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## Bilateral Comparison of 1.018 V and 10 V Standards between the NML/CSIRO (Australia) and the BIPM, October to December 2003 (part of the ongoing BIPM key comparisons BIPM.EM-K11a & b)

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As a part of the ongoing BIPM key comparison BIPM.EM-K11a and b, a comparison of the 1.018V and 10 V voltage reference standards of the BIPM and the National Measurement Laboratory (NML), Lindfield, Australia, was carried out between October and December 2003. Four BIPM 732B Zener diode-based travelling standards, Z BIPM 6 (Z6), Z BIPM 7 (Z7), Z BIPM 8 (Z8) and Z BIPM 9 (Z9), were transported by freight. Both the NML and the BIPM measurements of the travelling standards were carried out by direct comparison with the Josephson effect standard. Results of all measurements were corrected for the dependence of the output voltages on ambient temperature and pressure.

Figures 1 and 2 show the measured values obtained for the four standards by the two laboratories. The BIPM values and uncertainties, and those of the NML are calculated for the reference date from linear least-squares fits to all data from each laboratory.

Table 1 lists the results of the 1.018 V comparison and the component uncertainty contributions for the comparison NML-CSIRO/BIPM. Experience has shown that flicker or 1/f noise dominates the stability characteristics of Zener diode standards and it is not appropriate to use the standard deviation divided by the square root of the number of observations to characterize the dispersion of measured values. For the present standards, the relative value of the flicker floor voltage is about 1 part in  $10^8$ . Upon the return from Australia, two Zeners (Z8 and Z9) were found disconnected from the external batteries with the "ON CAL" lights off and the battery indicators on "LOW BAT". As a result, the 1.018 V reference voltages were changed significantly and so the 1.018 V results from those two Zeners were deleted from the comparison.

Table 2 lists the same information for the 10 V comparison. Upon the return to the BIPM the 10 V reference voltages of Zeners Z8 and Z9 were found to be nearly equal to the expected values and so the results from those two Zeners were included in the 10 V comparison.

In estimating the uncertainty we have calculated the *a priori* uncertainty based on all known sources except that associated with the stability of the standards when transported. We compare this with the *a posteriori* uncertainty estimated by the standard deviation of the weighted mean of the

results from the four (or two) travelling standards. With only four (or two) travelling standards, the uncertainty of the standard deviation of the mean is comparable to the value of the standard deviation of the mean itself. If the *a posteriori* uncertainty is significantly different from the *a priori* uncertainty, we assume that a standard has changed in an unusual way and we use the larger of these two estimates in calculating the final uncertainty.

In Tables 1 and 2, the following elements are listed:

(1) the predicted value  $U_{\text{NML}}$  of each Zener, computed using a linear least squares fit to all of the data from the NML-CSIRO and referenced to the mean date of the NML's measurements;

(2) the Type A uncertainty due to the instability of the Zener, computed as the standard uncertainty of the value predicted by the linear drift model, or as an estimate of the 1/f noise voltage level;

(3) the uncertainty component arising from the measuring equipment of the NML-CSIRO: this uncertainty is completely correlated between the different Zeners used for a comparison<sup>[1]</sup>;

(4-6) the corresponding quantities for the BIPM referenced to the mean date of the NML's measurements;

(7) the uncertainty due to the combined effects of the uncertainties of the pressure and temperature coefficients and to the difference of the mean pressures and temperatures in the participating laboratories; although the same equipment is used to measure the coefficients for all Zeners, the uncertainty is dominated by the Type A uncertainty of each Zener, so that the final uncertainty can be considered as uncorrelated among the different Zeners used in a comparison;

(8) the difference  $(U_{\text{NML}} - U_{\text{BIPM}})$  for each Zener;

(9) the uncorrelated part of the uncertainty;

(10) the result of the comparison, which is the weighted mean of the differences of the calibration results for the different standards; using as weights the reciprocal of the square of the uncorrelated part of the uncertainty components for each travelling standard;

(11 and 12) the uncertainty of the transfer, estimated by the following two methods: (11) the *a priori* uncertainty, which is the expected uncertainty from the different Zeners, taking into account only the uncorrelated uncertainties of the individual results; (12) the *a posteriori* uncertainty, which is the standard deviation of the weighted mean of the different results;

<sup>[1.]</sup> In fact, there is a high degree of correlation between these input quantities and we can assume a correlation coefficient of unity without significantly affecting the standard uncertainty of the result of this comparison.

(13) the correlated part of the uncertainty; and

(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty and of the larger of (11) and (12).

Table 3 summarizes the uncertainties due to the BIPM measuring equipment.Table 4 summarizes the uncertainties due to the NML-CSIRO measuring equipment.

The final results of the comparison are presented as the differences between the values assigned by the two laboratories to a 1.018 V and a 10 V standard, respectively. The difference between the value assigned to a 1.018 V standard by the NML, at the NML,  $U_{\text{NML}}$ , and that assigned by the BIPM, at the BIPM,  $U_{\text{BIPM}}$ , for the reference date is

$$U_{\rm NML} - U_{\rm BIPM} = +0.028 \ \mu \text{V};$$
  $u_{\rm c} = 0.026 \ \mu \text{V} \text{ on } 2003/11/12,$ 

and the difference between the value assigned to a 10 V standard by the NML, at the NML,  $U_{\text{NML}}$ , and that assigned by the BIPM, at the BIPM,  $U_{\text{BIPM}}$ , for the reference date is

$$U_{\rm NML} - U_{\rm BIPM} = +0.13 \ \mu \text{V};$$
  $u_{\rm c} = 0.14 \ \mu \text{V} \text{ on } 2003/11/12,$ 

where  $u_c$  is the combined Type A and Type B standard uncertainty from both laboratories.

This is a most satisfactory result. The differences between the values assigned to the mean voltage of the travelling standards at 1.018 V and 10 V by the two laboratories are not significantly different from the standard uncertainty associated with these differences. It is worth pointing out that these uncertainties are about the lowest that can be expected for such comparisons.

The results of these comparisons should provide a solid link, through the BIPM, between recent APMP comparisons and those of other RMOs.

If the analysis of the 10 V comparison had been limited to only the two travelling standards whose results were retained for the 1.018 V comparison (Z6 and Z7), the result would have been

 $U_{\rm NML} - U_{\rm BIPM} = -0.09 \ \mu \text{V};$   $u_{\rm c} = 0.16 \ \mu \text{V} \text{ on } 2003/11/12,$  result not retained, which is not significantly different from that given above.



Figure 1. Voltage of Z6, Z7, Z8 and Z9 at 1.018 V with arbitrary voltage origin as a function of time, with linear least-squares fits to the measurements of the BIPM (Z6 and Z7) or of the NML (Z8 and Z9).



Figure 2. Voltage of Z6, Z7, Z8 and Z9 at 10 V with arbitrary voltage origin, as a function of time, with linear least-squares fits to the measurements of the BIPM.

Table 1. Results of the NML-CSIRO (Australia)/BIPM bilateral comparison of 1.018 V standards using two Zener travelling standards (the results of Z8 and Z9, given in italics are not taken into account): reference date 2003/11/12. Uncertainties are 1  $\sigma$  estimates.

The uncorrelated uncertainty is  $w = [r^2 + t^2 + v^2]^{1/2}$ , the expected transfer uncertainty is  $x = [w_6^{-2} + w_7^{-2}]^{-1/2}$ , and the correlated uncertainty is  $y = [s^2 + u^2]^{1/2}$ .

		Z6	Z7	<i>Z8</i>	Z9	
1	NML-CSIRO (Australia) $(U_Z - 1.018V)/\mu V$	130.213	105.448	162.726	91.020	
2	type A uncertainty/µV	0.010	0.010	0.010	0.010	r
3	equipment uncertainty/µV	0.0	15			S
4	$BIPM \qquad (U_Z - 1.018V)/\mu V$	130.204	105.398	162.536	91.402	
5	type A uncertainty/µV	0.010	0.011	0.012	0.032	t
6	equipment uncertainty/µV	0.005			и	
7	uncertainty related to the corrections for the pressure and temperature differences/µV	0.000	0.000	0.000	0.000	v
8	$(U_{Z_{\rm NML}} - U_{Z_{\rm BIPM}})/\mu { m V}$	0.009	0.050	0.190	-0.381	
9	uncorrelated uncertainty/µV	0.014	0.015	0.016	0.033	w
10	$< U_{\rm NML} - U_{\rm BIPM} > /\mu V$	0.0	28			
11	expected transfer uncertainty/ $\mu V$	0.0	10			х
12	$\sigma_{\rm WM}$ of difference for two Zeners/ $\mu V$	0.0	20			
13	correlated uncertainty/µV	0.0	16			У
14	comparison total uncertainty/µV	0.0	26			

Table 2. Results of the NML-CSIRO (Australia)/BIPM bilateral comparison of 10 V standards using four Zener travelling standards: reference date 2003/11/12. Uncertainties are 1  $\sigma$  estimates.

The uncorrelated uncertainty is  $w = [r^2 + t^2 + v^2]^{1/2}$ , the expected transfer uncertainty is  $x = [w_6^{-2} + w_7^{-2} + w_8^{-2} + w_9^{-2}]^{-1/2}$ , and the correlated uncertainty is  $y = [s^2 + u^2]^{1/2}$ .

		Z6	Z7	Z8	Z9	
1	NML-CSIRO (Australia) $(U_Z - 10V)/\mu V$	-15.55	-24.36	-33.17	-27.05	
2	type A uncertainty/µV	0.10	0.10	0.10	0.10	r
3	equipment uncertainty/µV		0.0	1		S
4	BIPM $(U_{\rm Z} - 10 {\rm V})/\mu {\rm V}$	-15.32	-24.45	-33.50	-27.42	
5	type A uncertainty/µV	0.10	0.13	0.10	0.10	t
6	equipment uncertainty/µV	0.01				u
7	uncertainty related to the corrections for the pressure and temperature differences/µV	0.00	0.00	0.00	0.00	v
8	$(U_{Z_{\rm NML}} - U_{Z_{\rm BIPM}})/\mu { m V}$	-0.24	0.09	0.33	0.36	
9	uncorrelated uncertainty/µV	0.14	0.16	0.14	0.14	w
10	$< U_{\rm NML} - U_{\rm BIPM} > /\mu V$		0.1	3		
11	expected transfer uncertainty/ $\mu V$	0.08			x	
12	$\sigma_{ m WM}$ of difference for four Zeners/ $\mu  m V$	0.14				
13	correlated uncertainty/µV	0.01				у
14	comparison total uncertainty/µV		0.1	4		

Table 3 Estimated standard uncertainties for Zener calibrations with the BIPM equipment.

	Uncertainty/nV		
	1.018 V	10 V	
thermal EMFs	3	3	
Detector/EMI	3	0.5	
leakage resistance	3	0.3	
Frequency	< 0.1	0.3	
Pressure correction	0.4	3.8	
Temperature correction	1.5	1.3	
Total	5.4	5.1	

Table 4. Estimated standard uncertainties for Zener calibrations with the NML equipment.

	Uncertainty/nV		
	1.018 V	10 V	
thermal EMFs	14	14	
detector/EMI	0.1	0.1	
leakage resistance	4	2	
frequency	0.1	1.1	
pressure correction	0.1	0.8	
temperature correction	0.0	0.1	
total	14.6	14.2	