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Some remarks on the Galushka method

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Abstract

A method recently suggested for the on-line correction of dead-time losses in a Poisson process is analyzed. After correction of an erroneous mathematical description, the approach is recognized as sound. It may be of interest in applications where simplicity is essential, but provides no advantage for metrological purposes.

1. Introduction

Thanks to the kindness of V.A. Nazarov, an Ukrainian physicist and businessman, I recently received a copy of an article by A.N. Galushka [1]. In this, a method is described for restoring dead-time losses in real time so that at the output of a counter, constructed according to this new scheme, one obtains directly the number of events expected in the absence of a dead time. This is accomplished by inserting additional pulses into the actual series of registered events.

For a physicist, the message of this paper is not easy to grasp as the flourishing style does not compensate for a lack of clear information. The use of an "inversed projection" of pulses is described, but not explained. The suggested method remains opaque, has a touch of hocus-pocus and confronts the reader with a number of problems. Among them, the main question is whether the method has a sound basis.

2. Some elements on Poisson processes and dead-time losses

The assumptions of the Galushka method are not explicitly stated. Clearly, however, the incoming stream of pulses has to be purely Poissonian, i.e. it should not be distorted by the detector. One must also assume that the dead time τ remains constant and is strictly of the non-extendable type.

In such a situation, let the observed sequence of events (Fig. 1) be characterized by the arrival times T_0, T_1, T_2, ... Consecutive arrivals are separated at least by the dead time τ applied. If we put $T_0 = 0$, then pulse number k occurs at the instant

$$\begin{aligned} T_k &= (\tau + \delta_1) + (\tau + \delta_2) + \dots + (\tau + \delta_k) \\ &= k\tau + \sum_{j=1}^{k} \delta_j , \qquad \text{for} \quad k \geq 1. \end{aligned} \tag{1}$$

It is important to realize that the random time intervals δ_j may be considered as representative samples of the intervals appearing in the undisturbed, original process. This follows from the well-known peculiarity of the Poisson process that it has "no memory" so that an interval can start whenever we wish (see e.g. [2]). In our case, we choose to begin it at the end of the previous dead time.

If one eliminates in the observed sequence T_1, T_2, ... of arrival times all the dead times, a new sequence of arrivals

$$t_k = \sum_{j=1}^{k} \delta_j \tag{2}$$

is obtained (in "live time", with $t_0 = 0$). As a consequence of what has been said above, the arrivals t_k conform to a Poisson process in which the individual random intervals δ_j have a common density of the exponential form

$$f(\delta_j) = \rho\, e^{-\rho\, \delta_j} , \qquad \text{for} \quad \delta_j > 0 , \tag{3}$$

where ρ denotes the count rate of the original Poisson process. It follows from (3) that the expectation value of δ_j is $1/\rho$.

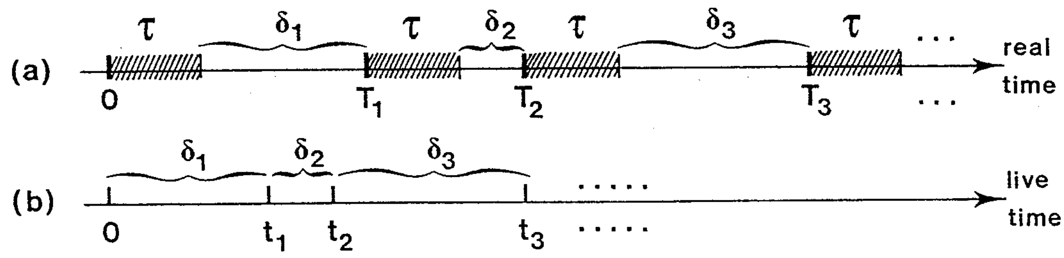

Fig. 1. Schematic representation of the relation between (a) the observed arrival times T_j and (b) the corresponding Poisson process t_j.

3. Principle of the on-line correction

How is it possible to compensate for counting losses ? Obviously, losses occur only during the dead times. One might think that to evaluate them we should know exactly how many pulses were lost in a given dead time, but this is of course impossible. However, it is easy to see that this is not really necessary. Our aim is not to reconstruct the original process; all we need is a reliable estimation of the original count rate, or of the number of original events for the interval of time considered. To do this, we need either an accurate value for the average loss per dead-time period, or a method which allows us to arrive at individual corrections which, in the mean or in the long run, have no bias.

Both approaches are possible. Traditional correction formulae use the first method: they are based on the observed count rate and are applied at the end of a measurement period. In contrast, methods of the second type work in a differential way by "instantly" correcting (or compensating) for losses, apparently without requiring a knowledge of the measured or calculated count rate. How can this work ?

It is possible, of course, to estimate the probability of losing a specific number k of counts in a dead time of length τ. Since we deal with a Poisson process, this probability is given by

$$P_k = \frac{(\rho\tau)^k}{k!} e^{-\rho\tau} , \qquad (4)$$

so that the total number of losses L in time τ amounts to

$$L = \sum_{k=1}^{\infty} k P_k = e^{-\rho\tau} \sum_k \frac{(\rho\tau)^k}{(k-1)!} = \rho\tau , \qquad (5)$$

as expected. However, the application of (4) or (5) requires that the count rate be known in advance, so we are captured in a vicious circle.

The question is whether we really have to know ρ. In one way or another, it is always required for the reconstruction of the original Poisson process, or at least some important feature of it. A careful look at Fig. 1 shows that what we are looking for exists already, for the sequence (b) is a realization of the required original process. All we need is some simple way to use the available information to determine and, if we wish, to correct the losses which occur during the dead-time periods.

A possible way to achieve this is to associate the dead time τ, which follows the registration T_j (see Fig. 1), with the preceding intervals δ_j, δ_{j-1},δ_{j-2}, ... This may be done by counting the number k of intervals that can be placed within one dead time. This leads, for a given registration at T_j, to the relation

$$\sum_{k=0}^{K_j-1} \delta_{j-k} < \tau , \quad \text{but} \quad \sum_{k=0}^{K_j} \delta_{j-k} > \tau . \tag{6}$$

It is convenient, for our purposes, to change the direction in which intervals are enumerated. By putting

$$\delta_{j-k} = \delta'_{k+1} , \quad \text{for } k = 0, 1, 2, \dots ,$$

the counting goes backwards in time (starting with T_j) and (6) becomes

$$\sum_{k=1}^{K_j} \delta'_k < \tau , \quad \text{but} \quad \sum_{k=1}^{K_j+1} \delta'_k > \tau . \tag{7}$$

In this form K_j is readily seen to be the largest possible number of original intervals within τ (Fig. 2). Since each interval has been initiated by an event, K_j can be considered as the number of pulses lost in the dead time τ starting at T_j. Obviously this association is arbitrary, but it is convenient; it also simplifies the electronic realization of the correction.

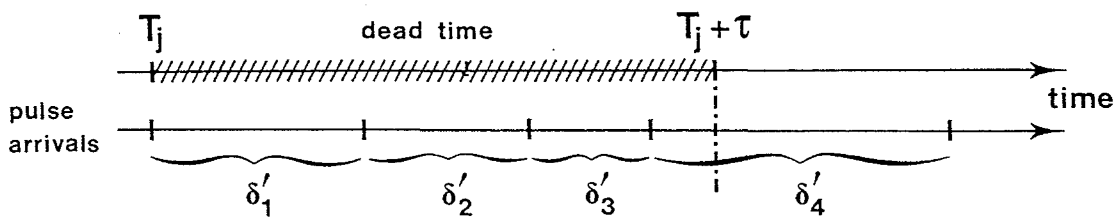

Fig. 2: Association of the time intervals δ prior to T_j with the number of pulses lost within the dead time τ (here $K_j = 3$).

If, in a real counting process, a sufficient number of arrival times T_j is registered and stored so that the required number of original intervals of type δ becomes available, a relation like (7) can be used to evaluate the number of lost events for every arrival T_j of a pulse. These losses can be compensated immediately ("on line", or in "real time") by injecting a corresponding number of artificial events. Exactly how this is done is of little interest, for the real behaviour of the original process cannot be recovered. It is unlikely that, for $K_j > 0$, they will be inserted at the moments $T_j - k\tau$, with k ranging from 1 to K_j, as is indicated in [1]. It seems easier to place them, for instance, in the dead time starting at T_j, time permitting, or in the subsequent one. The exact procedure does not change the final result.

Galushka's paper contains a single equation which reads (in the present notation and for $K_j = 1, 2, \dots$)

$$T_j - T_{j-K_j+1} < (K_j+1)\,\tau < T_j - T_{j-K_j} \,. \tag{8}$$

The question arises whether (8) can be related to (7) or to (6).

For such a comparison, we may start with (7) by writing it, with (2), in the simple form

$$t_{K_j} < \tau \quad \text{and} \quad t_{K_j+1} > \tau \,. \tag{9}$$

Adding $K_j\,\tau$ throughout leads to

$$K_j\,\tau + t_{K_j} < (K_j+1)\,\tau \quad \text{and} \quad K_j\,\tau + t_{K_j+1} > (K_j+1)\,\tau \,.$$

Recalling from (1) that $T_j - T_{j-k} = k\tau + t_k$, we obtain

$$T_j - T_{j-K_j} < (K_j+1)\,\tau \tag{10a}$$

and

$$T_j - T_{j-(K_j+1)} - \tau > (K_j+1)\,\tau \,,$$

or likewise

$$T_j - T_{j-K_j-1} > (K_j+2)\,\tau \,. \tag{10b}$$

Since both (10a) and (10b) disagree with the original relation (8) - which is given in [1] without any comment -, it follows that (8) is incorrect. All checks performed are in line with this conclusion.

In hindsight, the intended meaning of the relation is more transparent. It is not an inequality which is automatically true; rather, it is a condition for the intervals (or arrival times) prior to T_j which has to be fulfilled for a loss of exactly K_j events (associated, by convention, with the dead time initiated by the registration at T_j).

Our new relations (10), although incompatible with (8), do agree with the graphical reconstruction of pulses for compensating the dead-time losses, a procedure which can be inferred from Fig. 1 of [1].

4. Discussion

The Galushka method has several interesting features, the relative importance of which depends on the planned application. Uncritical users who just want a number as the result of a measurement may be impressed by the fact that no correction has to be applied to the reading. They will ask no further questions and will presumably be satisfied. They are no doubt a majority.

More demanding clients, those with a metrological conscience, will hesitate to apply a new method blindly. Their priorities will be precision and accuracy, not mere convenience. For them the precision of measurements made following this approach remains uncertain, even if the electronic circuits exactly implement the conditions (10).

Alternative methods roughly fall into two groups: those which apply a correction formula and those which adhere to the concept of live timing. Both approaches have the advantage that their results are based on a consideration of the whole measurement interval. This is to be distinguished from the Galushka method which only looks at the immediate neighbourhood of a registered event. This inevitably has an effect on the precision of the result. If, in the Galushka method, more than half of the final pulses originate from artificially injected events, then the average interval δ will be smaller than the dead-time value τ. This has the consequence that subsequent derived values of K_j are often based on the same measured intervals δ, with the result that the corrections applied in the form of added pulses are no longer independent. In the situation described in [1] where, for the highest count rates, K_j has a mean value of about 100, the problem of correlation becomes so serious that little can be said on the reliability of the results without a serious special study of this effect.

An instantaneous (or "differential") correction, as performed here, may be particularly advantageous in the case of a variable activity, for example as a result of decay. However, an effect such as this - if known to be real - can also be accounted for by a correction. In addition, live-timing methods would share the advantage.

To sum up, the Galushka method is interesting and has features which make it attractive to many users. From a metrological point of view, however, the approach is not attractive and may be even dangerous. In any event, it cannot be applied for a dead time of the extendable type.

If the above analysis is correct, we should expect that the electronic device used for implementing the correction for the dead-time losses differs markedly from similar procedures suggested previously. Here we have in mind in particular Westphal's method [3] of "loss-free counting", which is based on an idea described in [4]. However, this does not mean that these approaches are independent and it might be worthwhile examining their interrelations more in detail.

References

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