The stability of the measuring system is illustrated in figure 3, for a source with a count rate of $14\,800\text{ s}^{-1}$. The variation is smaller than 0.1 %, and thus within the normal statistical fluctuations of the source. The results of two typical situations are shown in figure 4, where the ratio R'/R is plotted against the values of γ . The intersections are obtained graphically; they yield the wanted experimental values of γ and their uncertainties.

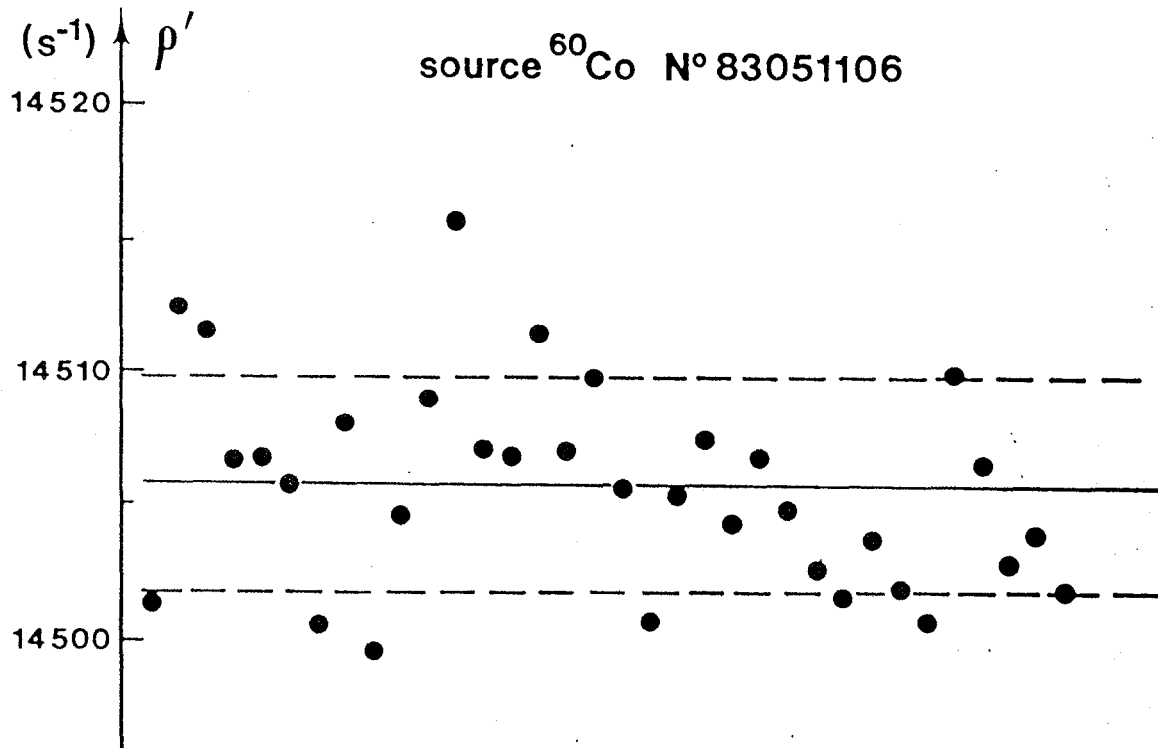


Fig. 3 - Results of a series of measurements for a source with a count rate of about $14\,800\text{ s}^{-1}$. Each experimental point was measured during 1 000 s and individual measurements were separated by an interval of 800 s.

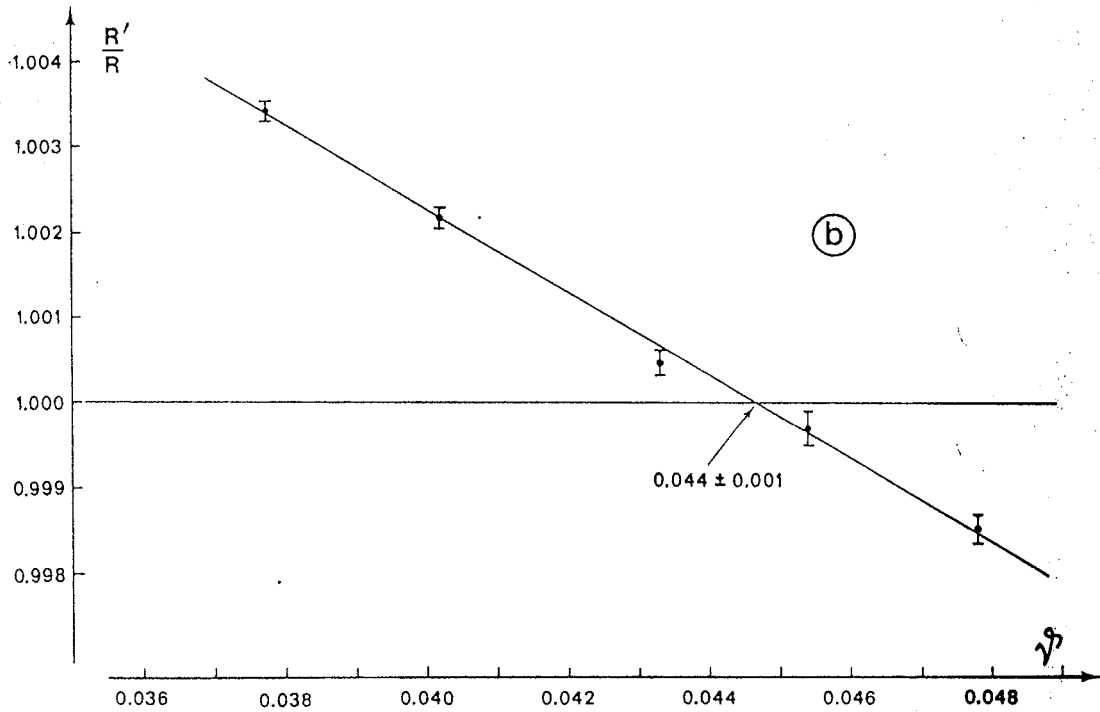
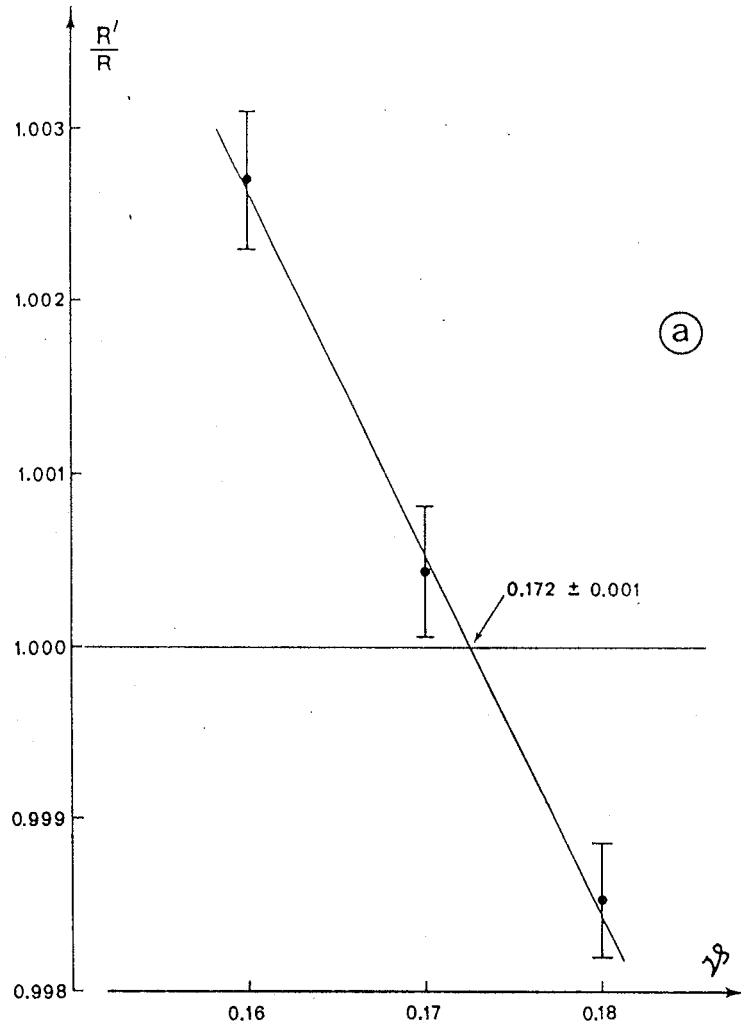


Fig. 4 - Typical plots of the relation between R'/R and γ^{η} . The intersection represents the value of γ^{η} for the relevant two parameters, which are $\alpha = \tau_1/\tau_2$ and $x = \rho\tau_2$.

Table 1 - Experimental (E) and theoretical(T) values of β^7 for an equivalent dead-time arrangement "E-N"

	$\rho = 10\,800\text{ s}^{-1}$ $x = 0.2$	$\rho = 2\,600\text{ s}^{-1}$ $x = 0.2$	$\rho = 5\,400\text{ s}^{-1}$ $x = 0.5$	$\rho = 10\,900\text{ s}^{-1}$ $x = 0.8$	$\rho = 14\,800\text{ s}^{-1}$ $x = 1.5$
$\alpha = 0.2$	E: 0.032 ± 0.002 T: 0.040	E: 0.040 ± 0.001 T: 0.040	E: 0.042 ± 0.001 T: 0.041		E: 0.044 ± 0.001 T: 0.044
$\alpha = 0.3$	E: 0.080 ± 0.002 T: 0.090			E: 0.094 ± 0.002 T: 0.095	
$\alpha = 0.4$	E: 0.158 ± 0.002 T: 0.163	E: 0.160 ± 0.002 T: 0.162	E: 0.165 ± 0.001 T: 0.166	E: 0.172 ± 0.001 T: 0.170	E: 0.183 ± 0.002 T: 0.180
$\alpha = 0.5$	E: 0.242 ± 0.001 T: 0.254			E: 0.264 ± 0.001 T: 0.268	
$\alpha = 0.6$	E: 0.358 ± 0.001 T: 0.364	E: 0.366 ± 0.002 T: 0.366	E: 0.376 ± 0.002 T: 0.375	E: 0.384 ± 0.001 T: 0.383	E: 0.402 ± 0.002 T: 0.401
$\alpha = 0.7$	E: 0.479 ± 0.001 T: 0.499			E: 0.515 ± 0.001 T: 0.517	
$\alpha = 0.8$	E: 0.614 ± 0.003 T: 0.627	E: 0.641 ± 0.004 T: 0.644	E: 0.656 ± 0.002 T: 0.655	E: 0.663 ± 0.002 T: 0.664	E: 0.669 ± 0.001 T: 0.670
$\alpha = 0.9$	E: 0.808 ± 0.002 T: 0.808			E: 0.827 ± 0.001 T: 0.826	

- The values indicated for ρ , α and x are rounded
- For the evaluation of the theoretical values the exact parameters have been used

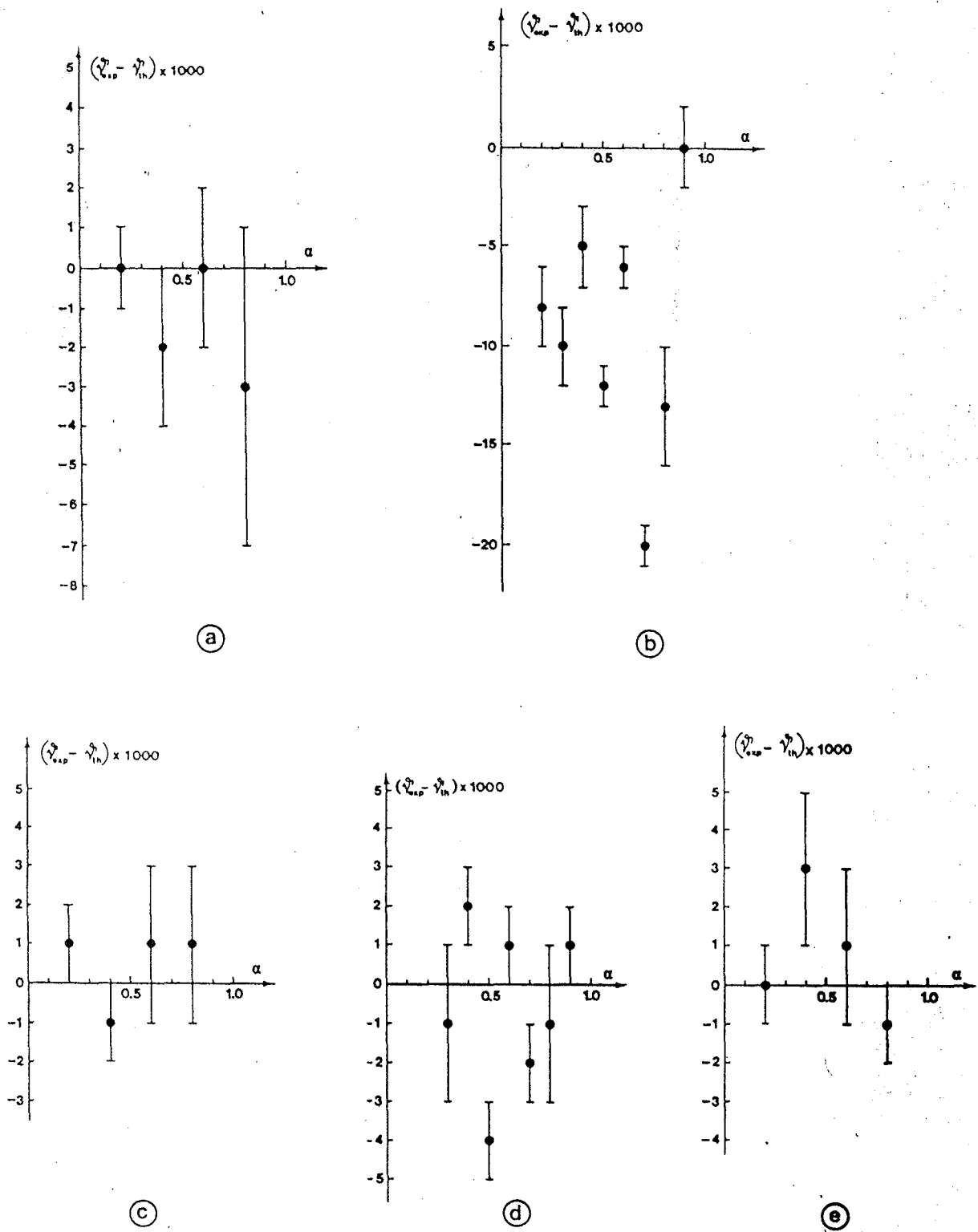


Fig. 5 - Differences between the experimental and theoretical values of γ_p for various values of the relevant two parameters, namely source count rate ρ and the product $x = \rho\tau_2$.

- a) $\rho \approx 2600 \text{ s}^{-1}$, $x = 0.2$ b) $\rho \approx 10800 \text{ s}^{-1}$, $x = 0.2$
 c) $\rho \approx 5400 \text{ s}^{-1}$, $x = 0.5$ d) $\rho \approx 10900 \text{ s}^{-1}$, $x = 0.8$
 e) $\rho \approx 14800 \text{ s}^{-1}$, $x = 1.5$.

The final results for all series of measurements together with the corresponding calculations are summarized in table 1. The agreement between the experimental and theoretical values of γ is satisfactory for all arrangements, except for the case with $\rho \cong 10\ 800\ \text{s}^{-1}$ and $x \cong 0.2$. It is easily seen that for this situation the deviations are always negative and it can be explained by the influence of τ_0 on τ_1 , since the latter is in this case only about two to seven times larger than the first one. Also the discrepancy tends to get smaller for increasing values of α , and becomes negligible for $\alpha = 0.9$.

The measured differences between the experimental and theoretical values of γ are indicated in figure 5 for all situations referred to in table 1. The statistical uncertainties are in all cases only a few parts in a thousand.

The influence of high-order terms on the theoretical values of γ is shown in figure 6, which explains the need for their evaluation and subsequent use. These differences, for $x = 0.8$, may reach values of up to $25 \cdot 10^{-3}$, with the main influence coming from the term proportional to x .

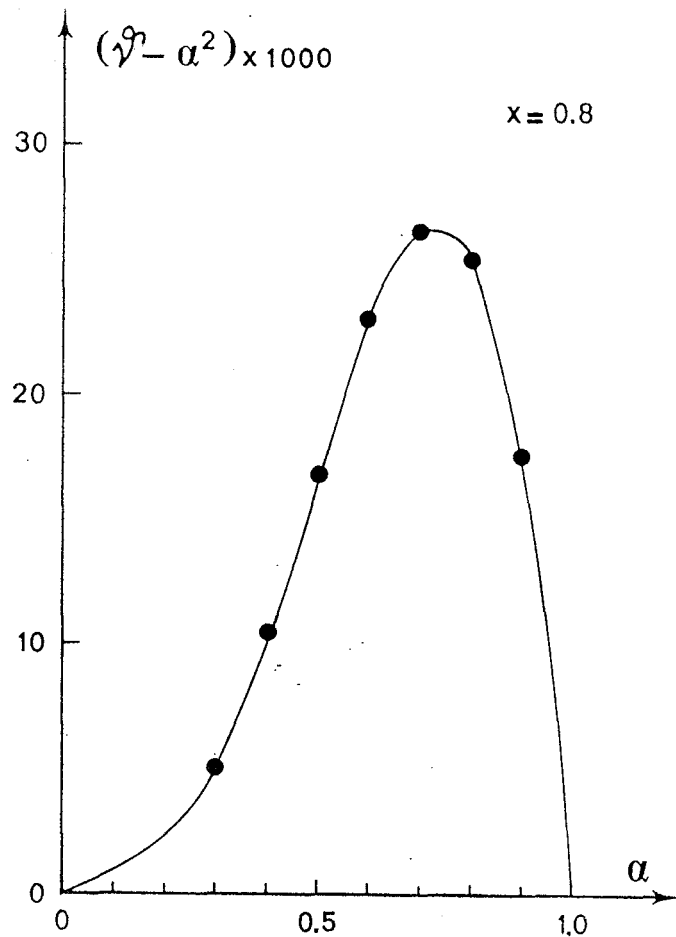


Fig. 6 - Typical example of the effect of higher-order terms in the theoretical values of γ .

5. Conclusions

The experimental results obtained from the arrangement type "E-N" indicate that the approach used for these measurements seems to be reliable and of sufficient accuracy. The good agreement between experimental and theoretical values gives clear support to the theoretical evaluation of the parameter τ , which can thus be reliably predicted, provided that we dispose of good numerical values for the dead times involved. In these evaluations the higher-order terms give non-negligible contributions.

Further measurements should be performed in order to verify if such an agreement is also found for the other dead-time arrangements, in particular the case "N-E" where the theory is already available.

This method, for which the practical feasibility has now been established, is likely to find useful applications in the medical and non-medical fields.

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