Rapport BIPM-2004/04

Bureau International des Poids et Mesures

Determining the temperature coefficient of remanent magnetization for the magnets used to servocontrol the FB-2 balance

R.S. Davis
BIPM

March 2004

Pavillon de Breteuil, F-92312 Sèvres cedex

Determining the temperature coefficient of remanent magnetization for the magnets used to servocontrol the FB-2 balance

by R.S. Davis

Bureau International des Poids et Mesures, F-92312 Sèvres cedex

Abstract
An experiment was devised to determine the reversible temperature coefficient of remanance, α, for small magnets operating near room temperature. Such magnets are used to servocontrol the FB-2 balance. The manufacturer specifies that α is within the limits $\pm 50 \times 10^{-6}$/°C and this has now been confirmed. The analysis also reveals an ambiguity in the customary specification of α.

1. Introduction

The 1 kg mass comparator known as the BIPM FB-2 balance [1] uses electromagnetic servocontrol to maintain the balance beam at a fixed angle. A cylindrical magnet having a diameter of 3 mm and height of 2 mm is fixed to either end of the beam. Torque is produced by passing current through coils fixed to the laboratory reference frame and located above each magnet. The coil and magnet axes are aligned. The change in magnetic moment as a function of the temperature of the permanent magnets is thus an important design parameter.

The reversible temperature coefficient, α, of magnet's remanence, B_r, may be defined to be [2] :

$$\alpha = \frac{1}{B_r}\frac{\partial B_r}{\partial T} \tag{1}$$

where T is the temperature. The magnets used for servocontrol of the FB-2 balance are specified as having α between the limits $\pm 50 \times 10^{-6}$/°C in the range –40 °C to +80 °C but this is achieved at the expense of reduced remanence (typically $B_r = 0.64$ T, according to the manufacturer's specifications), compared with pure $SmCo_5$. The very small temperature coefficient is attained by the substitution of a heavy rare earth element (HRE) for some of the samarium [3]. Note that only the differential temperature appears in (1) so that the units may be either °C or K.

Some results obtained using the FB-2 balance raised the possibility that the magnets used for servocontrol might have a thermal coefficient of order -200×10^{-6}/°C, instead of that specified by the manufacturer; i.e. roughly half the coefficient of magnets made of pure

$SmCo_5$. In order to test this hypothesis, we undertook a measurement of α between 22 °C and 40 °C. This report describes these measurements, which will be seen to confirm the manufacturer's specification, although there is some doubt about whether thermal expansion effects are to be included in α (see Section 2.2).

We describe the basic principle of the measurement, give important experimental details and show the final results. All equations are written for quantities expressed in SI units.

2. Principle of the measurement

2.1 *Basic equations*

We determine the change in magnetic force between two uniformly magnetized cylindrical magnets when both are placed on the same axis at a fixed distance L (figure 1). In the first measurement of force F_1, the magnets are both at temperature T. For the second force measurement F_2, the temperature of the upper magnet, B, is increased to $T+\Delta T$. For simplicity, we first suppose that the distance L is much greater than the dimensions of either magnet so that we may use the dipole approximation,

$$F_1 = \frac{6\mu_0}{4\pi L^4} m_\mathrm{A} M_\mathrm{B} V_{0,\mathrm{B}} \tag{2}$$

$$F_2 = F_1\left(1 + \frac{1}{M_\mathrm{B}}\frac{\partial M_\mathrm{B}}{\partial T}\Delta T + \beta_V \Delta T\right) \tag{3}$$

where m_A is the magnetic moment of A, M_B is the average magnetization of B over its total volume, $V_{0,\mathrm{B}}$ is the volume of B at the ambient temperature of the room and β_V is the volumetric thermal expansion of B. The last parameter, about $+30 \times 10^{-6}$/°C, can be computed from the published linear expansion coefficients for this type of magnet, taking account of anisotropy. The parameter μ_0 is the magnetic constant, $4\pi \times 10^{-7}$ N/A^2. We note that the external axial field strength to which magnet B is subjected is negligible: -20 A/m from magnet A and +40 A/m from the ambient field in the laboratory.

We limit ΔT to less than 20 °C in order to make the following approximation:

$$\frac{1}{M_\mathrm{B}}\frac{\partial M_\mathrm{B}}{\partial T} \approx \frac{1}{F_1}\frac{F_2 - F_1}{\Delta T} - \beta_V \,. \tag{4}$$

The magnet under test (magnet B) has an aspect ratio (height/diameter) of 2. Therefore, $B_\mathrm{r} \equiv \mu_0 M_\mathrm{r} \approx \mu_0 M$. This follows from the fact that the M vs. H curve is practically horizontal starting from H=0 and extending far into the second quadrant [3]. The operating point of the magnet is the intersection of this curve with the load-line, a line passing through the origin with slope about –5.6 for a cylinder whose height is twice its diameter [4]. This means that the self-demagnetizing field is about –91 kA/m, or only 4 % of the intrinsic coercivity given by the manufacturer. Thus the right-hand side of (4) gives us a reasonable estimate of α between T and $T+\Delta T$. A more exact analysis, used to calculate all results, assumes uniformly magnetized cylindrical magnets, but this does not change (4).

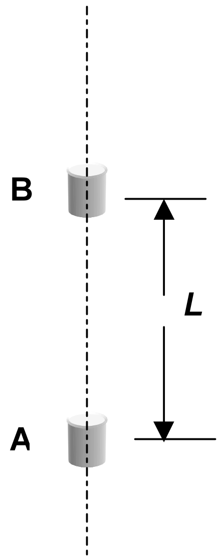

Figure 1. Basic geometry of the force measurement between two cylindrical magnets.

It should be noted that, for the FB-2 balance, it is the coefficient $\alpha + \beta_V$ that characterizes the reduction of the servocontrol force with respect temperature. This is because the magnets of the FB-2 balance are in an open magnetic circuit, just as in the experiment outlined above [1].

2.2 *Standard methods for determining α*

There is some ambiguity as to whether the appropriate thermal expansion of the magnet is already included in the values of α quoted by suppliers. For this reason, we review the two principal methods normally used to determine α. The question is less important for pure samarium cobalt magnets, where the quoted values of α are a factor of 5 to 10 times larger in magnitude than typical values of β_V. Even for "low-α" magnets, the question of whether or not to make a separate correction for thermal expansion is a secondary issue.

2.2.1 Hysteresigraph

The description and proper use of a hysteresigraph are given in international normalization standards [5,6]. The sample magnet to be measured is placed in a closed magnet circuit, so that, in the absence of externally applied field strength, the load-line is simply the vertical axis of the M vs. H curve. Magnetic field strength along the axis of the magnet can be increased or decreased by means of an electromagnet. In a typical configuration, the flux within the sample under test is determined by slowly ramping the applied field strength while integrating the voltage induced in a set of coils. As is clear from equation (9) of [5], the hysteresigraph may be used to determine $\mu_0 MA$, where M is the magnetization of the sample at the externally applied value of H and A is the cross-sectional area of the magnet. The question of measuring the thermal properties of the sample is not addressed in [5] and only mentioned briefly in [6]. Both [5] and [6] define A as the cross-sectional area of the sample but it is left to the reader to decide whether it is worthwhile to include the thermal expansion of A when making a series of measurements at different temperatures. Thus it is unclear whether β_A, the thermal expansion of A (approximately 24×10^{-6}/°C for the magnets that we tested), is routinely accounted for in the calculation of α. One supplier of low-α magnets has told us that the thermal expansion of A is ignored in their calculations and that their quoted values of α, some as small as -10×10^{-6}/°C, therefore include the thermal expansion of the cross-sectional area of the measured samples.

2.2.2 Vibrating Sample Magnetometer (VSM)
In this device [7], the sample to be measured forms part of an open magnetic circuit. Thus the VSM measures the magnetic moment of the sample just as we do and, consequently, it is the volumetric thermal expansion β_V that is either subsumed into the measurement of α or corrected. Because of the anisotropy of $SmCo_5$, the value of β_V for the magnets we tested is about 29×10^{-6}/°C, and thus rather similar to the coefficient for the cross-sectional area. The VSM, like the hysteresigraph, has provision for changing the external field strength.

The VSM has advantages over the hysteresigraph for determining temperature effects of magnetic materials over a wide temperature range. Indeed, a number of conference papers report research results using a VSM to obtain temperature coefficients for various intrinsic magnetic properties [8]. Again, it is not explicitly stated in any of the reports we have found whether the volumetric thermal expansion has been taken into account. Certainly, no value for volumetric thermal expansion is stated in such reports.

We now describe how our own measurement has been realized.

3. Experimental details

3.1 *Sample*
Our sample is a stack of three $HRE_xSm_{1-x}Co_5$ magnets from the same lot as used in the FB-2 balance. The cylindrical magnet thus formed is 3 mm in diameter and 6 mm in height. The aspect ratio is, therefore, 2 so that N_m, the magnetometric demagnetizing, factor is 0.18 [4]. The internal demagnetizing field H_d has been taken to be - $N_m M$, which leads to the load-line given in Section 2.1.

3.2 *Sample holder*
The sample is placed in the central cavity of a copper disc, as shown in figure 2. The disc always sits on a glass plate of 3 mm nominal thickness. The top surface of the disc is painted black so that its temperature may be read using a hand-held infrared thermometer [9]. We have assumed an emissivity of 0.95 for the painted surface.

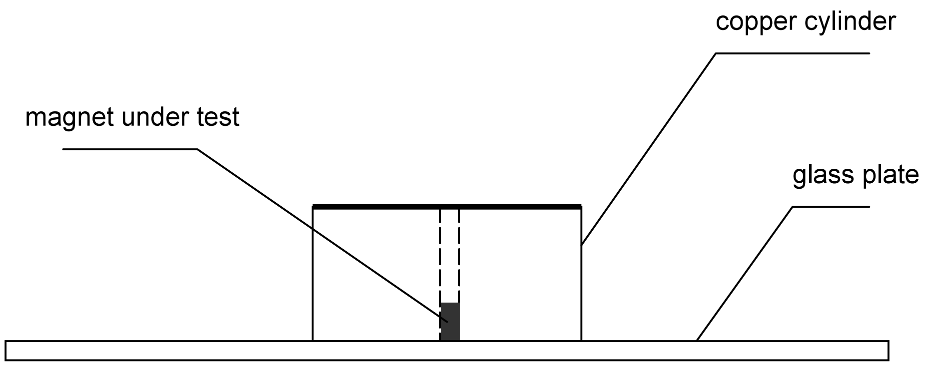

Figure 2. Schematic of sample holder.

For measurements at $T + \Delta T$, the sample holder (including glass plate) was placed on a nearby stand. The vertical surface of the copper was wrapped with a heating element and the temperature was slowly raised until it reached about 40 °C, as determined by the infrared thermometer. The heating element was removed and the sample temperature was read. The sample was then quickly positioned on the balance, a stable balance reading was obtained, and the temperature was again recorded. The second temperature reading is always about 2 °C below the first. The operation was repeated until five force measurements had been made with B at approximately 40 °C.

We assume that there are no significant temperature gradients within the copper and that the temperature read by the infrared thermometer is the same as the temperature of the magnets. Table 1 compares approximate thermal properties of copper, glass, $SmCo_5$ and aluminium.

	Thermal Conductivity/ W/(m·K)	Density/ kg/m^3	Volumetric Heat Capacity/ 10^6 J/(m^3·K)	Thermal diffusivity/ 10^{-8} m^2/s
Copper	390	8960	3.5	11000
Glass	1	2600	2.2	45
$SmCo_5$	11	8400	3.1	350
Aluminium	240	2700	2.4	10000

Table 1. Parameters that influence thermal response. Values are approximate.

3.3 *Force measurements*
The force measurement between two co-linear magnets is made according to [10,11]. Figure 3 shows the apparatus configured to measure susceptibility. For the measurements reported here, the outer support rests on additional 50 mm high aluminium-alloy blocks. The quantity L referred to in this report is defined as $L = Z_0+h_l+h_g+h_B/2$, where h_l is the height of the added blocks (50 mm), h_g is the thickness of the glass plate of the sample holder (3 mm) and h_B is the height of magnet B (6 mm). If the axes of A and B are aligned following the

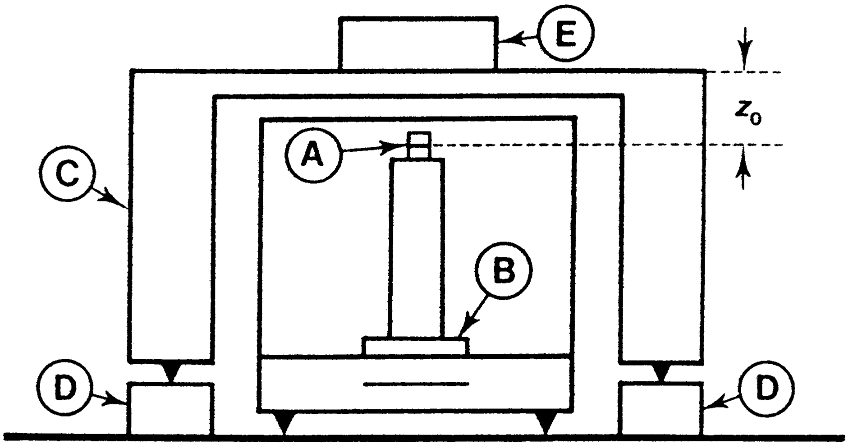

Figure 3. The BIPM susceptometer configured for measuring magnetic susceptibility. A: magnet A; B: balance pan; C: sample support; D: blocks to adjust the height of E with respect to A; E: the sample under test.

procedure of [11], then placement of the upper magnet is only sensitive to second order in δ/L, where δ is a possible displacement of magnet B in the horizontal plane with respect to the axis of magnet A.

Each data point is taken by first setting the balance reading to zero. The sample is then placed to be concentric with circles inscribed on the bridge beneath the glass plate. A stable reading, obtained after 30 s, is recorded. The sample is removed and the balance reading is again recorded. Any change between the first and third balance readings is assumed to be due to a linear drift and the appropriate correction is made to the second reading.

The balance used is a Mettler-Toledo UMT5, which has a capacity of over 5 g. The smallest division of this balance is 0.1 μg but, for the measurements reported below, the pooled standard deviation of a single measurement is approximately 0.6 μg (22 degrees of freedom). Note that the force in (2) is directly proportional to the mass displayed the balance. Therefore F_1 and F_2 may be replaced by the corresponding mass measurements.

3.4 *Interferences*

As shown by (1), any change in the temperature of magnet A or of the aluminium-alloy supports that largely define the distance L can lead to an error if this change is correlated with the temperature of the sample holder. Magnet A is made of a stack of two of the same type of magnets that are under test. Therefore, it is only necessary that the change in temperature within the weighing chamber of the UMT-5 balance be much smaller than ΔT. This condition is easily fulfilled.

An increase of 1 °C in the temperature of the structure supporting magnet B during the high-temperature measurements would lead to an error in α of about 10×10^{-6}/°C. For this calculation, we take account of the fact that the height of the structure supporting magnet B is approximately $2L$. Magnet A is placed on a support of approximate height L. The balance chamber is never opened throughout the course of the measurements. As a worst case, we consider that the support of magnet A remains at temperature T whereas the support of magnet B increases in temperature by an amount discussed in Section 4.

4. Data

The data obtained are summarized in Table 2 and figure 4. Measurements of m_B were made at the ambient room temperature, 21.3 °C, then at elevated temperature (39.3 °C) and finally at temperature 22.0 °C. Cooling between the second and third sets of measurements was accelerated by placing a massive copper plate on top of the sample holder. The room-temperature data (sets 1 and 3) were each taken over a period of 15 minutes. There was a delay of 30 minutes between the end of the first set of data and the start of the second set. Because the high-temperature data required reheating of the sample holder between successive balance readings, only five points were taken and these required a total of 70 minutes. There was a delay of 50 minutes between the end of the second data set and the start of the third. During the final set of measurements, the sample holder was still approaching the ambient temperature of the room. However, during the 15 minutes necessary to obtain the third set of data, the decrease in temperature was only 0.1 °C.

If there were an interference due to unexpectedly large heating of the structure supporting magnet B, it should be seen most easily as a slope in the data of set 2. If we force a linear fit to these data, we find a slope consistent with an increase in the temperature of 0.4 °C, although the standard uncertainty is larger than the slope itself. We have made no correction for an increase in temperature but have instead assigned a standard uncertainty (Type B) of 0.5 °C to allow for this possibility, giving a standard uncertainty component of 3×10^{-6}/°C in α.

Sequence number	Set 1 21.3 °C	Set 2 39.3 °C	Set 3 22.0 °C
1	95.0	93.2	94.5
2	94.0	93.6	95.1
3	94.4	93.2	94.6
4	94.4	93.4	95.6
5	94.6	93.0	96.6
6	93.6		96.7
7	95.3		95.6
8	94.2		96.1
9	94.6		95.8
10	95.1		94.2
Avg.	**94.5**	**93.3**	**95.5**
s	**0.52**	**0.23**	**0.87**
s_m	**0.16**	**0.10**	**0.27**

Table 2. Data discussed in this report. If x represents a balance reading in micrograms, then the numbers in the table are $x - 2500$.

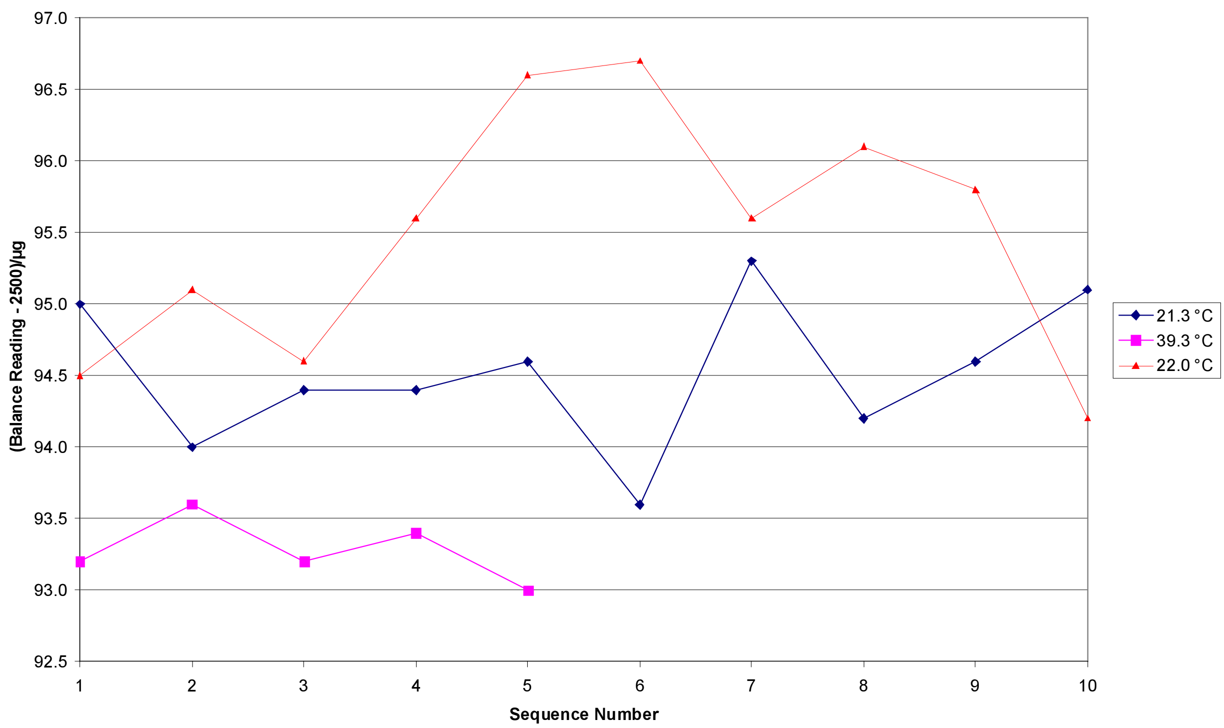

Figure 4. Data discussed in this report

The standard deviations of the means of data sets 1 and 3 are too small to account for the observed difference between the measurements. Based on experience with measurements of this magnetic moment, we estimate that the standard uncertainties of each mean is about 1 μg. This results in a standard uncertainty component in α of 25×10^{-6}/°C.

We estimate that ΔT (17.6 °C) has a standard uncertainty of 1 °C, leading to a standard uncertainty component of about 1×10^{-6}/°C in α.

We thus can compute our final result

$$\alpha + \beta_V = -12 \times 10^{-6}/°\mathrm{C}$$

with a combined standard uncertainty ($k = 1$) of 25×10^{-6}/°C.

It is also possible to measure the value of $\mu_0 M_B V_{0,B}$, the magnetic moment of magnet B at room temperature, using the technique described in [11]. This was done for a single magnet and stacks of two and three magnets. The results obtained are within the following limits:

$$\mu_0 M_{r,B} \cong \mu_0 M_B = 0.656 \text{ T} \pm 0.003 \text{ T},$$

compared with the manufacturers specification of B_r = 0.64 T for a typical magnet. In a previous comparison of our method for determining M_r with results obtained using a VSM, all discrepancies were within 3% [11]. Confirmation of the manufacturer's value of $\mu_0 M_{r,B}$ gives added assurance that the magnets tested also have the value claimed for α. This is because the reduction in M_r is expected for $HRE_xSm_{1-x}Co_5$ magnets, where HRE is a heavy rare earth such as gadolinium or erbium and x is the fraction that produces the smallest α at room temperature [3]. (It is interesting to note that $HRE_xSm_{1-x}Co_{8.5}$ can also achieve low α with significantly higher remanence.)

5. Summary

From the data in hand, we estimate that

$\alpha + \beta_V = -12 \times 10^{-6}$/°C, with a combined standard uncertainty of 25×10^{-6}/°C

The manufacturer had specified the value of α as $\pm 50 \times 10^{-6}$/°C. It is unknown whether this result was corrected for β_A or β_V. In any case, the inference that measurements with the FB-2 balance might be due to a value of $\alpha + \beta_V$ much larger than expected is not supported by the experiment reported here. For example, a coefficient of -200×10^{-6}/°C would mean that the points at 39.3 °C in figure 4 would be displaced downward by about 10 μg.

The largest uncertainty is due to an allowance of 1 μg for the reproducibility of the balance measurements. A more sophisticated oven for heating the magnets could also be envisaged, as well as a more conventional way of measuring the temperature of the sample. Nevertheless, the relative simplicity of using magnets and instruments at hand allowed us to test the hypothesis.

Finally, it must be remembered that one may define temperature coefficients for all interesting magnet parameters. Reducing $|\alpha|$ does not, in general, imply that other coefficients are also reduced

References

1. A. Picard, submitted to *Metrologia.*

2. S. Liu *et al*., *J. Appl. Phys.*, **67**, 1990, 4975-4977.

3. K.J. Strnat, *Proc. IEEE*, **78**, 1990, 923-946.

4. D.-X. Chen *et al*., *IEEE Trans. Mag.*, **MAG-27**, 1991, 3601-3619.

5. *Magnetic materials—Part 5: Permanent magnetic (magnetically hard) materials—Methods of measurement of magnetic properties,* IEC International Standard 404-5, 1993, 31 p.

6. *Standard Test Method for Magnetic Properties of High-Coercivity Permanent Magnetic Materials Using Hysteresigraphs*, **A977/A977M-02**, ASTM International, 2002, 12 p.

7. A. Zieba, S. Foner, *Rev. Sci. Instrum.*, **53**, 1982, 1344-1354; Erratum, 1991, **62**, 293.

8. For example, F.E. Camp *et al*., *IEEE Trans. Mag.*, **MAG-21**, 1985, 1970-1972.

9. Raynger® MX2™, Raytek Corp., Santa Cruz CA, USA

10. R.S. Davis, *J. Res. Natl. Inst. Stand. Technol.*, **100**, 1995, 209-225.

11. J.W. Chung *et al*., *Metrologia*, **38**, 2001, 535-541.