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Results of the 1988 NRC absolute and relative frequency measurements carried out on portable (He-Ne)/CH_4 lasers.

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Abstract

This note reports on a comparison of the performance of the frequency multiplication chains from the VNIIFTRI (Russian acronym of the State Research Institute of Physical - Technical and Radiotechnical Measurement), and the National Research Council of Canada (NRC), carried out at the NRC in Nov./Dec. 1988. This comparison was done by measuring the absolute frequency of a portable laser from the VNIIFTRI on both chains in turn. In order to avoid any ambiguity in the exploitation of the results, the absolute frequency of one of the two portable BIPM lasers was measured at the same time. Relative frequency measurements made between the three transportable lasers allowed these new results to be linked with others obtained, with the same lasers, at different places during the past five years.

1 Introduction

From Nov. 23rd to Dec. 9th, 1988, a series of frequency measurements were made at the NRC laboratory in order to compare the performance of the frequency-multiplication chains of the NRC, [1], with those of the VNIIFTRI, [2]. This was done by measuring the frequency (f=88 THz) of a transportable (He-Ne)/CH_4 laser from the VNIIFTRI, called M101, [3], on each chain in turn. The BIPM participated as a witness to this experiment using two systems similar to M101: B.3, [4, 5], and VB, [3]. A number of absolute frequency determinations were carried out on M101 and VB. Relative frequency measurements between M101 and both BIPM lasers, were also carried out so as to secure a link between these new measurements and those carried out with the same lasers in other laboratories.
Additional systematic measurements of the modulation and power effects (frequency shifts due to the respective changes in the amplitude of the modulation needed to stabilize the laser and in the intra cavity power density) were made on the BIPM lasers. In this report, we have compiled only those results which concern the measurements involving BIPM lasers.

2 Relative frequency measurements

2.1 General remarks

During the beat frequency measurements which were carried out between M101 and the two portable BIPM lasers, VB and B.3, the offset frequency between the master laser and the slave one of the VNIIFTRI device was determined twenty times. This frequency it appears was not as stable as one could expect since its mean was found to be (5000010,5 ± 56) Hz, where 56 Hz is the standard deviation for one measurement. This result perfectly reflects the frequency jumps of about 50 Hz that often observed between consecutive determinations of the offset frequency. These fluctuations, which could be due to the counting set-up, may explain some data which is too distant from the fitting curves that calculated for the VB-modulation and power effects.
In this paper, the modulation widths given refer to peak-to-peak values.

2.2 Measurement procedure

Each experimental datum used to draw the modulation and power-effect curves, is the mean of a series of twenty consecutive beat-frequency measurements with a 10 s sample time. In the curve-fitting calculation we made from the population of these experimental data, each experimental datum is weighted by $1/s_i^2$ where s_i is the corresponding standard deviation for one measurement in one series. We calculated, by the least-squares method, fitting curves having a parabolic form [$Y=a_0+a_1x+a_2x^2$] or, when it was possible, a hyperbolic one [$Y=a+b/(x+c)$]. This allowed us to determine the best values of the modulation and power-effect coefficients for a given set of operating conditions and, by using these coefficients, to estimate the uncertainty in the emitted frequency resulting from these parameters.

2.3 Results obtained with VB

2.3.1 Modulation effect

The modulation effect was measured for two values of the discharge current in the laser tube (2,0 and 3,5 mA). The results are interesting inasmuch as the curves of Fig. 1 show that there probably exists a value of the discharge current for which the emitted frequency would be insensitive to modulation widths greater than 500 kHz. This behaviour has already been observed on B.3 and on B.6 (another BIPM reference laser, [4, 5],), [6], and will be further investigated to improve the frequency stability of the laser.
From the experimental results given in Tables 1, 2, 3, we have determined the modulation-effect coefficients for modulation widths of 0,2 and 1,0 MHz, supposing the hyperbolic form to be an adequate model for the 3,5 mA-experimental data populations. The resulting values, as well as the corresponding uncertainties on the frequency emitted by the laser, are given in the following table.

Discharge current/mA	Operating mod. width/kHz	Mod. effect coefficients/(Hz/kHz)	Mod. width uncertainty/kHz	Frequency uncertainty/Hz
3,5	200	-14	10	140
3,5	1000	-1	50	50

2.3.2 Power effect

Figure 2 shows the change of frequency for different values of the discharge current in the laser tube. The modulation width was maintained at 1,0 MHz. The experimental results are given in Tables 4 and 5. By treating the hyperbolic form as giving the best fit to the data, the dependence on the power of the emitted frequency, at around the 1,0 MHz operating modulation width, is found to be -705 Hz/mA. As the discharge current was maintained within ± 20 μA, this leads to a resulting uncertainty of ± 14 Hz in the emitted laser frequency.

2.3.3 Short-term frequency stability

By considering the beat frequency measurements made with respect to M101, we have obtained for both lasers a relative Allan standard deviation of $1{,}8\times10^{-13}$ for a 10 s sample time and of $1{,}4\times10^{-13}$ for a 100 s sample time. These frequency stabilities are excellent and were not altered when the modulation width of VB was decreased to 200 kHz.

2.3.4 Mid-term frequency repeatability

Taking into account the above modulation and power effects, the error budget can be expressed as in the following table:

Influencing parameter	Resulting uncertainty/Hz.	
	Modulation width/kHz	
	200	1000
Modulation width	140	50
Discharge current in the laser tube	14	14
Zero servo locking	70	10
M101 fluctuations	56	56
Total uncertainty	167	77

We have to compare these total uncertainties with the results we obtained on the (M101-VB) frequency differences when VB was used under normal operating conditions (modulation width 1,0 MHz; discharge current in the laser tube: 3,5 mA).
We obtained:

December 1st. a.m. $f_{88}(\text{M101})-f_t(\text{VB}) = +(851 \pm 19)$ Hz
p.m. $f_{88}(\text{M101})-f_t(\text{VB}) = +(807 \pm 22)$ Hz

December 2nd. a.m. $f_{88}(\text{M101})-f_t(\text{VB}) = +(775 \pm 31)$ Hz

where the index t means that the photodetector was placed on the tube-side end of the resonator. The second number in brackets is the standard deviation of one measurement in one series, each series comprising twenty measurements. This leads to the mean frequency difference:

$$f_{88}(\text{M101})-f_t(\text{VB}) = +(811 \pm 38) \text{ Hz}$$

where the second number in brackets is now the standard deviation for one determination in a series of three determinations. This standard deviation suggests that the total uncertainty given above in the table is pessimistic, although they have both the same order of magnitude.

2.4 Results obtained with B.3

2.4.1 Modulation effect

The modulation effect was determined for the normal discharge current-operating conditions of the laser (5,0 mA). The experimental results are given in Table 6. We have, of course, only considered the parabolic form for the fitting curve of Fig. 3. That leads to a modulation-effect coefficient of -40 Hz/100 kHz around the 1,0 MHz modulation width. The resulting uncertainty in the emitted frequency is ± 20 Hz.

2.4.2 Power effect

The last morning, just before our Russian colleagues left the NRC, we made a quick measurement of the power effect for only three values of the discharge current in the laser tube at a 1,0 MHz modulation width. We found a power effect coefficient of -340 Hz/ mA. This gives a resulting uncertainty in the emitted frequency of ± 7 Hz.

2.4.3 Short-term frequency stability

From the beat-frequency measurements we made we can estimate the relative Allan standard deviation as being $5{,}8 \times 10^{-13}$ for a 10 s sample time. It is not as good as found for VB and is mainly due to particular features of B.3: 1) the higher pressure of methane; 2) the lower contrast of the methane peak.

2.4.4 Mid-term frequency repeatability

Three determinations of the frequency difference (M101-B.3) were made in the normal operating conditions of B.3 (modulation width 1,0 MHz, discharge current in the laser tube 5,0 mA). We found:

December 5th,	a.m.	f_{88}(M101)-f_t(B.3)	= + (660 ± 64)	Hz
	p.m.	f_{88}(M101)-f_t(B.3)	= + (735 ± 69)	Hz
December 6th,	a.m.	f_{88}(M101)-f_t(B.3)	= + (740 ± 66)	Hz

This gives the mean-frequency difference:

$$f_{88}(\text{M101})-f_t(\text{B.3}) = +(712 \pm 45)\ \text{Hz}$$

where the second number in brackets represents the standard deviation for one determination in one series of three determinations. The index t has the same meaning as before and the index 88 indicates the year of the measurements. By comparing this result with that of the following table, just as the results obtained on VB, the calculated total uncertainty is greater than the standard deviation for one determination.

Influencing parameter	Resulting uncertainty/Hz
Modulation width	20
Discharge current in the laser tube	7
Zero servo locking	100
M101 fluctuations	56
Total uncertainty	117

2.4.5 Long-term frequency repeatability

Laser B.3 has been used in all international comparisons of (He-Ne)/CH_4 systems in the past ten years and, in particular, was used with M101 in the last absolute frequency determinations carried out at the LPTF (Paris) in 1986 [2]. Our Russian colleagues gave us at the NRC, the change in the frequency of M101 with respect to the 1986 measurements. They obtained:

$$f_{86}(\text{M 101}) - f_{88}(\text{M 101}) = +(420 \pm 180)\ \text{Hz}.$$

From the 1986 LPTF measurements we have:

$$f_{86}(\text{M 101}) - f_{t,86}(\text{B.3}) = +(1250 \pm 110)\ \text{Hz}.$$

That gives: $$f_{88}(\text{M 101}) - f_{t,86}(\text{B.3}) = +(830 \pm 210)\ \text{Hz}.$$

From the NRC relative measurements we obtained:

$$f_{88}(\text{M 101}) - f_{t,88}(\text{B.3}) = +(712 \pm 45)\ \text{Hz}.$$

We can then deduce:

$$f_{t,88}(\text{B.3}) - f_{t,86}(\text{B.3}) = +(118 \pm 215)\ \text{Hz}.$$

The indexes 86 and 88 indicate the year of the measurements. The emitted frequency of B.3 is not significantly different from those at the LPTF in 1986. So as the frequency of B.3 obtained in 1986 was close to that found at the LPTF in 1983, we can conclude that the frequency of B.3 has been maintained to within 1 kHz for five years. This we consider to be very good.

3 Absolute frequency measurements

Laser VB was used for the determination of absolute frequencies with a modulation width of 200 kHz and a discharge current of 3,5 mA. By considering the modulation-effect results (see section 2.3.1), we have:

$$f_{t,0.2}(VB)-f_{t,1.0}(VB) \quad = +(3000 \pm 140)\ \text{Hz}$$

where the second number in brackets is the standard deviation for one measurement given in Table 3. The indexes 0.2 and 1.0 mean that the modulation widths were respectively of 0,2 and 1,0 MHz. To compare the absolute frequency chains we prefer to use the total uncertainty given in the error budget. We find then:

$$f_{t,0.2}(VB)-f_{t,1.0}(VB) \quad = +(3000 \pm 170)\ \text{Hz}.$$

From the relative determinations carried out at NRC we have:

$$f_{88}(M101)-f_{t,1.0}(VB) \quad = +(810 \pm 40)\ \text{Hz},$$

which leads to:

$$f_{t,0.2}(VB)-f_{88}(M101) \quad = +(2190 \pm 170)\ \text{Hz}.$$

By taking into account the M101 absolute frequency determinations carried out at the VNIIFTRI just before the NRC experiment:

$$f_{88}(M101) \quad = (88376181601{,}52 \pm 0{,}1)\ \text{kHz}.$$

We obtain for VB:

$$f_{t,0.2}(VB) \quad = (88376181603{,}71 \pm 0{,}2)\ \text{kHz},$$

while from the NRC chain we have:

$$f_{t,0.2}(VB) \quad = (88376181603{,}25 \pm 1{,}04)\ \text{kHz}$$

and: $$f_{t,0.2}(VB) \quad = (88376181603{,}08 \pm 0{,}95)\ \text{kHz}.$$

These values do not differ significantly. They indicate that there are no important systematic errors in either the VNIIFTRI chain or in the part of the NRC chain used for the measurements of the absolute frequency of VB.

4 Conclusion

The experiment carried out at the NRC was interesting in that from the results we can deduce the following:

- The BIPM is able to keep the long-term frequency stability of its transportable (He-Ne)/CH_4 systems within 1 kHz (1 part in 10^{+11} of its value). This is of great value considering the role it has to play in the international comparisons.
- The VNIIFTRI and BIPM reference lasers exhibit frequency differences within 1 kHz. This demonstrates that these standards would be very reproducible if practical rules could be established concerning their conditions of operation.
- The NRC chain does not show to present important systematic errors although if square feature obtained in the data population of the M101 absolute frequency determinations has yet to be explained.

References

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Figure captions and Tables

Fig. 1 Modulation effect in VB for two values of the discharge current in the laser tube. Horizontal axis: modulation width in kHz. Vertical axis: frequency difference with respect to M101, in kHz.

Fig. 2 Power effect in VB for a 1,0 MHz modulation width. Horizontal axis: exciting current in the discharge tube, in Ma. Vertical axis: frequency difference with respect to M101, in kHz.

Fig. 3 Modulation effect in B.3 for a 5,0 mA discharge current in the laser tube. Horizontal axis: modulation width in kHz. Vertical axis: frequency difference with respect to M101, in kHz. The solid line represents a parabolic function fitted to the data.

In the following tables $f_{meas.}$ is the mean value of a series of twenty consecutive beat-frequency measurements done with respect to M101, (sample time, 10 s); s_i is the corresponding standard deviation of one measurement; $f_{calc.}$ is the corresponding value either of the parabola or of the hyperbola-type fitting curve obtained by the least-squares method.

Table 1 Experimental results of the study of the VB modulation effect at a discharge current of 2,0 mA. Parabola-type fitting curve.

Table 2 Experimental results of the study of the VB modulation effect at a discharge current of 3,5 mA. Parabola-type fitting curve.

Table 3 Experimental results of the study of the VB modulation effect at a discharge current of 3,5 mA. Hyperbola-type fitting curve.

Table 4 Experimental results of the study of the VB power effect at a modulation width of 1,0 MHz. Parabola-type fitting curve.

Table 5 Experimental results of the study of the VB power effect at a modulation width of 1,0 MHz. Hyperbola-type fitting curve.

Table 6 Experimental results of the study of the B.3 modulation effect at a discharge current of 5,0 mA. Parabola-type fitting curve.

Table 1.

Modul. width/kHz	$f_{meas.}$/Hz	s_i/Hz	$(f_{calc.}-f_{meas.})$/Hz
1000	1209	24,3	21,6
900	888	20,6	-24,4
800	560	22,8	-4,2
700	293	21,2	14,3
600	103	22,3	14,9
500	32	20,6	-44,2
400	-101	16,0	17,9
300	-126	28,6	31,2
200	34	46,0	-81,2
1000	1228	39,0	2,6

Parabola coefficients		Standard deviation of coefficients	
a_0=	225,5 Hz	$s[a_0]$=	81,8 Hz
a_1=	-1,956	$s[a_1]$=	0,275
a_2=	0,0030 Hz^{-1}	$s[a_2]$=	0,00021 Hz^{-1}

Standard deviation for one measurement: s=31,8 Hz

Table 2.

Modul. width/kHz	$f_{meas.}$/Hz	s_i/Hz	$(f_{calc.}-f_{meas.})$/Hz
1000	-851	15,0	144,6
900	-569	13,0	-74,5
800	-383	22,0	-121,6
700	-82	17,0	-207,7
600	130	14,0	-128,7
500	345	12,0	23,3
400	602	14,0	209,4
300	1173	12,0	157,4
200	2416	29,0	-490,5
1000	-759	17,0	52,6
100	5125	86,0	2528,3
100	4253	103,0	1656,3

Parabola coefficients

a_0= 3343,8 Hz
a_1= -7,852
a_2= 0,0038 Hz^{-1}

Standard deviation of coefficients

$s[a_0]$= 507 Hz
$s[a_1]$= 1,77
$s[a_2]$= 0,00136 Hz^{-1}

Standard deviation for one measurement: s=256,2 Hz

Table 3.

Modul. width/kHz	$f_{meas.}$/Hz	s_i/Hz	$(f_{calc.}-f_{meas.})$/Hz
1000	-851	15,0	181,3
900	-569	13,0	9,9
800	-383	22,0	-40,7
700	-82	17,0	-172,3
600	130	14,0	-165,8
500	345	12,0	-88,6
400	602	14,0	65,5
300	1173	12,0	115,2
200	2416	29,0	-82,1
1000	-759	17,0	89,3
100	5125	86,0	-656,7
100	4253	103,0	215,3

Hyperbola coefficients		Standard deviation of coefficients	
$a=$	-1766,3 Hz	$s[a]=$	203.0 Hz
$b=$	$1{,}198*10^6$ Hz^2	$s[b]=$	208468 Hz^2
$c=$	92,1 Hz	$s[c]=$	40.21 Hz

Standard deviation for one measurement: s=140,1 Hz

Table 4.

Discharge current/mA	$f_{meas.}$/Hz	s_i/Hz	$(f_{calc.}-f_{meas.})$/Hz
3,5	- 809	17,2	58,2
4,5	-1349	12,6	-71,6
5,5	-1637	22,1	51,4
5,5	-1696	17,0	110,4
5,0	-1425	18,4	-141,2
4,0	-1067	26,9	-81,8
3,5	- 843	20,5	92,2
3,5	- 715	31,0	-35,8
3,0	- 286	26,7	59,4
2,5	176	22,3	247,9
2,0	1528	26,5	-327,5

Parabola coefficients			Standard deviation of coefficients		
a_0=	5569,1	Hz	$s[a_0]$=	640	Hz
a_1=	-2689,1		$s[a_1]$=	335	
a_2=	252,4	Hz^{-1}	$s[a_2]$=	41,9	Hz^{-1}

Standard deviation for one measurement: s=151,3 Hz

Table 5.

Discharge current/mA	$f_{meas.}$/Hz	s_i/Hz	$(f_{calc.}-f_{meas.})$/Hz
3,5	- 809	17,2	-5,6
4,5	-1349	12,6	15,8
5,5	-1637	22,1	2,3
5,5	-1696	17,0	61,3
5,0	-1425	18,4	-76,6
4,0	-1067	26,9	-46,5
3,5	- 843	20,5	28,4
3,5	- 715	31,0	-99,6
3,0	- 286	26,7	-98,4
2,5	176	22,3	112,0
2,0	1528	26,5	-40,9

Hyperbola coefficients			Standard deviation of coefficients		
a=	-2773,1	Hz	$s[a]$=	141.797	Hz
b=	5437,5	Hz^2	$s[b]$=	09.73.13	Hz^2
c=	-0,724	Hz	$s[c]$=	93	Hz

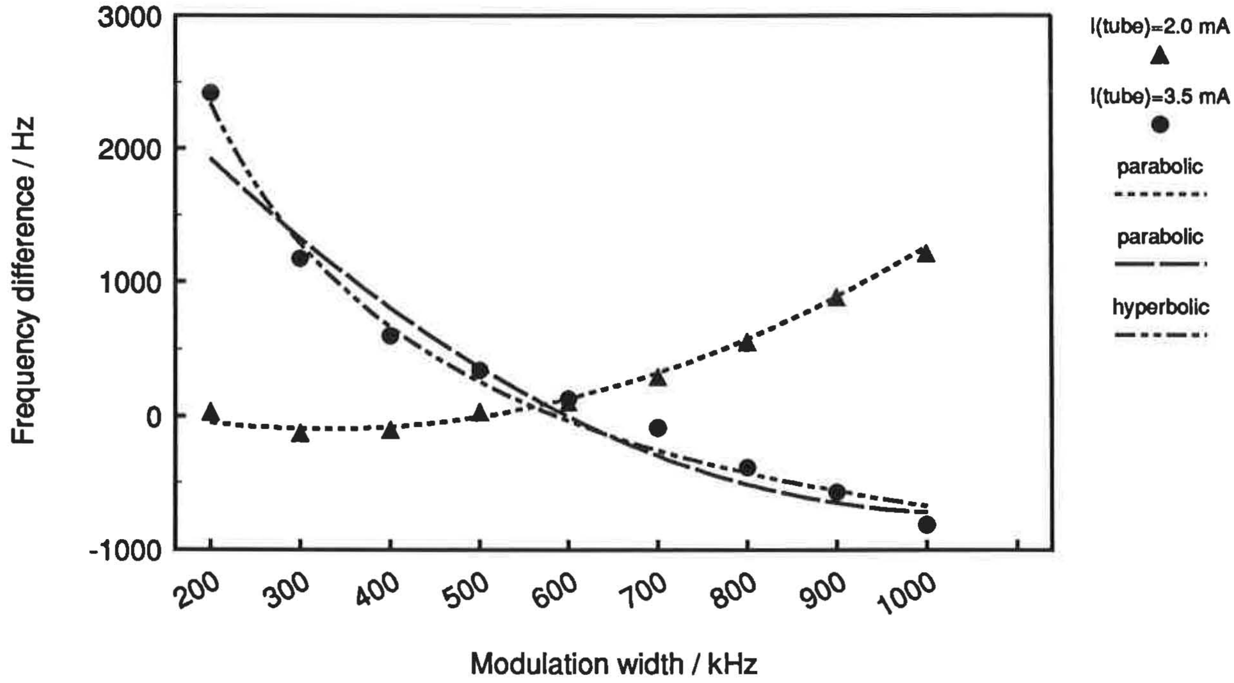
Standard deviation for one measurement: s=66,15 Hz

Table 6.

Modul. width/kHz	$f_{meas.}$/Hz	s_i/Hz	$(f_{calc.}-f_{meas.})$/Hz
1000	-740	48,7	-32,8
800	-695	49,6	-24,4
600	-690	82,4	-31,4
400	-958	133,6	179,2
1200	-852	45,9	-29,6
1400	-1101	64,7	55,2
1600	-1292	44,3	26,6
1800	-1558	76,1	17,7
2000	-1835	57,3	-35,7
1000	-813	54,7	40,2

Parabola coefficients			Standard deviation of coefficients		
a_0=	-1059,8	Hz	$s[a_0]$=	140	Hz
a_1=	0,9794		$s[a_1]$=	0,227	
a_2=	-0,00069	Hz^{-1}	$s[a_2]$=	0,00008	Hz^{-1}

Standard deviation for one measurement: s=48,9 Hz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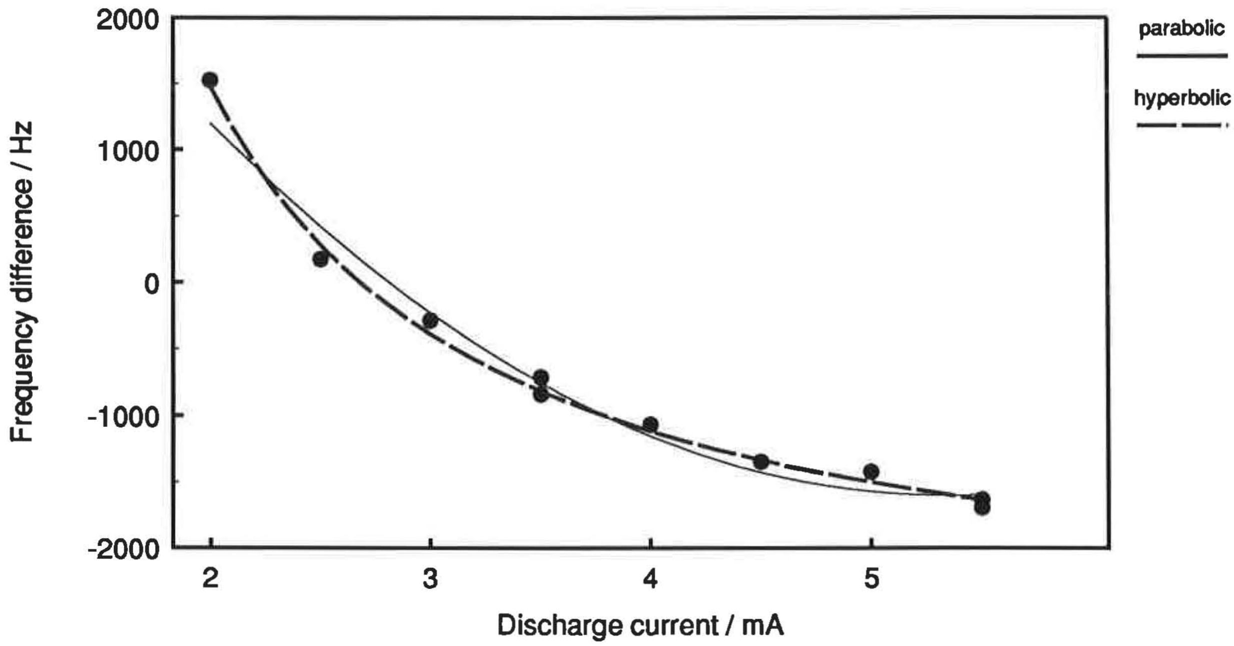
2000
1000
0
-1000
Frequency difference / Hz
200
300
400
500
600
700
800
900
1000
Modulation width / kHz
I(tube)=2.0 mA
I(tube)=3.5 mA
parabolic
parabolic
hyperbolic

Fig.1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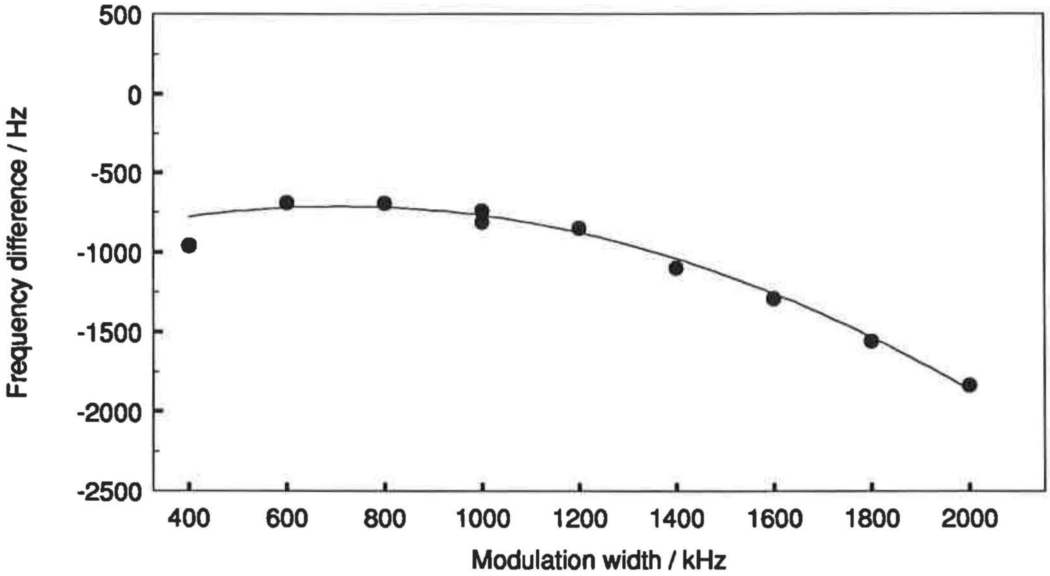
-1000
-2000
Frequency difference / Hz
2
3
4
5
Discharge current / mA
parabolic
hyperbolic

Fig.2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0
-500
-1000
-1500
-2000
-2500
Frequency difference / Hz
400
600
800
1000
1200
1400
1600
1800
2000
Modulation width / kHz

Fig.3