

**Bilateral Comparison of 1.018 V and 10 V Standards  
between the CMI, Czech Republic and the BIPM,  
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by A. Sebelá\*\*, D. Reymann\* and T. J. Witt\*

\*Bureau International des Poids et Mesures, F-92312 Sèvres Cedex

\*\* Czech Metrological Institute, Brno, Czech Republic

A comparison of the 1.018 V and 10 V voltage reference standards of the BIPM and the Czech Metrological Institute, Brno, Czech Republic, was carried out in April 2000. Two BIPM 732B Zener diode-based travelling standards, BIPM4 and BIPM5, were transported by automobile. The CMI measurements were carried out by comparison with the mean of the CMI voltage standard, linked to the BIPM Josephson standard via previous annual calibrations of a reference Zener. The BIPM measurements of the travelling standards were carried out by direct comparison to the Josephson effect standard or (1) at 10 V by dividing the 10 V outputs to 1.018 V by means of a resistive divider and comparing the divided voltages to the electromotive force of a standard cell, and (2) at 1.018 V by direct comparison to the electromotive force of a standard cell using a potentiometer. Results of all measurements were corrected for the dependence of the output voltage on ambient temperature and pressure.

Figures 1 and 2 show the measured values obtained for the two standards by the two laboratories. The BIPM values and uncertainties, and those of the CMI are calculated for the reference date from linear least-squares fits.

Table 1 lists the results of the 1.018 V comparison and the component uncertainty contributions for the comparison CMI/BIPM. Experience has shown that flicker or  $1/f$  noise dominates the stability characteristics of Zener-diode standards and it is not appropriate to use the standard deviation of the mean to characterize the dispersion of measured values. For the present standards, the relative value of the flicker floor voltage is about 1 part in  $10^8$ .

Table 2 lists the same information for the 10 V comparison.

In estimating the uncertainty we have calculated the *a priori* uncertainty based on all known sources except that associated with the stability of the standards when transported. We compare this with the *a posteriori* uncertainty estimated by the standard deviation of the mean of the results from the two travelling standards. With only two travelling standards, the uncertainty of the standard deviation of the mean is comparable to the value of the standard

deviation of the mean itself. If the *a posteriori* uncertainty is significantly different from the *a priori* uncertainty, we assume that a standard has changed in an unusual way and we use the larger of these two estimates in calculating the final uncertainty.

In tables 1 and 2, the following elements are listed:

- (1) the predicted value  $U_{\text{CMI}}$  of each Zener, computed using a linear least squares fit to all of the data from the CMI and referenced to the mean date of the CMI's measurements;
  - (2) the type-A uncertainty due to the instability of the Zener, computed as the standard uncertainty of the predicted value from the linear drift model, or an estimate of the  $1/f$  noise voltage level;
  - (3) the uncertainty component due the measuring equipment of the CMI. This uncertainty is completely correlated between the different Zeners used for a comparison;
  - (4-6) the corresponding quantities for the BIPM;
  - (7) the uncertainty due to the combined effects of the uncertainties of the pressure and temperature coefficients and to the difference of the mean pressures and temperatures in the participating laboratories; although the same equipment is used to measure the coefficients for all Zeners, the uncertainty is dominated by the type-A uncertainty of each Zener, so that the final uncertainty can be considered as uncorrelated among the different Zeners used in a comparison;
  - (8) the difference ( $U_{\text{CMI}} - U_{\text{BIPM}}$ ) for each Zener, and (9) the uncorrelated part of the uncertainty;
  - (10) the result of the comparison which is the mean of the differences of the calibration results for the two standards;  
the uncertainty of the transfer, estimated by two methods:
  - (11) the *a-priori* uncertainty, which is the standard deviation of the mean value of the results, from the different Zeners, counting only the uncorrelated uncertainties of the individual results;
  - (12) the *a-posteriori* uncertainty, which is the standard deviation of the mean of the different results;
  - (13) the correlated part of the uncertainty;
- and
- (14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty and of the larger of (11) and (12).

Table 3 summarises the uncertainties due to the BIPM measuring equipment.

Table 4 lists the uncertainties of maintenance and measuring equipment at the CMI.

The final results of the comparison are presented as the difference between the value assigned to a 1.018 V and a 10 V standard by each laboratory. The difference between the value assigned to a 1.018 V standard by the CMI, at the CMI,  $U_{\text{CMI}}$ , and that assigned by the BIPM, at the BIPM,  $U_{\text{BIPM}}$ , for the reference date is

$$U_{\text{CMI}} - U_{\text{BIPM}} = 0.10 \mu\text{V}; u_c = 0.78 \mu\text{V} \text{ on } 2000/04/20,$$

and the difference between the value assigned to a 10 V standard by the CMI, at the CMI,  $U_{\text{CMI}}$ , and that assigned by the BIPM, at the BIPM,  $U_{\text{BIPM}}$ , for the reference date is

$$U_{\text{CMI}} - U_{\text{BIPM}} = 0.77 \mu\text{V}; u_c = 0.73 \mu\text{V} \text{ on } 2000/04/20,$$

where  $u_c$  is the combined type-A and type-B standard uncertainty from both laboratories.

This is a satisfactory result. At 1.018 V, the difference between the values assigned to the travelling standards by the two laboratories is well within the standard uncertainty associated with the difference; at 10 V the difference between assigned values is approximately the standard uncertainty associated with the difference.



Table 1. Results of the CMI/BIPM bilateral comparison of 1.018 V standards using two Zener travelling standards: mean date 20 April 2000. Uncertainties are 1- $\sigma$  estimates. The uncorrelated uncertainty is  $w=[r^2+t^2+v^2]^{1/2}$ , the expected transfer uncertainty is  $x=[w_7^2+w_8^2]^{1/2}/2$  and the correlated uncertainty is  $y=[s^2+u^2]^{1/2}$ .

		BIPM4	BIPM5	
1	CMI $(U_Z - 1.018V)/\mu V$	136.875	138.309	r s
2	type-A uncertainty/ $\mu V$	0.010	0.011	
3	equipment uncertainty/ $\mu V$	0.78		
4	BIPM $(U_Z - 1.018V)/\mu V$	136.738	138.246	t u
5	type-A uncertainty/ $\mu V$	0.010	0.010	
6	equipment uncertainty/ $\mu V$	0.010		
7	pressure and temperature corrections uncertainty/ $\mu V$	0.012	0.024	v
8	$(U_{Z\_CMI} - U_{Z\_BIPM})/\mu V$	0.136	0.063	w
9	uncorrelated uncertainty/ $\mu V$	0.019	0.028	
10	$\langle U_{CMI} - U_{BIPM} \rangle / \mu V$	0.100		
11	expected transfer uncertainty/ $\mu V$	0.026		x
12	$s_M$ of difference for 2 Zeners/ $\mu V$	0.010		
13	correlated uncertainty/ $\mu V$	0.78		y
14	comparison total uncertainty/ $\mu V$	0.78		

Table 2. Results of the CMI/BIPM bilateral comparison of 10 V standards using two Zener travelling standards: mean date 20 April 2000. Uncertainties are 1- $\sigma$  estimates. The uncorrelated uncertainty is  $w=[r^2+t^2+v^2]^{1/2}$ , the expected transfer uncertainty is  $x=[w_7^2+w_8^2]^{1/2}/2$  and the correlated uncertainty is  $y=[s^2+u^2]^{1/2}$ .

		BIPM4	BIPM5	
1	CMI $(U_Z - 10V)/\mu V$	-35.62	-21.14	
2	type-A uncertainty/ $\mu V$	0.10	0.10	r
3	equipment uncertainty/ $\mu V$	0.72		s
4	BIPM $(U_Z - 10V)/\mu V$	-36.52	-21.78	
5	type-A uncertainty/ $\mu V$	0.10	0.10	t
6	equipment uncertainty/ $\mu V$	0.10		u
7	pressure and temperature corrections uncertainty/ $\mu V$	0.16	0.18	v
8	$(U_{Z\_CMI} - U_{Z\_BIPM})/\mu V$	0.90	0.63	
9	uncorrelated uncertainty/ $\mu V$	0.22	0.23	w
10	$\langle U_{CMI} - U_{BIPM} \rangle / \mu V$	0.77		
11	expected transfer uncertainty/ $\mu V$	0.09		x
12	$s_M$ of difference for 2 Zeners/ $\mu V$	0.10		
13	correlated uncertainty/ $\mu V$	0.73		y
14	comparison total uncertainty/ $\mu V$	0.73		

Table 3: estimated standard uncertainties for Zener calibrations with the BIPM equipment.

	1.018 V		10 V	
	Josephson value/nV	Potentiometer value/nV	Josephson value/nV	Comparator value/nV
thermal EMFs	2	2	2	2
detector/EMI	5	5	0.5	10
leakage resistance	5	5	0.3	10
frequency	<0.1		0.3	
reference cell		5		50
potentiometer/comparator		5		85
total	7.4	10	2.1	100

Table 4: estimated standard uncertainties for Zener calibrations with the CMI equipment.

	1.018 V	10 V
	value/ $\mu$ V	value/ $\mu$ V
reference standard	0.7	0.65
thermal EMFs	0.1	0.1
leakage resistance	0.2	0.2
electromagnetic interference	0.15	0.15
detector	0.2	0.15
rss total	0.78	0.72