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Bilateral Comparison of 10 kΩ Standards (ongoing BIPM key comparison BIPM.EM-K13.b) between the NML (Ireland) and the BIPM, April 2006

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A comparison of the 10 kΩ reference standards of the BIPM and the National Metrology Laboratory (NML, Ireland) was carried out from February to July 2006. Two BIPM 10 kΩ travelling standards, B10k08 and B10k09, were shipped to the NML by air courier. Both before and after the measurements at the NML, the standards were measured at the BIPM. All three measurement periods were of approximately one month duration.

The BIPM measurements were carried out by comparison with a 100 Ω reference resistor whose value is known with respect to the BIPM quantized Hall resistance (QHR) standard. The current in the 10 kΩ resistors during the measurements at the BIPM was 0.1 mA. The NML carried out measurements of the travelling standards against 10 kΩ reference resistors by a substitution method using a current comparator resistance bridge. The NML reference standards are maintained with respect to $R_{\mathrm{K-90}}$ by means of periodic calibrations and comparisons with the BIPM and by extrapolation of the behaviour over time of its reference group. The measuring current used at NML was 0.5 mA.

The reference conditions were a temperature of 23 °C and pressure of 101 325 Pa. Both NML and BIPM measurements were carried out at temperatures close to the reference temperature, to within 0.5 °C. Results of all NML and BIPM measurements were corrected to 23 °C and 101 325 Pa using the known temperature and pressure coefficients of the travelling standards and the recorded values of ambient temperature and pressure.

The reference standards of the two participants are clearly closely correlated, as the NML takes its traceability from the BIPM. The effect of this correlation is reduced by the size of the time lapse since the last comparison of NML's standards with those of the BIPM. This took the form of comparison BIPM.EM-K13.a in April 2004.

Figures 1 and 2 show the measured values obtained (after correction to the reference temperature) for the two standards by the two laboratories. The resistances of the standards are assumed to vary linearly over time, and linear least-squares fits to the BIPM data for each standard are used to provide the comparison reference. The results of the comparison are presented as the differences between the mean values measured at NML and the values calculated for the same date from the fitted BIPM data. The reference date of the comparison is thus the mean date of the NML measurements, 13 April 2006.

Table 1 lists the results for the comparison and the uncertainty contributions as separate Type A and B components for both laboratories. The following elements are listed:

(1) the mean resistance value R_{NML} of each resistor measured by the NML
(2) the Type A uncertainty due to the instability of the resistors and the measuring equipment, computed as the standard uncertainty of the mean value;
(3) the Type B uncertainty component due to the measuring equipment of the NML. This uncertainty is partially correlated between the different travelling standards used for a comparison and the contributions that are completely or partially correlated are indicated by asterisks (*) in Table 3;
(4-6) the corresponding quantities for the BIPM; the Type A uncertainty for the resistance value at the reference date is calculated from the least-squares linear fit;
(7) the difference ($R_{\mathrm{NML}} - R_{\mathrm{BIPM}}$) for each resistor, and (8) the clearly uncorrelated (Type A) part of the uncertainty;
(9) the result of the comparison, which is the mean of the differences of the calibration results for the two standards;
(10) the standard uncertainty on this mean difference (including Type A and Type B components from both laboratories).

Table 2 lists the Type B uncertainties associated with maintenance and measuring equipment at the BIPM and Table 3 lists the corresponding uncertainties for the NML.

No key comparison reference value (KCRV) is needed in this bilateral comparison to calculate the degree of equivalence (DoE) between the NML and the BIPM. The two differences (7 above), which allow for the behaviour of each standard individually, are combined as a simple mean (9) to give the DoE given below. The combined uncertainty of the mean difference (10) provides the uncertainty on this DoE. However, in order to link this result to results from other participants in comparison BIPM.EM-K13.b, the KCRVB is as usual taken to be the BIPM value.

The measured resistances and differences in table 1 are given with their corresponding absolute standard uncertainties (1 σ estimates, in mΩ). The final result of the comparison is presented as the degree of equivalence between the NML and the BIPM for values assigned to 10 kΩ standards, D_{NML}, and its expanded uncertainty, U_{NML} (k=2, 95% confidence level) both expressed in 10^{-8} (i.e. as relative values, where $1 \times 10^{-8} = 0.1$ mΩ on a measurement of 10 kΩ):

$$\boldsymbol{D}_{\mathbf{NML}} = (\boldsymbol{R}_{\mathbf{NML}} - \boldsymbol{R}_{\mathbf{BIPM}}) / \mathbf{10\ k\Omega} = \mathbf{+\,28 \times 10^{-8}}$$

$$\boldsymbol{U}_{\mathbf{NML}} = \mathbf{92 \times 10^{-8}}$$

This is a most satisfactory result. The difference between the values assigned to the travelling standards by the two laboratories is less than the expanded uncertainty associated with the difference. The calibration and measurement capability (CMC) of NML held in the BIPM key comparison database appendix C for measurements of 10 kΩ standards is 15 mΩ (i.e. 150×10^{-8}). This comparison result clearly supports this CMC.

Comments:

More qualitative information can also be gained from the comparison data, particularly by inspection of figures 1 and 2. Multiple standards and the ‘*A-B-A*’ pattern of measurements are used to reduce the effects of instabilities of the standards on the comparison. From figures 1 and 2, it is clear that, at least at the level of interest defined by the NML uncertainties, the behaviour of the standards is not a limiting factor in this comparison. The linear drift rates obtained for the standards during the period of this comparison have also been compared to the historical data maintained at the BIPM for each standard (covering approximately the last 5 years), and a satisfactory agreement was found.

The consistency of any difference observed for the two standards (i.e. the difference between the differences (7)) can be very revealing. In this case, both standards show a difference of the same sign and similar magnitude. The size of the offset is significant compared to the Type A uncertainties, but not compared to the total NML uncertainties. This comparison thus reveals a small discrepancy between the resistance values assigned at 10 kΩ at NML and BIPM, which could be due to a shift in the reference standards since the last calibration or the result of an uncorrected error in the measurement apparatus. The offset is adequately covered by the Type B uncertainties assigned by the NML, and is not significant compared to the laboratory CMC for this measurement.

The measurement current in the travelling standards during this comparison was 0.1 mA in one laboratory and 0.5 mA in the other. The type of travelling standards used have previously been characterised for possible power or voltage coefficients, however, and this difference does not significantly increase the uncertainty of the comparison. (An uncertainty contribution of 1×10^{-8} is included in table 3 to cover any possible effect.)

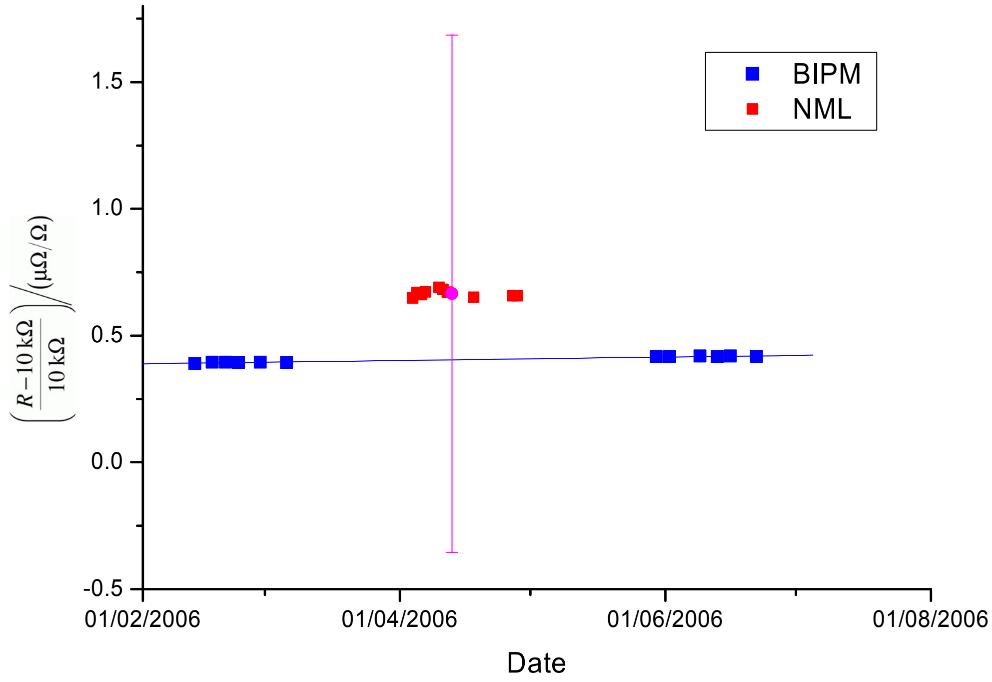

Figure 1: Relative deviation from the nominal 10 kΩ value of the resistance of B10k08 vs time: individual measurements of the BIPM and NML, a linear least-squares fit to the BIPM measurements and the reported mean and expanded uncertainty of the NML measurements.

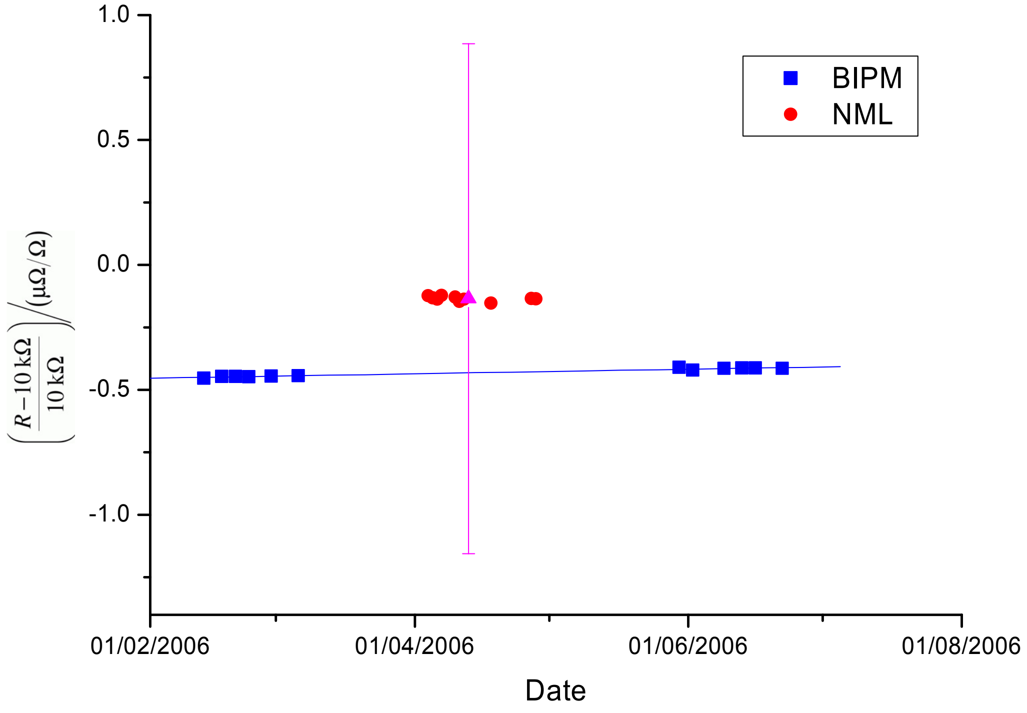

Figure 2: as figure 1, but for resistor serial number B10k09

Table 1. Results of the NML/BIPM bilateral comparison of 10 kΩ standards using two BIPM travelling standards: mean date 13 April 2006. Uncertainties are 1 σ estimates.

		B10k08	**B10k09**
1	**NML** **(R − 10 kΩ)** / mΩ	+6.69	−1.33
2	Type A uncertainty / mΩ	0.10	0.10
3	Type B uncertainty / mΩ	5.1	
4	**BIPM** **(R − 10 kΩ)** / mΩ	+4.05	−4.33
5	Type A uncertainty / mΩ	0.03	0.04
6	Type B uncertainty / mΩ	0.15	
7	**($R_{\mathrm{NML}} - R_{\mathrm{BIPM}}$)** / mΩ	+2.64	+3.00
8	combined Type A uncertainty/ mΩ	0.10	0.11
9	Mean difference: < **$R_{\mathrm{NML}} - R_{\mathrm{BIPM}}$**> / mΩ	**+2.8**	
10	Total uncertainty on mean difference / mΩ	**4.6**	

Table 2. Estimated Type B standard uncertainties, relative to the nominal value, for 10 kΩ calibrations with the BIPM equipment. A relative uncertainty of 1 × 10^{-9} corresponds to 0.01 mΩ.

Realization of $R_{\mathrm{H}}(2)$	2 × 10^{-9}
Ratio of 10 kΩ transfer resistor to $R_{\mathrm{H}}(2)$	6 × 10^{-9}
DC/AC difference (at 1 Hz) of the transfer resistor	2 × 10^{-9}
Comparison of transfer resistor to 10 kΩ reference resistors	5 × 10^{-9}
Imprecision in the values of the reference resistors (including uncertainties in extrapolated resistance values and residual power, temperature and pressure effects)	10 × 10^{-9}
Comparison of the travelling standards to the reference resistors	5 × 10^{-9}
Uncertainty in the temperature correction for the travelling standard	2 × 10^{-9}
Uncertainty in the pressure correction for the travelling standard	4 × 10^{-9}
rss total	**15 × 10^{-9}**

Table 3. Estimated Type B standard uncertainties, relative to the nominal value, for 10 kΩ calibrations with the NML equipment. A relative uncertainty of 1 × 10^{-8} corresponds to 0.1 mΩ. Asterisks (*) indicate components that are either completely correlated or probably significantly correlated when measuring different travelling standards.

Calibrated value and drift correction of the 10 kΩ reference standard*	40 × 10^{-8}
Bridge ratio	30 × 10^{-8}
Uncertainty in the temperature and pressure corrections of the reference standard *	1 × 10^{-8}
Possible change in value of travelling standard between 0.1 mA and 0.5 mA measuring current *	1 × 10^{-8}
Possible effect of leakage resistances	10 × 10^{-8}
rss total	**51 × 10^{-8}**

The following reasoning is applied to the calculation of the uncertainty on the mean difference. For n measurements of a variable x, $\overline{x} = n^{-1}\sum_{i=1}^{n} x_i$ and

$$\mathrm{var}(\overline{x}) = n^{-2}\sum_{i=1}^{n} \mathrm{var}(x_i) = n^{-2}\sum_{i=1}^{n} \sigma_i^2 = n^{-2}\sum_{i=1}^{n} [\sigma_{\mathrm{A}}^2(i) + \sigma_{\mathrm{B}}^2(i)] \ , \qquad (1)$$

where $\sigma_{\mathrm{B}}^2(i)$ is the Type B variance (one that is not evaluated by statistical methods) that is uncorrelated between different x_i and, for all i, is equal to σ_{B}^2. A second Type B variance $\sigma_{\mathrm{B*}}^2(i)$ is the correlated Type B variance equal to $\sigma_{\mathrm{B*}}^2$ for all i. The total variance of the mean difference is the sum of $\mathrm{var}(\overline{x})$ from (1) and $\sigma_{\mathrm{B*}}^2$.