Determination of absorbed dose to water for ^{60}Co by the scaling theorem

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Abstract

A simple indirect method has been used at the BIPM for the determination of absorbed dose to water. It is based on the so-called scaling theorem and on the experimental value of the absorbed dose measured in a graphite phantom. The results are in good agreement with those obtained by other methods.

1. Introduction

One of the tasks of the national laboratories is to measure the absorbed dose to water as accurately as possible. To achieve this goal, much effort has been devoted to the development of reliable experimental methods. The BIPM has recently developed a primary ionometric standard of absorbed dose to water (Boutillon and Perroche, 1992) and comparison of the results with those obtained at national laboratories gives consistent results which are compatible with the estimated uncertainties.

At present, the absolute methods used for the determination of absorbed dose to water are not easy to handle (both theoretically and experimentally). The purpose of the present work is to describe in detail a simple indirect method based on the so-called scaling theorem. This method, already used at the NPL (Burns et al., 1987), allows one to derive the absorbed dose to water from the value of the absorbed dose measured in another material. The BIPM has used its experimental value of absorbed dose to graphite to determine in this way the absorbed dose to water. Details of the experiment are given below and the results are compared with those obtained by other methods.

2. Principle of the method

The scaling theorem is based on a property of the Compton effect, predominant for ^{60}Co energy and in low-Z material, namely that scattering is proportional to the electronic density of the scattering material (Pruitt and Loevinger, 1982). Consider a source S in empty space and two phantoms (Fig. 1), one of graphite and the other of water, the

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electronic densities of which are denoted d_c and d_w, respectively. The linear dimensions of these phantoms and their distance to the source are "scaled" inversely to their respective electronic densities. Let P and Q be two corresponding points in the two phantoms, at depths t_c and t_w.
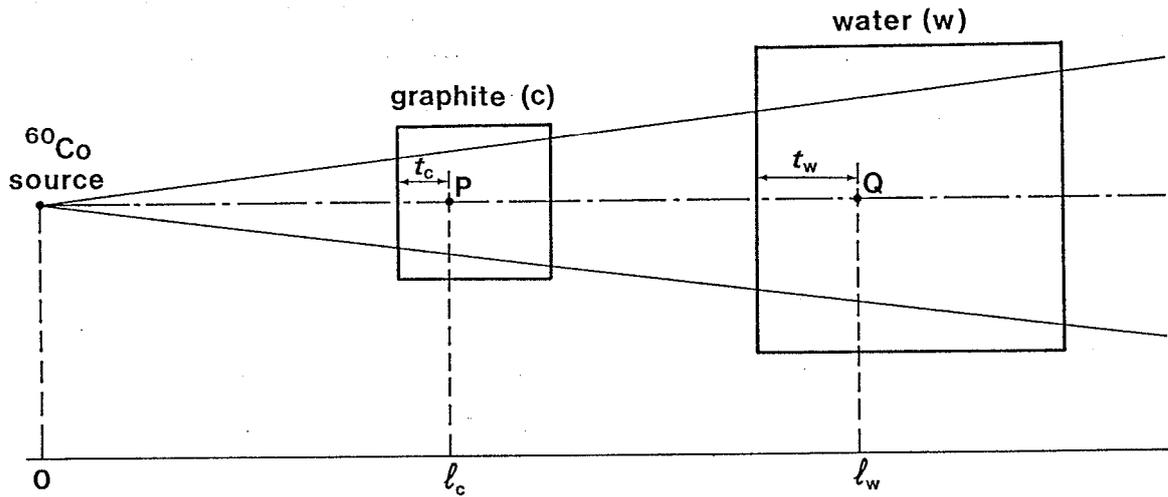

Fig. 1. Schematic experimental arrangement. The points P and Q are at distances l_c and l_w from the source and at depths t_c and t_w in their respective phantom. All distances are scaled inversely to their electronic densities.

The scaling theorem then states that the relation between the energy fluences $\Psi_c(P)$ at point P and $\Psi_w(Q)$ at point Q is

$$\Psi_w(Q)/\Psi_c(P) = (d_w/d_c)^2 . \tag{1}$$

Using the well-known relations between energy fluence, kerma and absorbed dose D, Eq. (1) can be rewritten as

$$\cdot D_w(Q) = D_c(P) \, (d_w/d_c)^2 \, [(\mu_{en}/r)_w / (\mu_{en}/r)_c] \, [(1 + \varepsilon_w)/(1 + \varepsilon_c)] , \tag{2}$$

where the subscripts w and c refer to water and graphite, respectively, μ_{en}/ρ is the mean mass-absorption coefficient, averaged over the energy photon spectra, and $(1+\varepsilon)$ is the ratio of absorbed dose to kerma.

However, in a practical situation correction factors are needed to take into account the deviation of a real arrangement from the ideal case considered above. Four are of significance :

- The scaling-theorem approximation. Interactions due to the photoelectric effect and pair production, which are of minor importance in the case of the ^{60}Co energy and for

low Z materials, are not proportional to the electronic density. Therefore, correction factors k_{pho} and k_{pair} are applied to take them into account.

- In empty space, the variation of the beam fluence with source distance shows a deviation from the inverse square law, due to the presence of radiation scattered by the source environment and the collimator system. This implies a correction factor k_{scat}.

- If air is present between source and phantom, a correction (k_{air}) should be made to account for beam attenuation.

- Geometric corrections may be required : the water phantom used in our experiment is slightly smaller than that required by the scaling theorem; this leads to a reduction of the absorbed dose at point Q by a factor k_{ph}. Another correction factor is needed for the non-equivalence to water of the front perspex plate of the phantom (k_{pf}).

As a consequence, the absorbed dose to water at point Q under experimental conditions may be derived from Eq. (2) as

$$D_w(Q) = D_c(P) \, (d_w/d_c)^2 [(\mu_{en}/\rho)_w/(\mu_{en}/\rho)_c] \, [(1 + \varepsilon_w)/(1 + \varepsilon_c)] \, \Pi k_i , \tag{3}$$

where $\Pi k_i = k_{pho} \, k_{pair} \, k_{scat} \, k_{air} \, k_{ph} \, k_{pf}$ is the product of the correction factors described above.

3. Experimental arrangement

The ^{60}Co source, of 2 cm diameter and 0.56 cm length, had an activity of 115 TBq on first of January 1991. The beam includes a scattered component which amounts to 14 % of the primary radiation, in terms of energy fluence.

The density of water is 0.9982 g cm^{-3} at 20 °C. As for graphite, it is experimentally known that the density is not uniform inside the phantom. It can be assumed, however, that the variation of the density behind the measuring point P has a negligible effect on the absorbed dose measured at that point. For this reason, the mean graphite density is taken to be the value averaged over that part of the graphite phantom located in front of point P. This value ($\rho = 1.784$ g cm^{-3}), determined experimentally, leads to a ratio for the electronic densities of graphite and water of 1.608, with an uncertainty estimated to 0.2 %. In the water phantom, point Q is at distance $l_w = 1.608$ m from the source. The distances of points P and Q from the source are measured with an uncertainty of 0.03 %. The main contribution to this uncertainty, which is common to both l_c and l_w, is in the distance from the source to the exit of the collimator and the uncertainty of the ratio l_c/l_w is estimated to be 0.02 %.

The first phantom (diameter 30 cm, thickness 20 cm), in graphite, is centred on the beam axis, with the point P of measurement at a distance $l_c = 1$ m from the source

where the beam cross section is 10 cm x 10 cm. The absorbed dose to graphite, $D_c(P)$, is measured with the BIPM ionometric standard with an uncertainty of 0.26 %. The reliability of this measurement is supported by previous comparisons with the calorimetric measurements of national laboratories (Boutillon, 1990). Measurements were made with point P at the depths of 5.018 g cm^{-2} and 16.928 g cm^{-2} in graphite. The corresponding depths of point Q in water were 4.517 g cm^{-2} and 15.233 g cm^{-2}, respectively.

The second phantom is made of a perspex tank, 35 cm side with walls 14 mm thick, filled with demineralized water (20 °C). The wall facing the beam has a thickness of 10 mm over a section of 20 cm x 20 cm to reduce the correction due to the non-equivalence of perspex with water.

4. Determination of $D_w(Q)$

The absorbed dose $D_w(Q)$ at point Q is deduced from the experimental value $D_c(P)$ using Eq. (3). The various factors entering this relation were determined by calculation, except for correction factor k_{scat} which was obtained experimentally. The numerical values, together with their estimated uncertainties, are given in Table 1 for point P at a depth of 5.018 g cm^{-2} in graphite and point Q at a depth of 4.517 g cm^{-2} in water.

4.1 Ratio of physical quantities

The value of the ratio $(\mu_{en}/\rho)_w/(\mu_{en}/\rho)_c$ at the point of measurement is based on the data of Hubbell (1982) and averaged over the energy spectrum of the photon, which is obtained by a Monte-Carlo calculation.

The absorbed dose to kerma factor $(1+\varepsilon)_w/(1+\varepsilon)_c$ is calculated by the moment method (Boutillon, 1981) and averaged over the electron spectrum at the point of measurement.

4.2 Correction factors

In what follows, the relevant correction factors are treated individually.

- The correction factors k_{pho} and k_{pair} are obtained by comparing the contributions to the total absorbed dose of the photoelectric and pair-production interactions at the points P and Q, respectively. This was done by a Monte-Carlo calculation, using the data of Hubbell (1982). The correction for pair production is negligible at the ^{60}Co energy, and that for the photoelectric effect is found to be small, of the order of 0.1 %, with an uncertainty estimated as 0.03 % .

- The air-attenuation correction factor k_{air}, calculated from the data of Hubbell (1982), is of the order of 0.4 %, with a negligible uncertainty.

- The deviation from the inverse square law of the beam fluence with distance to the source was obtained experimentally by measuring, with a small transfer ionization chamber (Φ = 11.5 mm), the air kermas at 1 m and 1.608 m, and comparing their ratio

to the inverse square ratio of these two distances. Five series of 30 measurements were performed at each distance. The calibration factor of the chamber was assumed to be constant over this range and the error introduced by this approximation is less than 0.05 % (Boutillon and Niatel, 1973). The correction for the leakage current was negligible, and the standard deviation of the mean value of the air kerma was less than 2×10^{-4}. As mentioned above, the experimental variation of the photon fluence with distance to the source was found to be larger than the one which is expected from the inverse square law, leading to a value of $k_{scat} = 0.9860$, with a total uncertainty of 0.07 %.

- The correction factors k_{ph} (for the insufficient size of the water phantom) and k_{pf} (for the non-equivalence of the front perspex plate with water) were obtained by a Monte-Carlo calculation, using a variance-reducing technique to improve the accuracy of the results. Deviation from unity is 0.16 % for k_{pf} and less than 0.05 % for k_{ph} . The uncertainty in these factors does not exceed 0.02 %.

The resulting total uncertainty of the absorbed dose $D_w(Q)$ is of the order of 0.4 % (see Table 1).

Table 1 - Factors entering in the determination of $D_w(Q)$
(for a depth in water of 4.5 g cm^{-2})

factor	value	uncertainty (%) *
$D_c(P)/\mathrm{mGy\ s^{-1}}$ **	9.540	0.26
d_c/d_w	1.608	0.20
l_w/l_c	1.608	0.02
$(\mu_{en}/\rho)_w/(\mu_{en}/\rho)_c$	1.114 4	0.14
$(1+\varepsilon)_w/(1+\varepsilon)_c$	1.000 5	0.06
k_{pho}	1.001 2	0.03
k_{pair}	1.000 0	<0.01
k_{scat}	0.986 0	0.07
k_{ph}	0.999 7	0.01
k_{pf}	1.001 6	0.02
k_{air}	0.995 6	0.01
$D_w(Q)/\mathrm{mGy\ s^{-1}}$ **	4.047	0.37

*   All uncertainties are standard deviations.
** Value for 1991-01-01.

5. Results and discussion

The present results were compared with those obtained by means of the BIPM ionometric standard of absorbed dose to water, by comparing the calibration factors of

transfer ionization chambers, determined by both methods, in terms of absorbed dose to water.

Two small thimble-type chambers, T1 and T2, with tissue-equivalent walls (Spokas chambers, manufactured by Exradin) were chosen for their high stability. Their diameters are 5.5 mm and 11.5 mm, respectively. The radial non-uniformity of the beam over the cross-section of the chambers, located in water, is small : its effect is negligible at a depth of 5 g cm^{-2} and is less than 0.02 % at a depth of 17 g cm^{-2} (Boutillon and Perroche, 1989). A large number of measurements (more than 300) were made using both methods. The correction for the leakage current was about 0.1 % for chamber T1 and was negligible for T2. The chambers showed an excellent stability (0.03 %).

For each method the calibration factor obtained has an uncertainty of order 0.4 %. We note, however, that the uncertainty in their ratio is smaller (0.2 %) since some factors are common to the two methods (for example the ratio of the mean mass-energy coefficients for graphite and air). The results, listed in Table 2, show that the calibration factors scarcely vary with depth in water. Furthermore, experiments with both chambers show the values obtained by the two methods to differ by less than 0.2 %. This is compatible with the estimated uncertainty.

Table 2 - Comparison of the results obtained by scaling theorem and ionometry

(N_w is the calibration factor (Gy μC^{-1}), in terms of absorbed dose to water, of ionization chambers T1 and T2.)

chamber	depth/ g cm^{-2}	N_w(scal)/ Gy μC^{-1}	N_w(iono)/ Gy μC^{-1}	N_w(scal)/N_w(iono)
T1	5.0	592.1	593.0	0.998 5
T2	5.0	55.12	55.19	0.998 7
T2	17.0	55.15	55.26	0.998 0

These results show the scaling-theorem method to be a simple and practical approach to the determination of absorbed dose to water that can readily be implemented in any laboratory which has access to a standard of absorbed dose for another material. Despite the fact that the method is indirect, it can be very useful as an alternative (perhaps temporary) measurement or as a check. The main difficulty of the method is the accurate determination of the ratio of the mass-absorption coefficients, which differ from unity by as much as 11 % for graphite and water. Nevertheless, experience shows that the results obtained by this particular method are of the same level of accuracy, in the present state of the art, as those used in international comparisons (BIPM, 1988; Shortt et al., 1992). This is confirmed by Fig. 2 which shows worldwide agreement to be at the 1 % level.
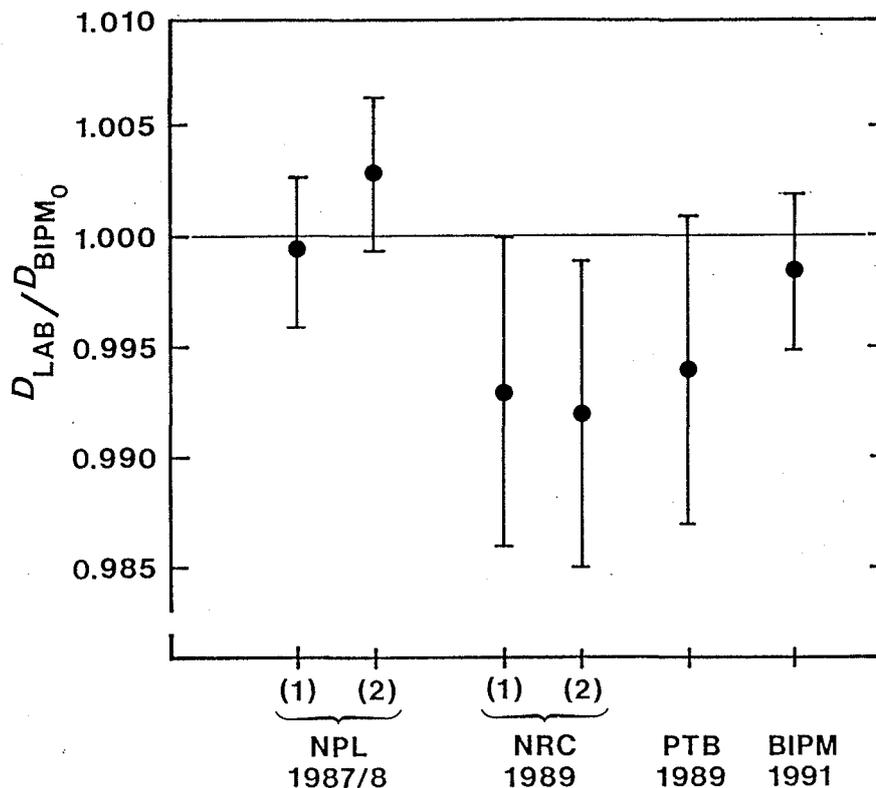
Fig. 2. Results of indirect comparisons of absorbed dose to water (^{60}Co).
The bars represent the relative uncertainty of D_{LAB} .

Determination of D_w:

$BIPM_0$	absolute measurements by ionometry (^{60}Co)
BIPM	scaling theorem from graphite ionometry present measurement (^{60}Co)
NPL (1) and (2)	scaling theorem from graphite calorimetry (^{60}Co)
NRC (1)	Fricke dosimetry from water calorimetry (20 MeV)
NRC (2)	ionometric transfer from graphite calorimetry (^{60}Co)
PTB	Fricke dosimetry from total absorption (5.6 MeV electrons).

The transfer instruments used by the national laboratories for the comparison were ionization chambers, except for NPL (2) which used a Fricke solution.

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