COMPARISON OF QUANTUM HALL EFFECT RESISTANCE STANDARDS OF THE NPL AND THE BIPM

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Abstract. An on-site comparison of the quantum Hall effect (QHE) resistance standards of the National Physical Laboratory (NPL) and of the Bureau International des Poids et Mesures (BIPM) was made in December 1997. Measurements of a 100 Ω standard in terms of the recommended value of the von Klitzing constant R_{K-90} agreed to 1 parts in 10^{10} with a relative combined standard uncertainty $u_c = 39 \times 10^{-10}$ when the NPL used an indirect method of measurement. Measurements of 10 000 Ω/100 Ω and 100 Ω/1 Ω ratios agreed to 33 parts in 10^{10} with $u_c = 32 \times 10^{-10}$ and 28 parts in 10^{10} with $u_c = 48 \times 10^{-10}$ respectively.

1. Introduction

The comparison reported here is part of a BIPM programme to verify the international coherence of primary resistance standards by comparing QHE standards of the national laboratories with that of the BIPM. The procedure used for the present comparison is the same as that used previously for this programme ([1],[2],[3]): the complete BIPM transportable QHE standard was taken to the NPL and, from 8 to 12 December 1997, measurements of a 100 Ω resistance standard in terms of the recommended value of R_{K-90} and of 10 000 Ω /100 Ω and 100 Ω/1 Ω ratios were carried out with the QHE standards of the two laboratories. The BIPM measurements were made at 1 Hz and those of the NPL with dc. The 1 Hz-dc differences of the three resistance- ratios were determined at the BIPM before the comparison. For this purpose, the three ratios were measured with the ac bridge at 1 Hz and with the BIPM cryogenic current comparator (CCC) bridge [4] operated with dc. The measured differences were applied as corrections to the BIPM ac measurements carried out at the NPL before comparing them with the corresponding NPL dc measurements, a procedure which has the effect of using the ac bridge as a transfer instrument referenced to the BIPM's CCC.

2. Equipment

2.1 QHE samples

For this comparison the BIPM used two GaAs based heterostructures fabricated by the Laboratoires d'Électronique Philips (LEP, Limeil-Brévannes) [5] and diced from an unprotected wafer (reference 900514). Samples from this wafer have mobilities of about 30 T^{-1} and carrier concentrations of about 5.1×10^{15} m^{-2}. The samples were operated on the i=2 plateau at a temperature of 1.3 K, with a current of 40 μA, and with a magnetic flux density of about 10.5 T. The residual values of the longitudinal resistivity did not exceed 50 μΩ.

The NPL used a GaAs device kindly donated by PTB [6]. The mobility was of order 54 T^{-1} and the carrier concentration was about 4.6×10^{15} m^{-2}. For the indirect measurements (see **3.** below) the sample was operated on the i=2 plateau with a magnetic flux density B=9.5 T and at a temperature of 0.45 K. For the direct measurements it was operated on the i=4 or i=2 plateau. Generally the current was 50 µA except for the direct measurement of 100 Ω when it was 25 µA for the i= 2 plateau. Residual values of the longitudinal resistivity did not exceed 10 µΩ.

2.2 Measurement systems

The NPL measurement system is based on two cryogenic current comparator (CCC) bridges operated with dc. The system used for quantized Hall resistance (QHR) measurements, CCC2, [7] relates $R_{K\text{-}90}$ for i=2 and/or 4 to two 100 Ω resistors which are connected in series and located in a thermo-regulated enclosure. The same bridge can be configured as a 10:1 ratio bridge and was used for the measurement of the 10 kΩ /100 Ω ratio in two stages using an intermediate 1 kΩ resistor.

The NPL normally uses a second CCC bridge, CCC1, [8] for the calibration of customers' artifacts. For the present comparison it was used with 2:1 or 1:1 ratios to measure a 100 Ω buffer resistor in terms of the two 100 Ω resistors connected in series and to relate the 100 Ω buffer to the BIPM 100 Ω standard. This bridge was also used with a 10 :1 ratio to measure the 100 Ω/1 Ω ratio using an intermediate 10 Ω resistor.

The uncertainty budgets corresponding to the NPL measurements carried out with these two CCC bridges are given in Annex 1.

The BIPM transportable measurement system includes a complete QHE resistance standard based on an ac-bridge operating at 1 Hz [9] as well as three conventional standard resistors of 100 Ω, 10 000 Ω and 1 Ω. The uncertainty budget for resistance-ratio measurements carried out with the BIPM ac-bridge, including the uncertainty on the corrections for the 1 Hz-dc differences, can be found in Table 2 of reference [3].

3. Comparison results

3.1 Measurements of the 100 Ω resistance standard in terms of $R_{K\text{-}90}$

3.1.1 Results based on NPL indirect measurements and on BIPM direct measurements of the 100 Ω standard

On 9 December the NPL carried out three measurements of the resistance R of the 100 Ω standard using their normal procedure. These measurements are referred to as indirect because in this procedure the 100 Ω standard is compared, via the NPL 100 Ω buffer resistor, to the NPL QHE 100 Ω secondary standard using CCC1. This secondary standard is itself measured in terms of $R_{K\text{-}90}$ using the NPL QHE system and CCC2. This technique is representative of the NPL's measurement chain from $R_{K\text{-}90}$ to customers' 100 Ω standard resistors. On the same day the BIPM carried out two measurements of R directly in terms of the quantized Hall resistance $R_H(2)$ as realized by the two BIPM samples. No significant difference was noted for results from the two samples. The relative difference between values R_{NPL} and R_{BIPM} attributed to R by the two laboratories was found to be $(R_{NPL} - R_{BIPM}) / R = -58 \times 10^{-10}$. Later the NPL found that a small additional correction should be applied to account for the resistance in the leads when a calibration shunt is connected to one of its bridges. This correction brings the NPL results into closer agreement with those of the BIPM. Figure 1 shows the three corrected NPL results and the two BIPM results. The final comparison result, obtained by calculating the difference between the mean of the three corrected NPL results and that of the two BIPM results, is :

$(R_{NPL} - R_{BIPM}) / R = 1 \times 10^{-10}$ with $u_c = 39 \times 10^{-10}$ (9/12/1997)

The combined standard uncertainty u_c is the square root of the sum of the squares of the NPL (31×10^{-10}) and BIPM (15×10^{-10}) type B standard uncertainties, of a standard uncertainty of 5×10^{-10} due to residual power and temperature effects in the 100 Ω standard, and of the type A standard uncertainty of the measurements (17×10^{-10}).

3.1.2 Results based on NPL and BIPM direct measurements of the 100 Ω standard

On 10 December the NPL and the BIPM measured the resistance R directly in terms of R_{K-90} . The relative difference $(R_{NPL} - R_{BIPM}) / R$ between the mean of NPL and BIPM direct measurements was found to be about -1×10^{-8}. However the NPL measurements showed an unusually high dispersion and were not representative of the normal behaviour of the NPL QHR and CCC2 system. NPL measurements in the weeks following the comparison did not show this behaviour and it was not possible to trace its origin.

3.2 Measurements of the 10 000 Ω/100 Ω ratio

The BIPM measured the ratio K of the 10 000 Ω resistance to the 100 Ω resistance in a single step using its 100/1 ratio resistance bridge. The NPL measured this ratio in two steps using a 10/1 ratio on bridge CCC2 and an intermediate resistor of 1000 Ω. For both laboratories the measuring current in the 10 000 Ω standard was 50 μA to within a few percent. The comparison result is:

$(K_{NPL} - K_{BIPM}) / K = 33 \times 10^{-10}$ with $u_c = 32 \times 10^{-10}$ (9/12/1997)

The combined standard uncertainty u_c is the square root of the sum of the squares of the NPL (24×10^{-10}) and BIPM (15×10^{-10}) type B standard uncertainties, of a standard uncertainty of 5×10^{-10} due to residual power and temperature effects in the resistances, and of the type A standard uncertainty of the measurements (15×10^{-10}).

3.3 Measurements of the 100 Ω/1 Ω ratio

The BIPM measured the ratio K' of the 100 Ω resistance to the 1 Ω resistance in a single step using its 100/1 ratio resistance bridge. The NPL measured this ratio in two steps using a 10/1 ratio on bridge CCC1 and an intermediate NPL resistor of 10 Ω. For both laboratories the measuring current in the 1 Ω standard was 50 mA to within a few percent. A number of difficulties occurred during the measurements. An unintentional leakage resistance was discovered between the BIPM 1 Ω resistor and its housing case. The cause of the leakage resistance was readily located and removed. Also the first series of NPL measurements were carried out with a rather short time delay of about 1s between the reversal of the dc current in the resistors and the start of the sequence of data acquisition. In the presence of a Peltier effect (in the 1 Ω standard and also in the 10 Ω standard) the use of a short delay results in a measured resistance value that differs slightly from the zero frequency or dc value. The dc value is defined as the dc voltage to current ratio once the thermal emf induced by the Peltier effect across the resistor has reached a stable value. This may take several seconds with the resistors used during the comparison. Subsequent NPL measurements were taken with an increased delay (about 15 s), similar to that used at the BIPM for dc measurements. Unfortunately, due to lack of time, only 10 Ω/1 Ω ratio-measurements were carried out with the increased delay. The preliminary comparison result, based on NPL 100Ω/10Ω measurements with short delay and on NPL 10 Ω/1 Ω measurements with increased delay was: $(K'_{\mathrm{NPL}} - K'_{\mathrm{BIPM}}) / K' = 125 \times 10^{-10}$. After the comparison the NPL carried out measurements of the ratio between the NPL 10 Ω resistor and a NPL 100 Ω resistor, with short and increased delays. The measured ratio increased by 97 parts in 10^{10} when the delay was increased. As the Peltier effect in the NPL or BIPM 100 Ω resistors is believed to be negligible compared with that in the 10 Ω resistor, a correction of same value was applied to the preliminary result, leading to the final comparison result:

$(K'_{\mathrm{NPL}} - K'_{\mathrm{BIPM}}) / K' = 28 \times 10^{-10}$ with $u_c = 48 \times 10^{-10}$ (12/12/1997)

The combined standard uncertainty u_c is the square root of the sum of the squares of the NPL (36×10^{-10}) and BIPM (20×10^{-10}) type B standard uncertainties, of a standard uncertainty of 10×10^{-10} due to residual power and temperature effects in the resistances, and of the type A standard uncertainty of the measurements (22×10^{-10}).

4. Conclusion

Despite a number of difficulties this comparison demonstrated good agreement between the NPL and BIPM measurements. Results of BIPM measurements and NPL indirect measurements of the 100 Ω resistance standard in terms of $R_{\mathrm{K-90}}$, as well as BIPM and NPL results of measurements of the 10 000 Ω/ 100 Ω and 100 Ω/1 Ω ratios agree to within a few parts in 10^9, a value consistent with the estimated uncertainties. It was demonstrated that to limit the influence of the Peltier effect on the 100Ω/1 Ω ratio measurement, it is essential that both laboratories use identical and sufficiently long delays after reversing the current in the dc measurements.

References

1. Delahaye F., Witt T. J., Piquemal F., Genevès G., *IEEE Trans. Instrum. Meas.*, 1995, **44**, 258-261.
2. Delahaye F., Witt T. J., Jeckelmann B., Jeanneret B., *Metrologia*, 1995/96, **32**, 385-388.
3. Delahaye F., Witt T. J., Pesel E., Schumacher B., Warnecke P., *Metrologia*, 1997, **34**, 211-214.
4. Delahaye F., Bournaud D., *IEEE Trans. Instrum. Meas.*, 1991, **40**, 237-240.
5. Piquemal F., Genevès G., Delahaye F., André J. P., Patillon J. N., Frijlink P., *IEEE Trans. Instrum. Meas.*, 1993, **42**, 264-268.
6. Pierz K., Schumacher B., preprint *Com. Consult. d'Électricité,* 1997.
7. Hartland A., *Metrologia*, 1992, **29**, 175-190.

8. Williams J.M., Hartland A., *IEEE Trans. Instrum. Meas.*, 1991, **40**, 267-270.
9. Delahaye F., Bournaud D., *IEEE Trans. Instrum. Meas.*, 1993, **42**, 287-291.

Annex 1 : Uncertainty budgets for the NPL measurements.

All uncertainties are standard uncertainties and are expressed in parts in 10^9.

1) Uncertainty budget for the NPL indirect measurement of the 100 Ω standard in terms of the quantized Hall resistance, via a NPL 100 Ω resistor (R100/1) and a buffer resistor (222025) :

	Uncertainty and type
<u>QHR to R100/</u>1	
Standard deviation of the mean (10 results):	1.9 (A)
Temperature stability of R100/1, (0.75×10^{-9}/mK, ± 1 mK rectangular):	0.4 (B)
Calibration using shunt, (1 mΩ on 2 leads, 23×10^{-6} deviation, rectangular):	0.2 (B)
Ratio accuracy of CCC bridge ($\pm3\times10^{-9}$, rectangular):	1.7 (B)
<u>R100/1 to 222025</u>	
Standard deviation of the mean (12 results):	0.2 (A)
Temperature stability of 222025, (0.3×10^{-9}/mK, ± 1 mK rectangular):	0.2 (B)
Calibration using shunt, (1 mΩ on 2 leads, 7×10^{-6} deviation, rectangular):	0.1 (B)
Ratio accuracy of CCC bridge ($\pm3\times10^{-9}$, rectangular):	1.7 (B)
<u>222025 to BIPM100</u>	
Standard deviation of the mean (14 results):	0.9 (A)
Calibration using shunt, (1 mΩ on 2 leads, 35×10^{-6} deviation, rectangular):	0.3 (B)
Ratio accuracy of CCC bridge ($\pm3\times10^{-9}$, rectangular):	1 .7 (B)
Total Type A uncertainty	2.1 ν=12
Total Type B uncertainty	3.1
Total combined uncertainty	3.7

2) Uncertainty budget for the NPL measurements of the 10 000 Ω/100 Ω ratio via an intermediate 1000 Ω resistor :

	Uncertainty and type	
Standard error of the mean (3 results):	1.7	(A)
Temperature stability of 1 kΩ is included in the above.		
Calibration using shunt, (1 mΩ on 2 leads, 20×10^{-6} deviation, rectangular):	0.02	(B)
Ratio accuracy of CCC bridge, 10 kΩ to 1 kΩ ($\pm3\times10^{-9}$, rectangular):	1.7	(B)
Ratio accuracy of CCC bridge, 1 kΩ to 100 Ω ($\pm3\times10^{-9}$, rectangular):	1.7	(B)
Total Type A uncertainty	1.7	
Total Type B uncertainty	2.4	
Total combined uncertainty	2.9	

3) Uncertainty budget for the NPL measurements of the 100 Ω/1 Ω ratio via an intermediate 10 Ω resistor :

	Uncertainty and type	
<u>BIPM 100 Ω to 10 Ω 239136</u>		
Standard deviation of the mean (5 results):	1.8	(A)
Temperature stability of 239136, (2×10^{-9} /mK, ± 1 mK rectangular):	1.2	(B)
Calibration using shunt, (0.5 mΩ on 2 leads, 28×10^{-6} deviation, rectangular):	1.2	(B)
Ratio accuracy of CCC bridge ($\pm3\times10^{-9}$, rectangular):	1.7	(B)
Peltier effect in 10 Ω (correction of 10×10^{-9})	2.0	(B)
<u>10 Ω 239136 to BIPM 1 Ω</u>		
Standard error of the mean (2 results):	1.0	(A)
Calibration using shunt, (0.5 mΩ on 2 leads, 0.3×10^{-6} deviation, rectangular):	0.01	(B)
Ratio accuracy of CCC bridge (±3×10-9, rectangular):	1.7	(B)
Total Type A uncertainty	2.1	ν=7
Total Type B uncertainty	3.6	
Total combined uncertainty	4.2	

Figure 1: NPL indirect measurements and BIPM direct measurements of the 100 Ω resistance R in terms of R_{K-90}. The error bars correspond to estimations of the type A standard uncertainties.