

Bureau International des Poids et Mesures

**Bilateral Comparison of 100 pF Capacitance Standards  
(ongoing BIPM key comparison BIPM.EM-K14.b)  
between the NML, Ireland and the BIPM, January/April 2004**

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## **Introduction**

This bilateral comparison between the NML and the BIPM was carried out in January/April 2004. Two 100 pF travelling standards belonging to the BIPM were used. The measurand was the two terminal-pair capacitance at a frequency of 1000 Hz and for a measuring voltage of 15 V. The BIPM was the pilot laboratory.

## **Travelling standards**

The two BIPM travelling standards are Andeen-Hagerling model AH11A capacitance modules having nominal values of 100 pF (S/N 01312 and S/N 01313) mounted in a frame model AH1100 (S/N 00105). The effect of ambient temperature on the standards mounted in the frame has been tested in the range 20 °C to 25 °C. No changes in the capacitance values greater than 1 part in  $10^8$  have been detected in this range. Both laboratories measured the travelling standards at ambient temperatures between 21.5 °C and 23 °C. In these conditions, the temperature corrections are negligible. The NML and the BIPM used measuring voltages of 15 V and 10 V, respectively. The capacitance change from 10 V to 15V was evaluated at the BIPM and is negligibly small.

## **Measurement principle**

### *NML capacitance standard and measurement method*

The NML standard for capacitance is composed of three commercially produced thermo-regulated fused silica capacitance standards ( 2 x Gen Rad 1408, 1 x AH 11A), each of nominal value 100 pF. The mean capacitance value ascribed to this group standard has been established by periodic calibration of a travelling standard by the National Physical Laboratory (UK). Prior to 1995, a gas filled capacitor (Gen Rad 1404), was used as a travelling standard. Since 1995, a fused silica standard (AH 11A) has been used. The period between calibrations was approximately 2 years. Based on the historic calibration data, the

drift rate of the NML capacitance standards has been established. Frequent internal comparisons between the individual standards are used to detect any anomalous drift behaviour.

The ratios of the BIPM travelling standards to the NML reference standards were measured using a substitution method. The measurement instrument was a high resolution digital capacitance meter (Andeen Hagerling Type 2500A Capacitance Bridge). Two 1 metre long coaxial cables, fixed to a rigid support, were used to connect the standards to the input of the capacitance bridge. Each bridge reading was corrected for the effects of the inter-cable capacitance and the bridge offset. The measured ratios, together with the predicted values of the NML reference standards were subjected to a weighted least-squares analysis to arrive at best estimates of the capacitance of the travelling standards.

#### *BIPM capacitance standard and measurement method*

The BIPM maintains a reference group of four fused silica 10 pF capacitors (one of the NBS type and three of the GR 1408-A type). Since 1999, the mean value of the group has been measured twice a year using a measurement chain linking the 10 pF capacitances to the recommended value of the von Klitzing constant,  $R_{K-90} = 25\,812.807\ \Omega$ . The chain includes a capacitance bridge with ratio 10/1, a multi-frequency quadrature bridge, an ac-dc coaxial resistor with calculable frequency dependence of resistance, and a quantum Hall device operated at 1 Hz. The drift rate of the mean value of the reference group is about -3.5 parts in  $10^8$  per year.

The BIPM travelling standards were measured in terms of the BIPM 10 pF reference group using a coaxial bridge for two terminal-pairs capacitances with calibrated 10/1 ratio.

### **Results**

Tables 1 and 2 give the values measured at 1000 Hz by the two laboratories for the two travelling 100 pF capacitors. All values are deviations from the nominal 100 pF capacitance value, expressed in parts in  $10^6$ . The results are presented in graphical form in Figures 1 and 2. The last line of each table gives the NML and BIPM evaluations of the capacitance values at the reference date of the measurements, the mean date of the NML measurements (03/03/2004). The NML evaluation is the simple mean of the NML measurements, and the BIPM evaluation is obtained by applying linear least-squares fits to the BIPM data. Also given in the last line is the value of  $D_{\text{NML-BIPM}}$ , the difference between the NML and BIPM measurement results at the reference date, and an estimate of its standard uncertainty,  $u_D$  (in this report, all uncertainties are one standard-deviation estimates).

The standard uncertainty,  $u_D$  is equal to  $0.56 \times 10^{-6}$  and is the rss sum of the uncertainty of the NML measurements ( $0.56 \times 10^{-6}$ , see Annex 1) and of that of the BIPM measurements ( $0.04 \times 10^{-6}$ , see Annex 1). Taking into account the good agreement between the results obtained with the two travelling standards (9 parts in  $10^9$ ), the transfer uncertainty is considered to be negligible.

The final comparison result is obtained by taking the mean of the two results and is:

$$D_{\text{NML-BIPM}} = -0.261 \times 10^{-6} \text{ with a combined standard uncertainty of } 0.56 \times 10^{-6}.$$

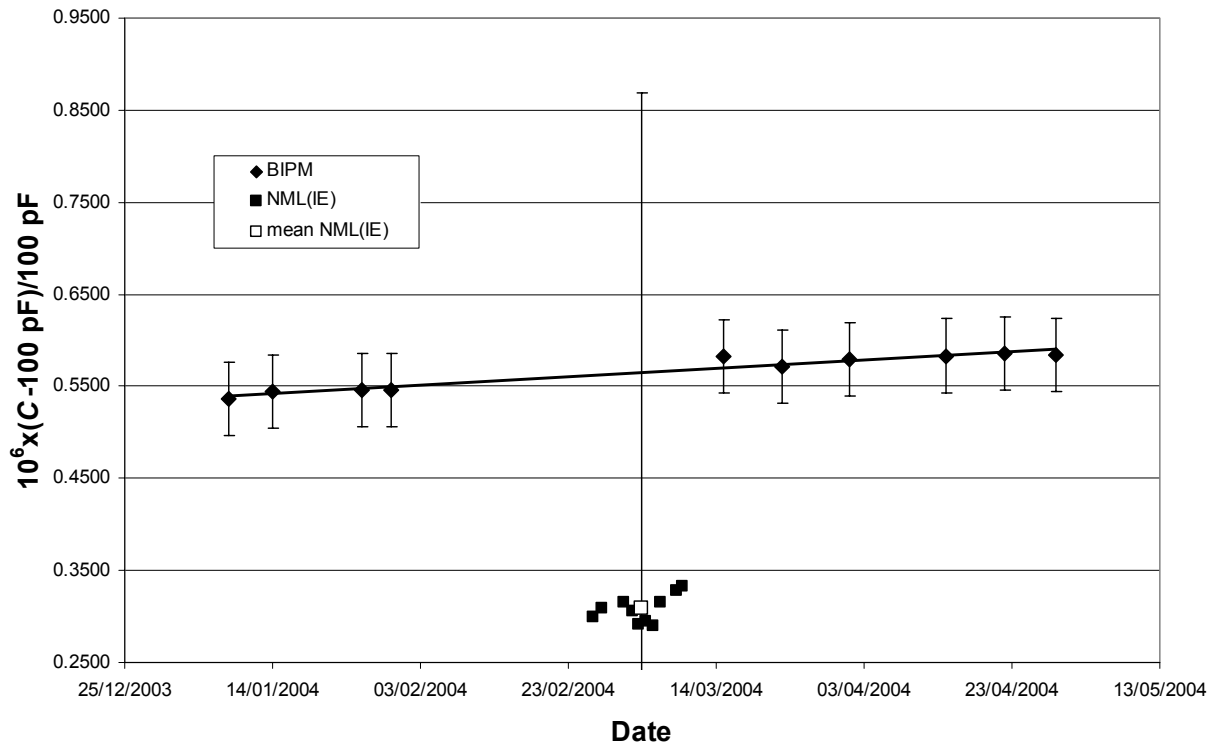
**Table 1: Relative value of capacitance  $C$  for 100 pF capacitance S/N 01312 at 1000 Hz.**

Date	$10^6 \cdot (C - 100 \text{ pF}) / 100 \text{ pF}$ measured at the BIPM	$10^6 \cdot (C - 100 \text{ pF}) / 100 \text{ pF}$ measured at the NML		
08/01/2004	0.5360			
14/01/2004	0.5446			
26/01/2004	0.5457			
30/01/2004	0.5464			
26/02/2004		0.2988		
27/02/2004		0.3094		
01/03/2004		0.3158		
02/03/2004		0.3053		
03/03/2004		0.2909		
04/03/2004		0.2946		
05/03/2004		0.2899		
06/03/2004		0.3150		
08/03/2004		0.3280		
09/03/2004		0.3331		
15/03/2004	0.5825			
23/03/2004	0.5717			
01/04/2004	0.5791			
14/04/2004	0.5831			
22/04/2004	0.5852			
29/04/2004	0.5842		$D_{\text{NML-BIPM}}/10^{-6}$	$u_D/10^{-6}$
<b>03/03/2004</b>	<b>0.5648</b>	<b>0.3081</b>	<b>-0.2567</b>	<b>0.56</b>

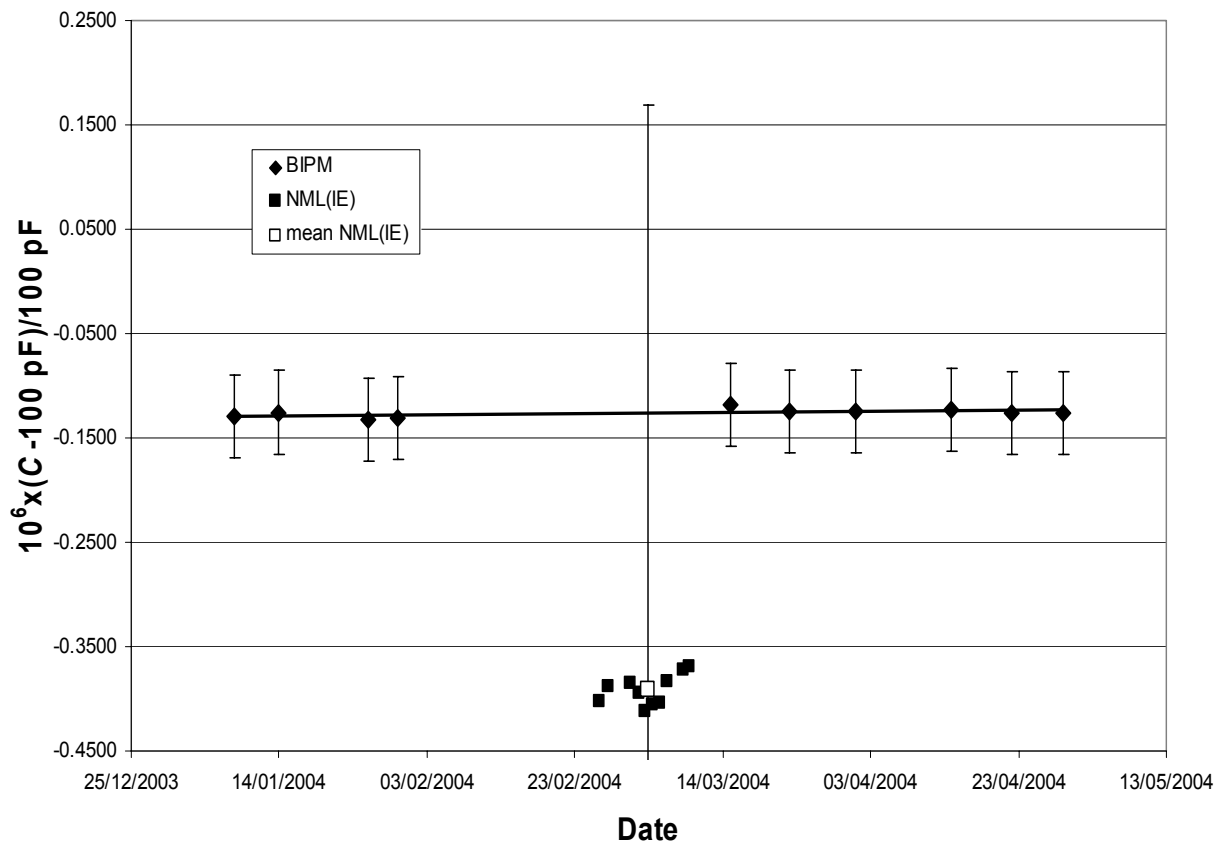
**Table 2: Relative value of capacitance  $C$  for 100 pF capacitance S/N 01313 at 1000 Hz.**

Date	$10^6 \cdot (C - 100 \text{ pF}) / 100 \text{ pF}$ measured at the BIPM	$10^6 \cdot (C - 100 \text{ pF}) / 100 \text{ pF}$ measured at the NML		
08/01/2004	-0.1293			
14/01/2004	-0.1255			
26/01/2004	-0.1324			
30/01/2004	-0.1310			
26/02/2004		-0.4027		
27/02/2004		-0.3881		
01/03/2004		-0.3842		
02/03/2004		-0.3947		
03/03/2004		-0.4124		
04/03/2004		-0.4054		
05/03/2004		-0.4038		
06/03/2004		-0.3830		
08/03/2004		-0.3720		
09/03/2004		-0.3686		
15/03/2004	-0.1183			
23/03/2004	-0.1251			
01/04/2004	-0.1243			
14/04/2004	-0.1227			
22/04/2004	-0.1260			
29/04/2004	-0.1259		$D_{\text{NML-BIPM}}/10^{-6}$	$u_D/10^{-6}$
<b>03/03/2004</b>	<b>-0.1262</b>	<b>-0.3915</b>	<b>-0.2653</b>	<b>0.56</b>

**Figure 1: 100 pF SN 01312 at 1000 Hz**  
**values and standard uncertainties of the measurements at each laboratory**



**Figure 2: 100 pF SN 01313 at 1000 Hz**  
**values and standard uncertainties of the measurements at each laboratory**



## Annex 1. Uncertainty budgets

### NML uncertainty budget:

For convenience, the experimental data were analysed using matrix techniques. For the uncertainty analysis, the variance-covariance matrices of the input vectors were constructed and propagated through the measurement model to produce the variance-covariance matrix of the output vector. The variance and covariance of the measured values of the BIPM travelling standard were read from this output variance-covariance matrix.

The following table gives the uncertainty components used to make up the variance-covariance matrices of the input vectors:

Input Quantity	Uncertainty Contribution expressed as a standard uncertainty
Weighted Mean Value of NML Reference Group Capacitance Standard (May 2002)	45 aF
Drift Correction to NML Standard	20 aF
Temperature Correction to NML Standard	10 aF
Measured Ratio of BIPM Standard to NML Standard	30 aF

The resultant standard uncertainty associated with a single reported value of a BIPM 100 pF travelling standard is 56 aF. Taking into account the relatively low dispersion of the NML measurements, the standard uncertainty associated with the mean of the NML measurement is not significantly different from 56 aF, or 0.56 parts in  $10^6$ .

The correlation coefficient of the measured values of the two travelling standards was 0.9.

BIPM uncertainty budget:

The relative standard uncertainties ( $1\sigma$ ) in the measurement at 1000 Hz of the 100 pF capacitances with respect to the quantized Hall resistance measured at 1 Hz are, in parts in  $10^9$  :

Source of uncertainty	Relative standard uncertainty in parts in $10^9$
<b>Comparison with 10 pF BIPM group:</b>	
Repeatability	5
Extrapolation of the mean value of the group	5
Change of 10 pF reference from 1541 Hz to 1000 Hz	15
Bridge imperfection including divider ratio	10
	<b>(19)</b>
<b>10 pF/2000 pF link :</b>	
Repeatability	2
Voltage divider calibration	6
Voltage injection	5
Imperfect current equalizer	4
	<b>(9)</b>
<b>Quad bridge (2000 pF to 51.6 k<math>\Omega</math>)</b>	
Repeatability	5
Effect of harmonics	5
Two-terminal pair definition of capacitors	10
Imperfect current equalizers	5
	<b>(13)</b>
<b>Measurement of 51.6 k<math>\Omega</math>/ <math>R_K</math> ratio at 1 Hz</b>	
Repeatability	10
$R_K/100 \Omega$ ratio measurement	5
400 $\Omega$ / 100 $\Omega$ Hamon divider	5
51.6 k $\Omega$ / 400 $\Omega$ ratio measurement	5
	<b>(13)</b>
<b>1 Hz-1541 Hz difference for 51.6 k<math>\Omega</math> resistances</b>	
Reference coaxial 1290.6 $\Omega$ resistance	2
Link coaxial resistance to intermediate 12.6 k $\Omega$ resistance	20
Link 12.6 k $\Omega$ to 51.6 k $\Omega$	20
	<b>(28)</b>
<b>Combined standard uncertainty</b>	<b>40</b>