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A new approach to the Eötvös experiment

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Abstract

A new method of performing the Eötvös experiment is suggested which uses a superconducting suspension. Its feasibility and advantages are discussed.

We have reviewed[1] the recent experimental efforts aimed at establishing the weak equivalence principle at ever higher levels of precision and the searches for new forces. We wish here to propose a new method for performing such experiments which uses a superconducting suspension. As will be seen below this offers the advantages of a high data rate, high Q (low thermal noise) together with high thermal and mechanical stability.

Figure 1 shows a schematic of the apparatus. The float is fabricated in the form of a cylinder or cone from halves of copper and

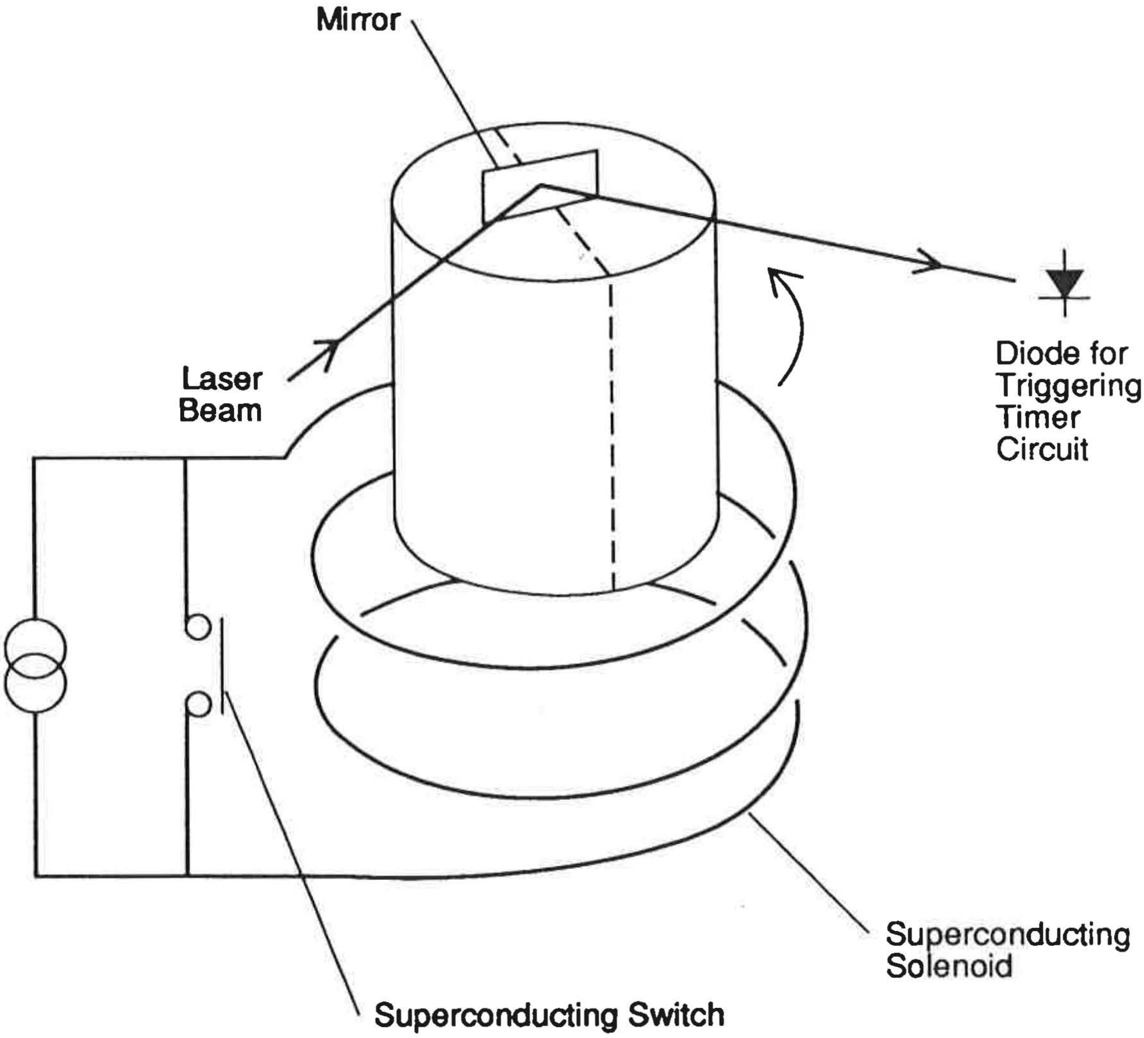

Fig. 1.- A schematic of the proposed experimental apparatus. The float is constructed from copper and beryllium, say, and coated with lead.

beryllium, say, and is coated with lead. It is suspended in the field due to the persistent current flowing through a superconducting solenoid. The float is constructed so that its center of mass coincides with the center of magnetic buoyancy and its geometry is chosen so that the magnetic "metacentric" height is located above the center of mass. A small mirror, reflecting on both sides, is fixed to the top of the float. The float is made to slowly rotate about a vertical axis and laser beams and photodiodes are used to time the rotation through successive angles of 180°. Before discussing some of the possible sources of uncertainty in the experiment we will give a simple theory of the dynamics of the float. We shall limit our discussion below to a test of the weak equivalence principle of the type undertaken by Eötvös, although the generalization to searches for finite-range, composition-dependent forces is straightforward and would also be envisioned to be part of the scientific program that could be carried out.

The float can be considered as a torsion balance suspended at the magnetic metacenter and hanging with its center of mass vertically below. The torsion constant of the float will be extremely small and will be defined by departures from azimuthal symmetry of the float and coil and by any trapped flus in the float. For simplicity, consider the torsion constant to be zero, then in that case the rotation of the float will be described by the following equation

$$I\ddot{\phi} + \Gamma \cos \phi = 0 \qquad (1)$$

Where I is the moment of inertia of the float and all losses have been ignored. In reference 1 we derived an expression for a composition dependent torque in the Eötvös experiment and found

$$\Gamma = \Omega^2 R_\oplus \frac{\sin 2\theta}{2} mr\, \Delta\kappa \qquad (2)$$

where Ω is the spin angular velocity of the earth and $R_\oplus$ is its radius. The latitude of the location of the experiment is θ and 2r is the centre of mass separation of the two test objects of mass m. Each test mass experiences a static gravitational acceleration

$$g_{ni} = (1 + \kappa_i) \frac{Gm_\oplus}{R_\oplus^2} \qquad (3)$$

and $\Delta\kappa$ is therefore the fractional difference in the gravitational acceleration which is experienced by each object. The solution to equation 1 is expressed in terms of elliptical integrals but in the limit that the energy associated with the external torque is small in comparison to the initial kinetic energy of rotation, we have a solution:

$$\phi = \omega_0 t + \phi_0 \cos \omega_0 t \qquad (4)$$

where ω_0 is the angular rotation rate and

$$\phi_0 = \frac{\Gamma}{I\omega_0^2}. \qquad (5)$$

The times for orientations 180° apart is given as $T/2 \pm \delta$ where T is the rotation period. For small values of $\phi_0 = \omega_0 \delta$, we have

$$\delta = \frac{\Gamma}{I\omega_0{}^3} \qquad (6)$$

or

$$\delta \sim \frac{\Delta\kappa}{r} \frac{\Omega^2 R_\oplus}{\omega_0^3} \qquad (7)$$

where r is the radius of gyration of the float about the vertical axis. The difference between successive time intervals of angular motion of 180° will be 4δ. If we take $\Delta\kappa$ as 10^{-12}, r as 1 cm, and a rotation period of 10 minutes (see below), we find a total time assymetry of a few μs which is easily measured.

Discussion

In order to develop the above conceptual model into a successful experiment many problems will have to be overcome.

1. The most serious problems will be associated with trapped flux. In practice, even with type I superconductors, it is difficult to achieve a perfect Meissner state and usually a small fraction (α) of the magnetic flux density (B) which threads the float above the superconducting phase transition remains trapped. We can associate a permanent magnetic dipole moment (M) with this remaining fields,

$$M = 2\,\alpha \frac{B}{\mu_0} V \qquad (8)$$

where V is approximately equal to the volume of the float. The ambient field will then create a torque of magnitude

$$\Gamma_m = \frac{2\alpha B^2 V}{\mu_0} \qquad (9)$$

which will mimic a violation of the equivalence principle if it has a vertical component and appropriate phase. The only sure way of eliminating this problem is to create an ultra-low magnetic field region around the float before it undergoes the superconducting phase transition. Comparing equations (2) and (9), assuming that $\alpha \sim 10^{-2}$ (see Ref. 2), $\Delta\kappa \sim 10^{-12}$, $m = 1$ g and $V \sim 1$ cm^3, we find that $B \approx 3 \times 10^{-9}$ T (3γ). Magnetic flux densities of 10^{-12} T have been achieved over large volumes (10^{-2} m^3) by Cabrera[3] using expandable superconducting shielding and 10^{-10} T can be achieved using nested μ-metal shielding. Therefore this problem can be avoided. It would be preferable to use type I superconductors to minimize the residual trapped flux, however as the critical fields of type I materials are less than those for type II, this would restrict the mass of the float. There are many equivalent ways of calculating the buoyancy force on a float[(5-7)]; perhaps the most straightforward method is to find the change in magnetic energy in the float-coil system for a small vertical displacement. The mass which can be suspended is then given as

$$m = \frac{1}{g}\frac{d}{dz}\int \frac{B^2}{2\mu_0}\, d^3 r. \qquad (10)$$

Typically, a field of 100 gauss is required to support a few grams of test mass and the critical field of lead (type I) and niobium (type II) are 500 gauss and 1.6 kgauss respectively at 4 K. If the float is not constructed completely of superconductor then the critical field decreases, for example, for a float with an outer conducting wall equal to 10% of its outer radius, the critical field is reduced by about 50%[(3)].

2. The magnitude of the timing asymmetry varies as the cube of the rotation period and therefore the feasibility of the experiment depends critically on the possibility of achieving long periods. The lowest rotation rate will depend on the residual torques on the float: if a residual magnetic torque Γ_m, for example, acts then the minimum rotation rate will be set by the kinetic energy required to escape the potential well due to this torque. Again with a residual magnetic torque Γ_m of the order of the signal torque of $\Delta\kappa = 10^{-12}$, we have

$$\frac{1}{2} I\omega_0^{\,2} \sim \Gamma_m \qquad (11)$$

or a rotation period of the order of one month ! Our original estimate of 10 minutes would then appear quite conservative, if, indeed, we can eliminate the residual torques to the required level. Rates of energy loss have been reported[(4)] of 10^{-17} W for small (~ 1 mm diameter), magnetically suspended niobium spheres. If we assume a rotation period of 10 minutes and the parameters given above, the rotation would decay at the rate

$$\frac{d\omega_0}{dt} \sim \frac{10^{-17}}{I\omega_0} \qquad (12)$$

or 10^{-18} rad s^{-2}. The float would complete approximately 10^3 revolutions before coming to rest. Of course a monotonic increase in the time intervals for successive 180° rotations can be easily eliminated by fitting procedures.

3. In the argument above we assume that the two surfaces of the mirror were perfectly parallel; of course this will never be the case. However, by using two timing systems approximately 180° apart which use both sides of the mirror and averaging their time-difference signals this error is eliminated.

4. Any asymmetries in the magnetic field due to the geometry of the coil and float will probably give rise to torques which are larger than the signal torque. These can be investigated by rotation of the coil in the laboratory frame and it may prove necessary to use three or four pairs of mirror surfaces to distinguish these torques from any true signal.

5. Torques due to ground vibration are minimized by ensuring that the center of mass is located at the center of magnetic buoyancy.

6. The effect of local gravity gradients can be investigated by changing the local mass distribution and by performing the experiment with a float fabricated from one material with a known exaggerated quadrupole moment.

7. The float cannot be electrically grounded and thus will acquire electric charge. Again, departures from azimuthal symmetry will lead to electrostatic torques and, again, rotation of the experiment relative to the east-west direction can be used to discriminate between such torques and a true signal.

8. The design of stable coil-float systems has been discussed by many authors[4-7].

Conclusion

It appears that the above scheme is a feasible way of increasing the precision of the Eötvös experiment. The advantages associated with the high modulation frequency, high uniformity of rotation, low torsion constant and inherent mechanical and thermal stability of the superconducting system make this approach attractive despite the technological complexity.

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