Bilateral Comparison of 10 V Standards
between the OFMET, Wabern, Switzerland and the BIPM,
March to May 1999

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A comparison of the 10 V voltage reference standards of the BIPM and the Office Fédéral de Métrologie (OFMET), Wabern, Switzerland, was carried out from March to May 1999. Three BIPM 732B Zener diode-based travelling standards, BIPM6, BIPM7 and BIPM8, were transported by car. The BIPM measurements of the travelling standards were carried out by direct comparison to the Josephson effect standard (March 19 to April 2) or by dividing the 10 V outputs to 1.018 V by means of a resistive divider and comparing these values to the electromotive force of a standard cell (April 6 to May 4). Results obtained from this indirect method were checked against those obtained by measuring the 10 V outputs directly with the BIPM Josephson array and the two were found to differ by less than 0.1 μV. Results of all measurements were corrected for the dependence of the output voltage on ambient temperature and pressure.

Figures 1 to 3 show the measured values obtained for the three standards by the two laboratories. The BIPM values and uncertainties, and those of the OFMET are calculated for the reference date from the least-squares fits.

Table 1 lists the results of the 10 V comparison and the component uncertainty contributions for the comparison OFMET/BIPM. Experience has shown that flicker or 1/*f* noise dominates the stability characteristics of Zener-diode standards and it is not appropriate to use the standard deviation of the mean to characterize the dispersion of measured values. For the present standards, the flicker floor voltage is about 0.1 μV.

In estimating the uncertainty we have calculated the *a priori* uncertainty based on all known sources except that associated with the stability of the standards when transported. We compare this with the *a posteriori* uncertainty estimated by the standard deviation of the mean of the results from the three travelling standards. With only travelling standards, the uncertainty of the standard deviation of the mean is comparable to the value of the standard deviation of the mean itself. If the *a posteriori* uncertainty is significantly different from the *a* *priori* uncertainty, we assume that a standard has changed in an unusual way and we use the larger of these two estimates in calculating the final uncertainty.

Remarks on the Items in Table 1

Items 1, 2, 3 and 4 are the OFMET values, the type-A, type-B and the combined standard uncertainties, respectively, for each Zener, expressed in µV. Item 2 is estimated by noting that the stability of the Zeners can be described by flicker noise (1/*f* noise) with a floor value of about 1 part in 10^8. This means that successive measured values are correlated and that the standard deviation of the mean is greater than the standard deviation divided by the square root of the number of measurements. As estimates of the standard deviation of the mean, taking correlation into account, we take 0.1 µV for all three Zeners.

Table 2 summarises the estimated type-B standard uncertainty for the BIPM measurements using the resistive divider. These estimates are deduced by comparing values obtained for Zener calibrations by the resistive divider method with those obtained by direct measurements with a BIPM 10 V array. Table 3 summarises the estimated type-B standard uncertainty for the BIPM measurements using the Josephson array voltage standard. Table 4 lists the type-B uncertainties of the OFMET.

Items 5, 6, 7 and 8 of Table 1 are the BIPM values, the type-A, type-B and the combined standard uncertainties, respectively, for each Zener.

Uncertainties for the voltage corrections are estimated from the uncertainties in the BIPM measurements of the temperature and pressure coefficients. In each case the voltage correction uncertainty is split into two parts, one associated with fluctuations of the pressure (or temperature) about the local mean value and the other associated with the difference between the local mean pressures (or temperatures) of the laboratories. The first part is assumed to be included in the type-A uncertainty. The second part is listed separately in Item 9, not as a contribution associated with one laboratory but as one associated with the two laboratories. This value, due to uncertainties in the pressure and temperature coefficients, is the root-sum-square (rss) of the temperature and pressure uncertainties related to the differences between the mean temperatures and pressures of the two laboratories.

Item 10 is the total *a priori* combined *type-A* uncertainty for each Zener. This is the rss of Items 2, 6 and 9.

Item 11 is the comparison result obtained from each Zener and Item 12 is the mean, $\overline{R}$, of the comparison results for all ($n = 3$) Zeners, R_1, R_2 and R_3. Then, from elementary statistics, if the R's are independent, $\mathrm{var}(\overline{R}) = \mathrm{var}\left[\left(\sum_{i=1}^{3} R_i\right)/3\right] = 3^{-2}\sum_{i=1}^{3}\mathrm{var}(R_i)$ or $\sigma(\overline{R}) = 3^{-1}\left[\sum_{i=1}^{3}\mathrm{var}(R_i)\right]^{1/2}$. This is given in Item 13 and it is the expected or *a priori* type-A uncertainty of the mean (Item 12).

In contrast, Item 14 is the *a posteriori* type-A standard uncertainty of the comparison and is the standard deviation of the mean, s_M of the results obtained from the three Zeners. With only three travelling standards, the uncertainty of s_M, is comparable to the value of s_M itself. Item 14 should be compared with Item 13, which we would expect to contain the same uncertainty components except for transport effects. In cases where the two are not consistent, we use the *larger* of the two estimates as the type-A uncertainty.

Item 15 is the total combined uncertainty of the comparison calculated from the rss of Item 3, Item 7, and either Item 13 or Item 14.

The final result of the comparison is presented as the difference between the value assigned to a 10 V standard by each laboratory. The difference between the value assigned to a 10 V standard by the OFMET, at the OFMET, U_OFMET, and that assigned by the BIPM, at the BIPM, U_BIPM, for the reference date is

$$U_\mathrm{OFMET} - U_\mathrm{BIPM} = -\,0.14\ \mu\mathrm{V};\ u_\mathrm{c} = 0.14\ \mu\mathrm{V}\ \text{on 1999/04/18},$$

where u_c is the combined type-A and type-B standard uncertainty from both laboratories.

This is an excellent result. Not only is the total standard uncertainty near the limit of that imposed by the noise and stability of the Zener standards but also the results agree to within this uncertainty.

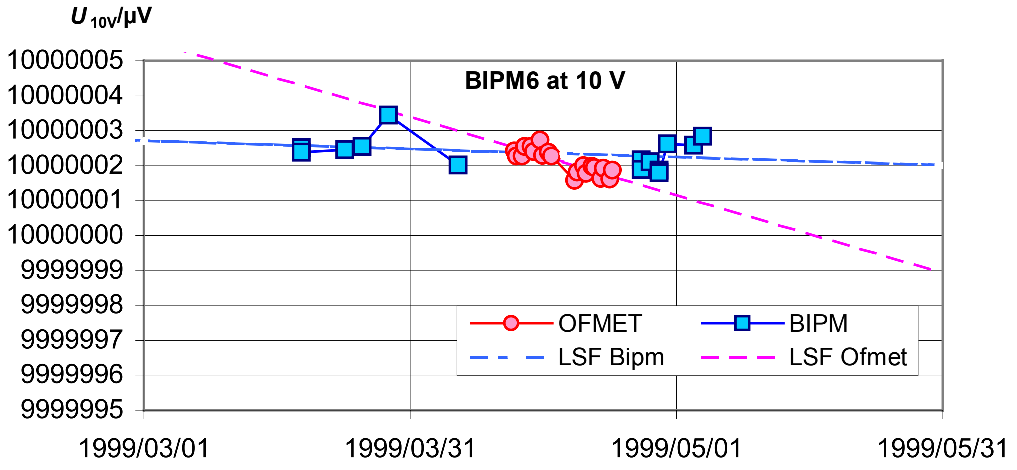

Figure 1. Voltage of BIPM6 vs time with linear least-squares fits (LSF) to the measurements of each laboratory

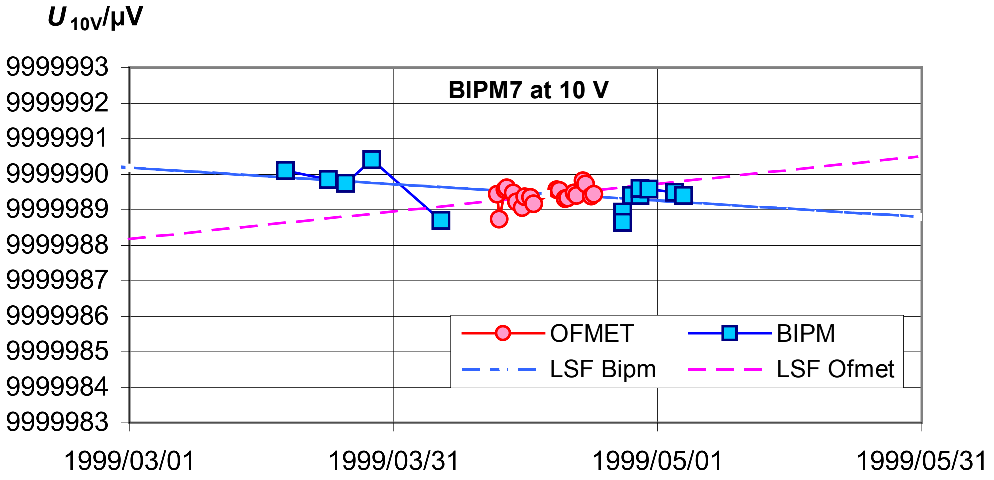

Figure 2. Voltage of BIPM7 vs time with linear least-squares fits (LSF) to the measurements of each laboratory

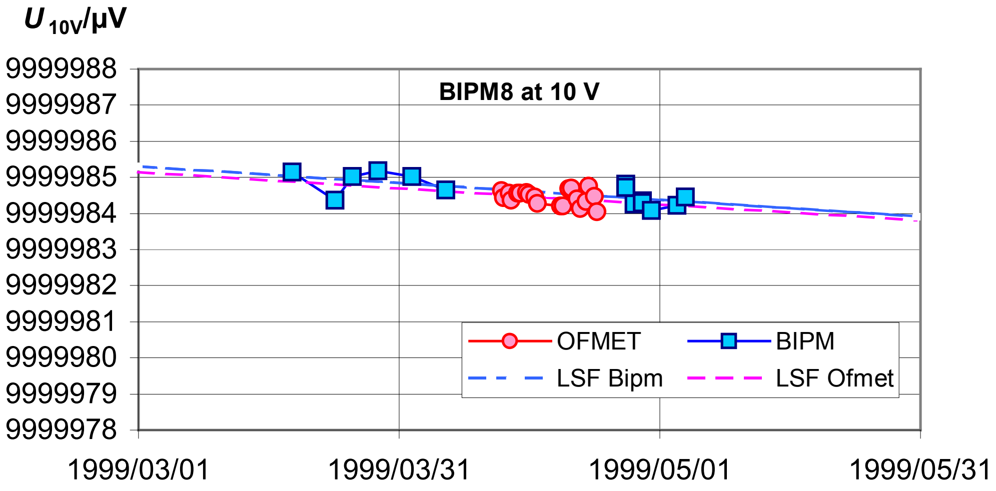

Figure 3. Voltage of BIPM8 vs time with linear least-squares fits (LSF) to the measurements of each laboratory

Table 1. Results of the OFMET/BIPM bilateral comparison of 10 V standards using Zener travelling standards: Mean Date 18 April 1999. Uncertainties are 1-σ estimates.

OFMET/BIPM Bilateral voltage comparison using travelling Zener standards
BIPM6, BIPM7 and BIPM8

Units are µV

		BIPM6	BIPM7	BIPM8	
1	OFMET value,U_{OFMET}	10000002.11	9999989.39	9999984.44	
2	OFMET unc (A)	0.10	0.10	0.10	r
3	OFMET unc (B)	0.006	0.006	0.006	s
4	OFMET unc (total)	0.10	0.10	0.10	
5	BIPM value,U_{BI}	10000002.35	9999989.46	9999984.58	
6	BIPM unc (A)	0.12	0.13	0.10	t
7	BIPM unc (B)	0.10	0.10	0.10	u
8	BIPM unc (tot)	0.16	0.16	0.14	
9	pc & tc unc.	0.06	0.06	0.04	v
10	tot rss uncorr for each Zener	0.17	0.17	0.15	$w=[r^2+ t^2+ v^2]^{1/2}$
11	$U_{OFMET}-U_{BIPM}$	-0.23	-0.06	-0.13	
12	mean $U_{OFMET}-U_{BIPM}$	-0.14			
13	Expected unc of transfer	0.09			$y=[w_6^2+ w_7^2+ w_8^2]^{1/2}/3$
14	s_M of difference for 3 Zeners	0.05			
15	Total unc of comparison	0.14			
	mean date yy/mm/dd	99/04/18	99/04/18	99/04/18	

Table 2. Estimated type-B standard uncertainties for Zener calibrations with the BIPM resistive divider system. The uncertainty evaluations follow from comparisons of results of Zener calibrations with the divider system with those obtained by direct measurements with a BIPM 10 V array.

	Value/nV
Uncertainty in the difference in calibration results by the two methods	50
Temporal stability of divider calibration	90
Total	100

Table 3. Estimated type-B standard uncertainties for Zener calibrations with the BIPM Josephson array voltage standard.

	BIPM/nV
Thermal emfs	1.0
Frequency	0.3
Leakage resistance	0.2
Detector	0.5
rss total	1.2

Table 4. Estimated type-B standard uncertainties for the OFMET from information supplied by the participant. In cases where the specification was a uniform distribution of full width $2a$, the corresponding standard deviation is taken to be $3^{-1/2}\, a$; units are nV.

	OFMET/nV
Thermal emfs & others (*)	5.0
Frequency	0.7
Leakage resistance	1.0
Detector	2.0
rss total	5.5

(*) By measuring a short circuit with the 0 V step, this comprises the following sources of uncertainty:

Detector bias current.
Detector offset, input impedance, non linearity and noise.
Uncorrected thermal voltages in the measurement circuit.
Rectification of the reference frequency.