Bilateral Comparison of 10 V and 1.018 V Standards between the SMU and the BIPM, May through October 1998

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A comparison of the voltage reference standards of the BIPM and the SMU was carried out from May to October 1998. Two 732B Zener diode-based travelling standards, BIPM4 and BIPM5, were shipped as freight via courier service. The BIPM measurements of the travelling standards were carried out by dividing the 10 V outputs to 1.018 V using a resistive divider and comparing these values to the electromotive force of a standard cell. The 1.018 V outputs were measured with respect to a reference standard cell. In both cases the reference cell value is known with respect to the BIPM Josephson voltage standard with a combined standard uncertainty of 10 nV. The SMU carried out direct measurements of the travelling standards with its Josephson array voltage standard. Results of all measurements were corrected for the dependence of the output voltage on ambient temperature and pressure.

Figure 1 shows the results of the 10 V measurements of BIPM4 in both laboratories. The measurements were analyzed using a linear least-squares fit to the voltages as a function of time. The straight lines on the graph show the predicted values. The dashed lines represent the standard uncertainties of the predicted points. The results are referenced to the mean date of the SMU measurements, 22 August 1998. In this way, the values and uncertainties of the SMU measurements are essentially the same whether we use a least-squares fit or a simple average. The BIPM value and uncertainty for the reference date are calculated from the least-squares fit. Figure 2 shows the results for BIPM5.

Figure 3 shows the results of the 1.018 V measurements of BIPM4 in both laboratories. The measurements were analyzed using a linear least-squares fit to the voltages as a function of time. The straight lines on the graph show the predicted values. The dashed lines represent the standard uncertainties of the predicted points. The results are referenced to the mean date of the SMU measurements, 21 August 1998. The BIPM value and uncertainty for the reference date are calculated from the least-squares fit. Figure 4 shows the results for BIPM5 at 10 V.

Table 1 lists the results of the 10 V comparison and the component uncertainty contributions. Experience has shown that flicker or $1/f$ noise dominates the stability characteristics of Zener-diode standards and it is not appropriate to use the standard deviation of the mean to characterize the dispersion of measured values. For the present standards, the flicker floor voltage is about 2 parts in 10^8 of the output voltage at both 10 V and 1.018 V.

In estimating the uncertainty we have calculated the *a priori* uncertainty based on all known sources except that associated with the stability of the standards when transported. We compare this with the *a posteriori* uncertainty estimated by the standard deviation of the mean of the results from the two travelling standards. With only two travelling standards, the uncertainty of the latter is comparable to the uncertainty itself. If the *a posteriori* uncertainty is significantly greater than the *a priori* uncertainty, we assume that a standard has changed in an unusual way and that we must use the *a posteriori* uncertainty in calculating the final uncertainty. This was done in the case of the 1.018 V comparison.

Uncertainties in the BIPM measurements of the temperature and pressure coefficients would lead to a type B uncertainty if only one travelling standard was used. In the case of more than one, we do not expect significant correlation among the corrections for different standards and in our uncertainty table they are treated as type A uncertainties.

The final results of the comparison are presented as the difference between the value assigned to a 10 V standard by the SMU, U_{SMU}, and that assigned by the BIPM, U_{BIPM}, on the reference date. The result is

$$U_{\mathrm{SMU}} - U_{\mathrm{BIPM}} = -0.31\ \mu\mathrm{V}; \qquad u_{\mathrm{c}} = 0.32\ \mu\mathrm{V} \text{ on 1998/8/21,}$$

where u_{c} is the combined type A and type B standard uncertainty from both laboratories. This is a very satisfactory result.

Table 2 lists the results of the 1.018 V comparison and the component uncertainty contributions. In this case the travelling standards were less well behaved and we have used the *a posteriori* uncertainty to estimate the type A component. The final results of the comparison are presented as the difference between the value assigned to a 1.018 V standard by the SMU, U_{SMU}, and that assigned by the BIPM, U_{BIPM}, on the reference date. The result is

$$U_{\mathrm{SMU}} - U_{\mathrm{BIPM}} = -0.07\ \mu\mathrm{V}; \qquad u_{\mathrm{c}} = 0.11\ \mu\mathrm{V} \text{ on 1998/8/20,}$$

where u_{c} is the combined type A and type B standard uncertainty from both laboratories. Our experience leads us to expect typical values of the order of 0.050 μV for the transfer uncertainty.

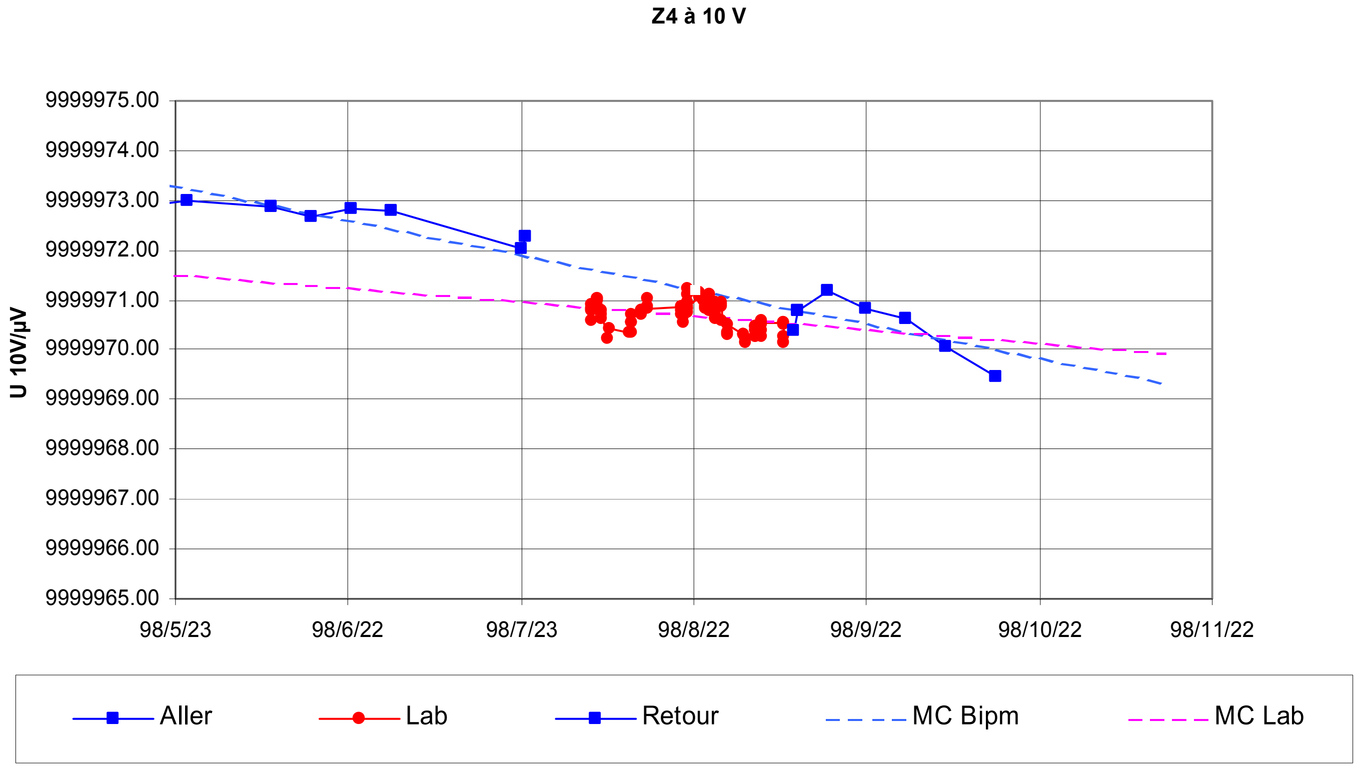

Fig. 1 Values of the 10 V output of BIPM4 during the comparison: Corrections for temperature and pressure have been applied. Straight lines indicate the result of linear least-squares adjustments.

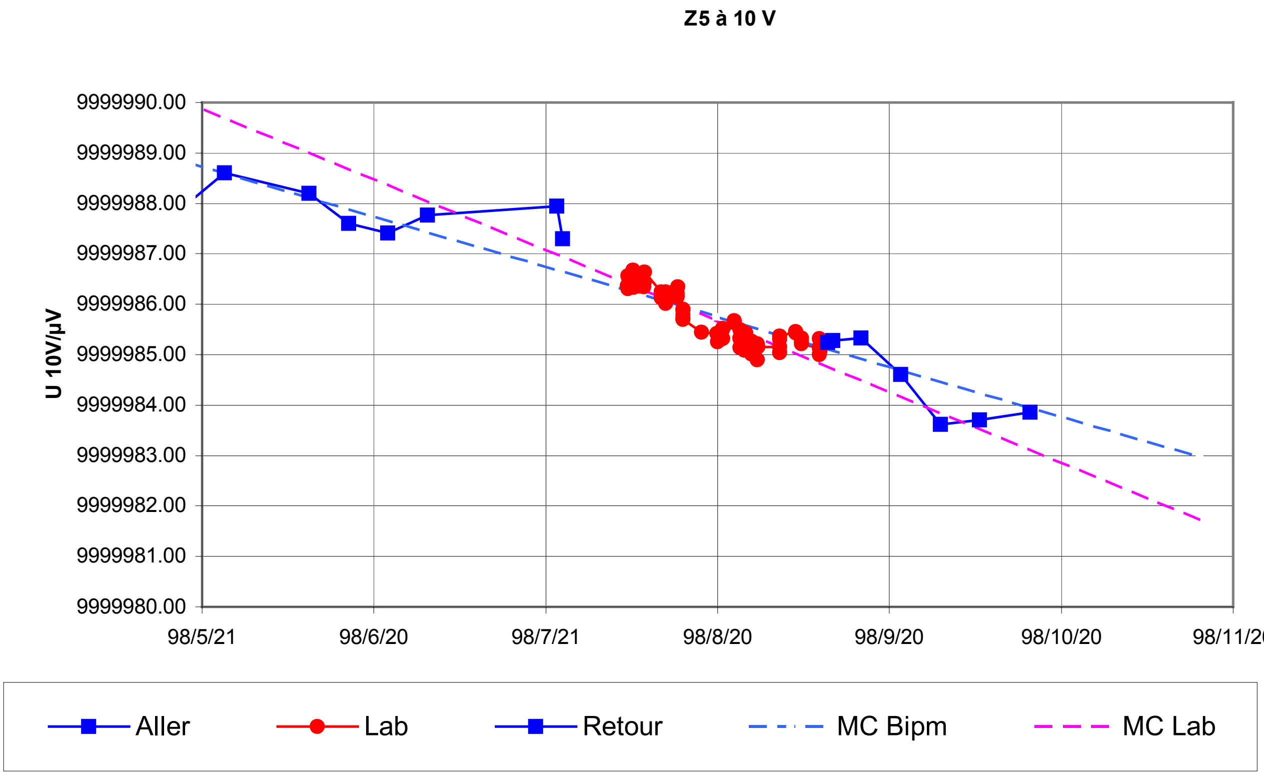

Fig. 2 Values of the 10 V output of BIPM5 during the comparison: Corrections for temperature and pressure have been applied. Straight lines indicate the result of linear least-squares adjustments.

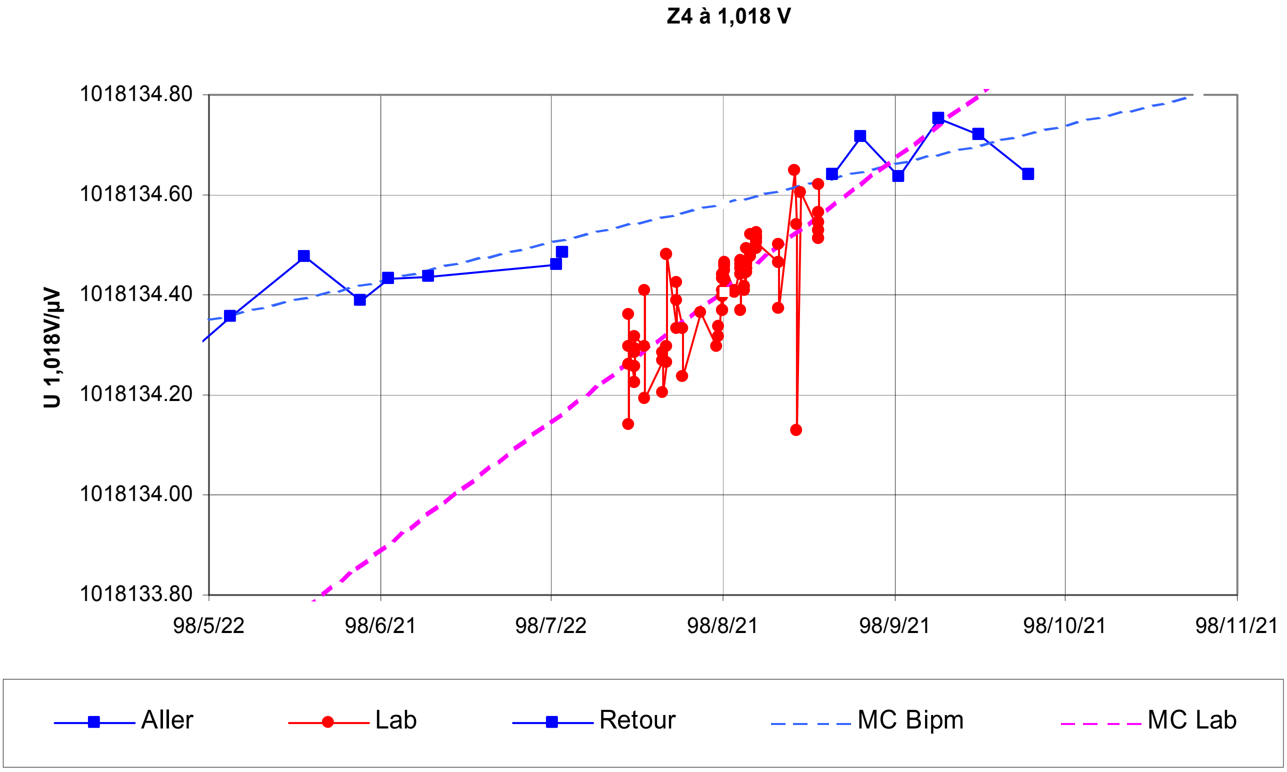

Fig. 3 Values of the 1.018 V output of BIPM4 during the comparison: Corrections for temperature and pressure have been applied. Straight lines indicate the result of linear least-squares adjustments.

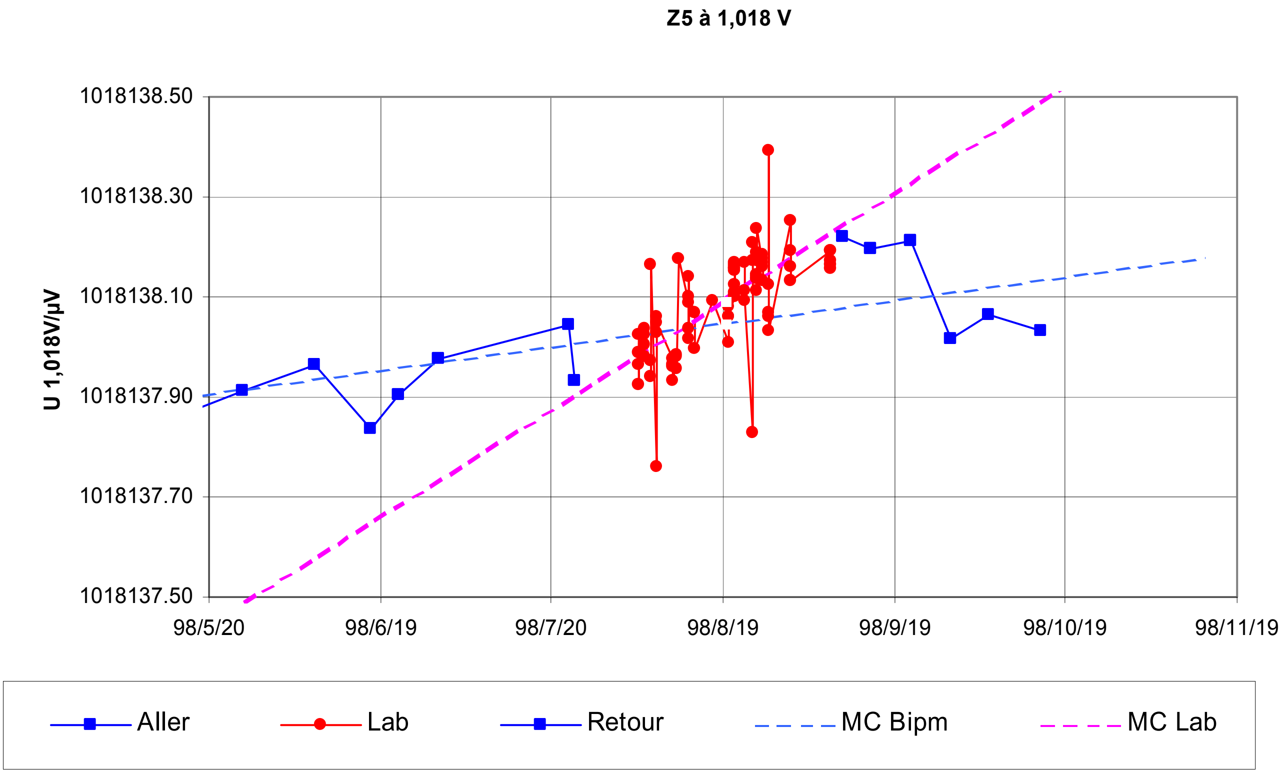

Fig. 4 Values of the 1.018 V output of BIPM5 during the comparison: Corrections for temperature and pressure have been applied. Straight lines indicate the result of linear least-squares adjustments.

Table 1. Results of the SMU/BIPM bilateral comparison of 10 V standards using Zener travelling standards: Mean Date 21 August 1998.

SMU/BIPM Bilateral voltage comparison using travelling Zener standards BIPM4 and BIPM5.

Units are µV

		BIPM4@10 V	BIPM5@10 V	
1	SMU value, U_{SMU}	9999970.70	9999985.68	
2	SMU unc (A)	0.20	0.20	*r*
3	SMU unc (B)	0.02	0.02	*s*
4	SMU unc (total)	0.20	0.20	
5	BIPM value, U_{BI}	9999971.24	9999985.76	
6	BIPM unc (A)	0.20	0.20	*t*
7	BIPM unc (B)	0.10	0.10	*u*
8	BIPM unc (tot)	0.22	0.22	
9	pc & tc unc, uncorrelated	0.30	0.34	*v*
10	tot rss uncorr for each Zener	0.42	0.44	w =rss(r,t,v)
11	U_{SMU} - U_{BIPM}	-0.54	-0.08	
12	mean U_{SMU} - U_{BIPM}	-0.31		
13	unc of transfer	0.23		*x*
14	*a priori* type A unc for 2 Zeners	0.30		y=1/{sqrt[w_4^{-2} + w_5^{-2}]}
15	Total unc of comparison	0.32		sqrt[y^2+s^2+u^2]
	mean date yy/mm/dd	98/8/22	98/8/20	

References to Table 1.

1, 2, 3 and 4 are the SMU value, type A, type B and combined uncertainties;

2, the stability of the Zeners can be described by flicker noise (1/*f* noise) with a floor value of about 2 parts in 10^8.

5, 6, 7 and 8 are the BIPM value, type A, type B and combined uncertainties.

9 is the the root-sum-square (rss) total uncertainty associated with the corrections for temperature and pressure; uncertainties in the pressure and thermistor measurements are negligible and that the total uncertainty is dominated by that of the coefficients.

10 is the total *a priori* combined type A uncertainty for each Zener. This is the rss of 2, 6 and 9.

11 is the comparison result from each Zener.

12 is the mean difference for all *n* (=2) Zeners.

13 is the *a posteriori* type A uncertainty and includes effects due to transport. It is the standard deviation of the results from both travelling standards. Since there were only two travelling standards, this uncertainty has a rather large uncertainty.

14 is the *a priori* type A standard uncertainty of the comparison and is the uncertainty of the mean or the results from the two Zeners. This should be compared with 13 which contains the same uncertainty components and transport effects. Since 13 and 14 are consistent, it may be concluded that no unusual change in the standards occurred in transport. Because 14 is a more accurate estimate, it is used in the final uncertainty estimate.

15 is the total uncertainty of the comparison calculated from the rss of 3, 7 and 14.

Table 2. Results of the SMU/BIPM bilateral comparison of 1.018 V standards using Zener travelling standards. Mean Date 20 August 1998.

SMU/BIPM Bilateral voltage comparison using travelling Zener standards BIPM4 and BIPM5.

Units are µV

		BIPM4@1.018 V	BIPM5@1.018 V	
1	SMU value, U_{SMU}	1018134.41	1018138.09	
2	SMU unc (A)	0.04	0.03	*r*
3	SMU unc (B)	0.02	0.02	*s*
4	SMU unc (total)	0.028	0.028	
5	BIPM value, U_{BI}	1018134.58	1018138.05	
6	BIPM unc (A)	0.02	0.02	*t*
7	BIPM unc (B)	0.01	0.01	*u*
8	BIPM unc (tot)	0.022	0.022	
9	pc & tc unc, uncorrelated	0.034	0.034	*v*
10	tot rss uncorr for each Zener	0.044	0.044	w =rss(r,t,v)
11	U_{SMU} - U_{BIPM}	-0.17	0.04	
12	mean U_{SMU} - U_{BIPM}	-0.07		
13	unc of transfer	0.11		*x*
14	*a priori* type A unc for 2 Zeners	0.03		y=1/{sqrt[w_4^{-2} + w_5^{-2}]}
15	Total unc of comparison	0.11		Same as 13
	mean date yy/mm/dd	98/8/21	98/8/19	

References to Table 2.

1, 2, 3 and 4 are the SMU value, type A, type B and combined uncertainties;

2, The stability of the Zeners can be described by flicker noise (1/*f* noise) with a floor value of about 2 parts in 10^8. If the SMU results for BIPM 4 from each day are averaged and used in a linear least-squares fit, the standard deviation of the residuals is 0.047. The standard deviation of the value assigned by the SMU on the mean date of the measurements, which is taken as the mean date of the SMU measurements, is the standard deviation divided by the square root of the number of degrees of freedom (number of daily measurement results minus two) or about 0.01, if the daily measurement values are uncorrelated. But, in fact, they are correlated in a way that is difficult to model from the data. We therefore estimate a value between the two limits of 0.047 and 0.01 for the type A uncertainty and set it at 0.04. A similar argument was applied for the estimated type A uncertainty for BIPM 5.

5, 6, 7 and 8 are the BIPM value, type A, type B and combined uncertainties.

9 is the the root-sum-square (rss) total uncertainty associated with the corrections for temperature and pressure; uncertainties in the pressure and thermistor measurements are negligible and that the total uncertainty is dominated by that of the coefficients.

10 is the total *a priori* combined type A uncertainty for each Zener. This is the rss of 2, 6 and 9.

11 is the comparison result from each Zener.

12 is the mean difference for all *n* (=2) Zeners.

13 is the *a posteriori* type A uncertainty and includes effects due to transport. It is the standard deviation of the results from both travelling standards. There were only two travelling standards and so this uncertainty has a rather large uncertainty.
14 is the *a priori* type A standard uncertainty of the comparison and is the uncertainty of the mean or the results from the two Zeners. This should be compared with 13 which contains the same uncertainty components and transport effects. Since 13 is much greater, we assume that transport-related effects dominate the uncertainty.
15 is the total uncertainty of the comparison calculated from the rss of 3, 7 and 13.