

On the evaluation of the correction factor $\mu(\rho', \tau')$
for the periodic pulse method

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1. Introduction to the problem

Among the various techniques suggested for measuring the dead time of an electronic counting device, the periodic pulse method [1] is one of the more recent ones. It is a seemingly simple variant of the classical two-source method. However, and in spite of repeated efforts, no "exact" mathematical treatment of the corresponding stochastic problems has been possible, at least for the practically important case of a non-extended dead time.

On the other hand, an approximate solution to the problem of determining the value of a dead time τ has been known from the very beginning. The simple formula reads

$$\tau = \frac{1}{r} \left[1 - \sqrt{\frac{r\nu - r}{\nu}} \right], \quad (1)$$

where r is the measured count rate of the source alone, while r_ν denotes the corresponding rate with the oscillator pulses of frequency ν superimposed. This formula, currently applied by the laboratories utilizing this method, seems indeed to give very satisfactory numerical results for the usual experimental conditions. Nevertheless, it has always been taken, at least by some purists, as a possible source of discomfort that (1) was derived on the basis of probabilistic arguments which cannot withstand a high degree of scrutiny. In particular, it is obvious that the formula will break down when $r \equiv 1/\nu$ approaches the dead time τ .

In 1973, when a new attempt was made for obtaining a "rigorous" solution, we found at last the exact reason for the difficulties. A closer look at the problem revealed that four interval densities had to be known in order to determine the total loss of pulses in the superposition due to dead times (Table 1).

Case	Time origin given by a registered event of	End of interval given by the arrival of events from the original series of
a	type ρ	type ρ
b	type ρ	type ν
c	type ν	type ρ
d	type ν	type ν

Table 1 - Characteristics of the "start" and "stop" events for the relevant time intervals.

Type ρ means that the pulse originates from the source, whereas ν refers to the oscillator.

While three of the densities corresponding to the intervals defined in Table 1 can be readily stated (cases a and c give rise to a step function, case d results in a Dirac comb), case b is much less obvious; the description of a possible way to its numerical evaluation is the subject of the present study.

If we begin the reasoning, as may seem natural, with a registered ρ -pulse, then determining the arrival time of the next ν -event is equivalent to stating when the last ν has arrived, as a result of their strict periodicity. However, it is easy to realize that the density for this last ν -event cannot be uniform, for any arrival at a time less than τ before the ρ -event would have been a serious obstacle for its registration. Nevertheless, the inhibition is not an absolute one because the ν -event may have fallen into the dead time of a previously registered ρ -event. Whether this takes place or not depends on the sequence of all the previously counted pulses, and it will become obvious why reasonings of this type are uncomfortable: at each step the "prehistory" should be known, but the number of possibilities is so rapidly increasing that this way of approach looks hopeless even to an optimist.

This disparate state of affairs has stopped any real progress on this problem for years and it is only quite recently that we believe we have found a way out of the vicious circle.

Although in general the superposition of renewal processes results in a series of events which is lacking this convenient feature, it may still happen that some of the arrival times conserve the property of being renewal points. In Feller's terminology [2] these instants are called renewal epochs. Their significance lies in the fact that from such a point onwards, the previous development of the process, i.e. all its "history", has no influence whatsoever on the future evolution. In our case, as is easily verified, the arrival times of registered ν -events have this remarkable property.

The main problem is thus reduced to an evaluation of the probability density, denoted by $D(t)$ in what follows, of the time interval between a registered ρ - and the following ν -pulse, which may be counted or lost. For the reason given above it is more practical, however, to start the consideration with a ν -event. For the calculation we shall first suppose that we know the value τ of the non-extended dead time as well as the original count rate ρ of the source (i.e. for $\tau = 0$), the emissions of which are assumed to form a Poisson process. At a later stage (not included here) it will be possible to restrict ourselves to observable quantities alone, as was the case in (1). Once the density $D(t)$ is known, it is a simple matter to obtain the corresponding correction factor μ .

2. Derivation of the interval density $D(t)$

It is convenient to derive first two related density functions, called $A(t)$ and $B(t)$, from which $D(t)$ can then be readily obtained. Both intervals start with a ν -event which will be recorded in one case, but suppressed in the other. For convenience we restrict ourselves to the region of most interest where

$$T = 1/\nu \gg \tau. \quad (2)$$

This condition is easy to fulfil by a corresponding choice of the oscillator frequency and avoids the case where ν -pulses fall into their own dead times.

a) Case I where ν has been registered

If we start (at $t = 0$) with a registered ν -pulse, the total arrival density for the subsequently registered ρ -events is known to be given by [3]

$$A(t) = \sum_k f_k(t) = \rho \sum_{k=1}^K \frac{[\rho(t - k\tau)]^{k-1}}{(k-1)!} \cdot e^{-\rho(t - k\tau)}, \quad (3)$$

with $K \equiv \lceil t/\tau \rceil$ and at least for $0 < t < T$.

$f_k(t)$ is the normalized density for the arrival time of event number k in a Poisson process of original rate ρ , which has been distorted by a non-extended dead time τ . The values of $A(t)$ are calculated at regular intervals (up to 1 000) and stored for later use. The domain of t may be restricted from 0 to $T + \tau$. For a ν -pulse chosen at random, the probability for this case I to happen is given by the transmission factor T_ν which corresponds to the chance of an original oscillator pulse to "survive".

b) Case II where ν falls into a dead time

When ν (at $t = 0$) has been suppressed by a preceding ρ -pulse - the mutual effacement of ν -pulses is excluded by (2) -, the situation is somewhat more complicated. As a matter of fact, the rigorous treatment of this case II would suppose a knowledge of the arrival time of the preceding ρ -pulse which is responsible for the suppression of ν . However, this is just given by the final density $D(t)$ we want to derive. Instead of trying to solve the corresponding integral equation, we prefer a recursive approach which lends itself more directly to a numerical treatment by computer. The total density $B(t)$ for registered ρ -pulses, when the time origin is given by a suppressed ν -event, can therefore be written for the iteration number j ($j = 1, 2, 3, \dots$) in the following form of a convolution

$$B_j(t) = \frac{1}{\tau} \int_0^{\tau} D_j(t') \cdot A(t + t') dt' . \quad (4)$$

For the initial step where no density D is yet available, we may start* with the uniform distribution $D_1(t) = 1/T$, at least for $\tau < T/2$. Again the values of $B_j(t)$ are calculated at sufficiently many points and stored. Since it is only the ratios

$$\rho' \equiv \rho/\nu = \rho T \quad \text{and} \quad \tau' \equiv \tau/T = \nu \tau \quad (5)$$

which have a physical meaning, any arbitrary value may be assigned to T . The simplest choice is $T = 1$, taking thereby the period of the oscillator as the unit of time.

c) Determination of $D(t)$

Once the numerical values for the distributions A and B are available, the density $D(t)$ for the arrival of a ν -event after a registered ρ -pulse (at $t = 0$) can be evaluated recursively by the linear superposition

$$D'_{j+1}(t) = \nu T_j \cdot A(T - t) + (1 - \nu T_j) \cdot B_j(T - t) , \quad (6)$$

from which we form the normalized density

$$D_{j+1}(t) = D'_{j+1}(t) / S'_{j+1} , \quad (7)$$

where

$$S'_{j+1} = \int_0^T D'_{j+1}(t) dt .$$

* cf. Appendix

Thereby in the initial round the simple approximation

$$\nu T_1 = [1 + \rho \tau (1 - \nu \tau)]^{-1} \quad (8)$$

may be used for the transmission factor of the oscillator pulses*. In subsequent iterations this value is steadily improved by using the semi-empirical quantity

$$\nu T_i = \frac{1 - (\mu_i - 1) \rho \tau}{1 + \rho \tau (1 - \mu_i \nu \tau)} \quad (9)$$

The meaning of μ_i will be explained below. The special forms chosen in (8) and (9) for the ν -transmission result from the complete theory for the superimposed process; its justification has to be deferred to a subsequent report.

Figure 1 gives an example for the shapes of the (final) densities A, B and D.

3. Calculation of the correction factor μ

After each new result $D_i(t)$ in the recursive evaluation of the normalized density $D(t)$, we determine the effective loss νl_i of ν -events produced by the dead time τ of the registered ρ -pulse at $t=0$. This loss is given by

$$\nu l_i = \int_0^{\tau} D_i(t) dt \quad (10)$$

The ratio of this value to the loss corresponding to a hypothetical uniform density $D''(t) = U(t) \cdot \nu$ for the subsequent oscillator pulses will be called μ_i for iteration number i , hence

$$\mu_i(\rho', \tau') = \frac{\nu l_i}{\int_0^{\tau} D''(t) dt} = \frac{1}{\nu \tau} \int_0^{\tau} D_i(t) dt \quad (11)$$

This is the factor appearing in (9).

For given values of ρ , ν and τ , or more precisely for the two corresponding ratios ρ' and τ' defined in (5), the auxiliary densities A, B_i and D_i are determined recursively and for each round the corresponding correction factor μ_i as defined in (11) is calculated. This evaluation is repeated till μ_i becomes stable (at μ , say), as is easily controlled by comparing consecutive values. In this way, the values for μ listed in Table 2 have been calculated on our IBM 1130 computer. The accuracy of the values given has been checked in various ways and it is believed that only in some rare cases the uncertainty will exceed one unit of the last decimal given.

* cf. Appendix

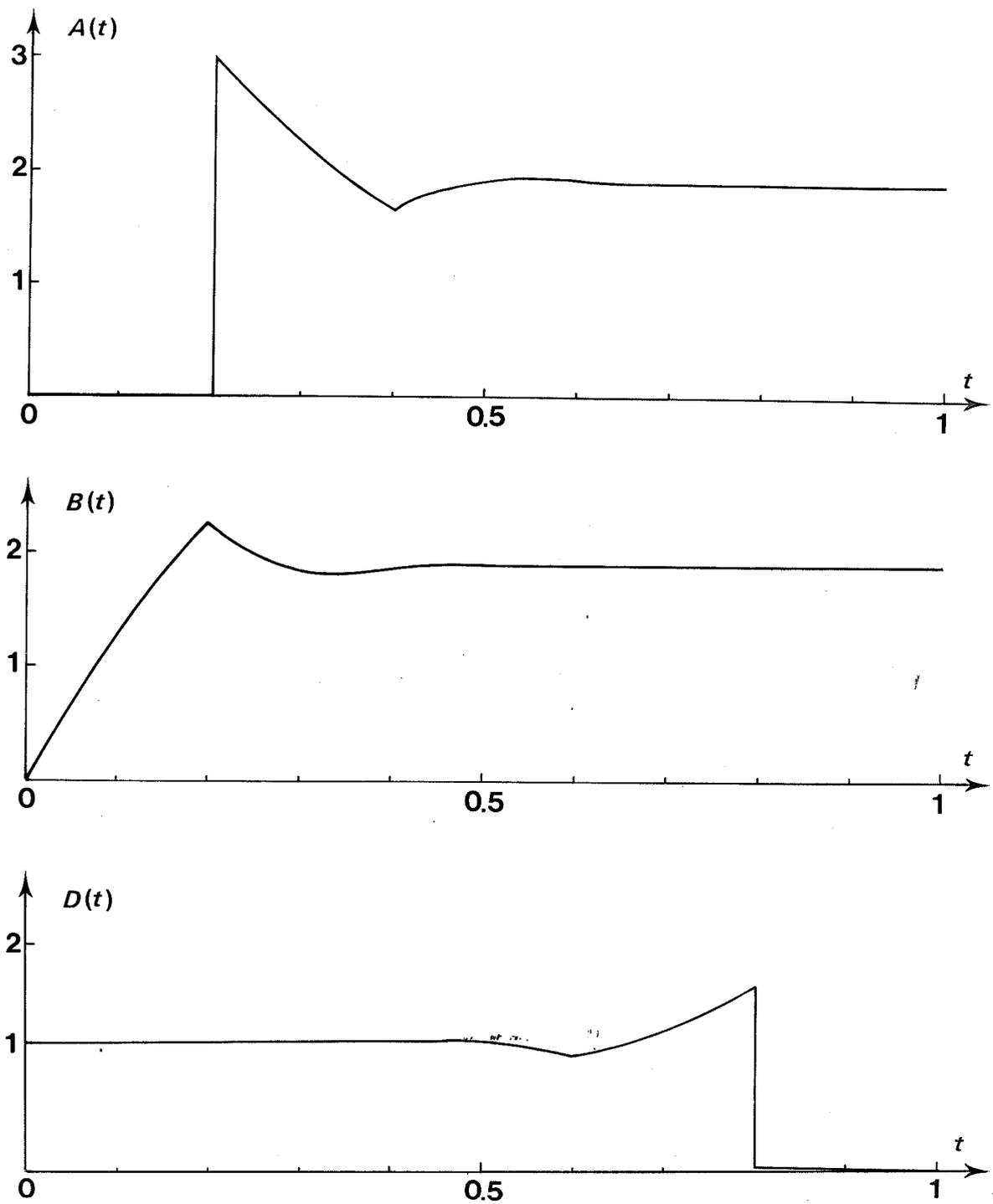


Fig. 1 - The densities $A(t)$, $B(t)$ and $D(t)$. The example chosen corresponds to the parameters $\rho' = 3$ and $\tau' = 0.2$.

τ'	$\rho' = 0.00$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.05	1.0526	1.0526	1.0526	1.0526	1.0525	1.0525	1.0525	1.0524	1.0524	1.0524
0.10	1.1111	1.1110	1.1109	1.1107	1.1106	1.1105	1.1104	1.1103	1.1101	1.1100
0.15	1.1765	1.1762	1.1759	1.1755	1.1752	1.1749	1.1746	1.1743	1.1740	1.1737
0.20	1.2500	1.2494	1.2488	1.2481	1.2475	1.2469	1.2463	1.2457	1.2451	1.2445
0.25	1.3333	1.3322	1.3311	1.3300	1.3290	1.3279	1.3268	1.3257	1.3247	1.3236
0.30	1.4286	1.4267	1.4249	1.4231	1.4213	1.4196	1.4178	1.4161	1.4143	1.4126
0.35	1.5385	1.5356	1.5328	1.5300	1.5273	1.5245	1.5218	1.5191	1.5164	1.5137
0.40	1.6667	1.6631	1.6595	1.6560	1.6524	1.6489	1.6454	1.6420	1.6386	1.6352
0.45	1.8182	1.8140	1.8098	1.8057	1.8015	1.7974	1.7934	1.7893	1.7853	1.7813
0.50	2.0000	1.9950	1.9901	1.9852	1.9803	1.9754	1.9706	1.9658	1.9611	1.9563
0.55	1.8182	1.8157	1.8132	1.8108	1.8083	1.8058	1.8033	1.8008	1.7984	1.7959
0.60	1.6667	1.6658	1.6650	1.6641	1.6632	1.6624	1.6615	1.6605	1.6596	1.6587
0.65	1.5385	1.5384	1.5383	1.5383	1.5382	1.5381	1.5380	1.5379	1.5378	1.5376
0.70	1.4286	1.4286	1.4286	1.4286	1.4286	1.4286	1.4286	1.4285	1.4285	1.4285
0.75	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333	1.3333
0.80	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500
0.85	1.1765	1.1765	1.1765	1.1765	1.1765	1.1765	1.1765	1.1765	1.1765	1.1765
0.90	1.1111	1.1111	1.1111	1.1111	1.1111	1.1111	1.1111	1.1111	1.1111	1.1111
0.95	1.0526	1.0526	1.0526	1.0526	1.0526	1.0526	1.0526	1.0526	1.0526	1.0526
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 2a - Numerical values of the correction factor $\mu(\rho', \tau')$ for $\rho' = 0.00 (0.01) 0.09$ and $\tau' = 0.00 (0.05) 1.00$.

The limiting values for $\rho' = 0$ and for $\tau' = 0$ or 1 are given for interpolation.

τ'	$\rho' = 0.1$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.05	1.0524	1.0521	1.0518	1.0516	1.0513	1.0510	1.0508	1.0505	1.0502	1.0500
0.10	1.1099	1.1087	1.1075	1.1064	1.1053	1.1042	1.1031	1.1020	1.1010	1.1000
0.15	1.1734	1.1705	1.1676	1.1648	1.1622	1.1596	1.1571	1.1546	1.1523	1.1500
0.20	1.2439	1.2381	1.2326	1.2273	1.2222	1.2174	1.2128	1.2083	1.2041	1.2000
0.25	1.3226	1.3125	1.3030	1.2942	1.2858	1.2779	1.2704	1.2633	1.2565	1.2501
0.30	1.4109	1.3946	1.3795	1.3654	1.3525	1.3404	1.3292	1.3188	1.3091	1.3001
0.35	1.5111	1.4857	1.4620	1.4401	1.4197	1.4008	1.3832	1.3667	1.3515	1.3373
0.40	1.6318	1.5992	1.5687	1.5402	1.5135	1.4884	1.4650	1.4430	1.4224	1.4030
0.45	1.7773	1.7390	1.7030	1.6693	1.6376	1.6078	1.5798	1.5534	1.5286	1.5052
0.50	1.9516	1.9064	1.8639	1.8242	1.7870	1.7520	1.7192	1.6883	1.6594	1.6321
0.55	1.7934	1.7687	1.7441	1.7199	1.6960	1.6727	1.6500	1.6279	1.6064	1.5857
0.60	1.6577	1.6477	1.6368	1.6250	1.6125	1.5994	1.5860	1.5722	1.5581	1.5440
0.65	1.5375	1.5357	1.5330	1.5294	1.5251	1.5200	1.5141	1.5076	1.5005	1.4929
0.70	1.4285	1.4283	1.4278	1.4271	1.4261	1.4249	1.4232	1.4212	1.4189	1.4161
0.75	1.3333	1.3333	1.3333	1.3333	1.3332	1.3331	1.3329	1.3326	1.3323	1.3318
0.80	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500	1.2500
0.85	1.1765	1.1765	1.1765	1.1765	1.1765	1.1765	1.1765	1.1765	1.1765	1.1765
0.90	1.1111	1.1111	1.1111	1.1111	1.1111	1.1111	1.1111	1.1111	1.1111	1.1111
0.95	1.0526	1.0526	1.0526	1.0526	1.0526	1.0526	1.0526	1.0526	1.0526	1.0526
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

∞

Table 2b - As in Table 2a, for $\rho' = 0.1 (0.1) 1.0$

τ'	$\rho' = 1$	2	3	4	5	6	7	8	9	10
0.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.05	1.0500	1.0476	1.0455	1.0435	1.0417	1.0400	1.0385	1.0370	1.0357	1.0345
0.10	1.1000	1.0909	1.0833	1.0769	1.0714	1.0667	1.0625	1.0588	1.0556	1.0526
0.15	1.1500	1.1304	1.1154	1.1035	1.0937	1.0857	1.0789	1.0730	1.0680	1.0636
0.20	1.2000	1.1667	1.1431	1.1253	1.1113	1.0996	1.0896	1.0809	1.0734	1.0671
0.25	1.2501	1.1995	1.1642	1.1382	1.1190	1.1052	1.0956	1.0892	1.0847	1.0814
0.30	1.3001	1.2361	1.1982	1.1706	1.1474	1.1272	1.1096	1.0945	1.0816	1.0708
0.35	1.3373	1.2391	1.1910	1.1667	1.1522	1.1408	1.1297	1.1182	1.1082	1.0985
0.40	1.4030	1.2620	1.1833	1.1390	1.1148	1.1032	1.0987	1.0975	1.0966	1.0949
0.45	1.5052	1.3324	1.2309	1.1676	1.1262	1.0982	1.0785	1.0644	1.0543	1.0470
0.50	1.6321	1.4323	1.3168	1.2455	1.1987	1.1664	1.1429	1.1250	1.1113	1.1003
0.55	1.5857	1.4174	1.3089	1.2392	1.1930	1.1613	1.1385	1.1214	1.1081	1.0974
0.60	1.5440	1.4107	1.3099	1.2402	1.1924	1.1590	1.1350	1.1173	1.1040	1.0938
0.65	1.4929	1.4016	1.3149	1.2491	1.2019	1.1681	1.1431	1.1243	1.1097	1.0980
0.70	1.4161	1.3695	1.3060	1.2484	1.2036	1.1701	1.1451	1.1259	1.1111	1.0993
0.75	1.3318	1.3180	1.2848	1.2424	1.2035	1.1720	1.1476	1.1286	1.1137	1.1018
0.80	1.2500	1.2486	1.2410	1.2224	1.1965	1.1704				
0.85	1.1765	1.1765	1.1763	1.1748	1.1694	1.1583				
0.90	1.1111	1.1111	1.1111	1.1111	1.1111	1.1110				
0.95	1.0526	1.0526	1.0526	1.0526	1.0526	1.0526				
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000				

Table 2c - As in Table 2a, for $\rho' = 1$ (1) 10

τ'	$\rho' = 10$	20	30	40	50	60	70	80	90	100
0.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.05	1.0345	1.0256	1.0204	1.0170	1.0145	1.0127	1.0112	1.0101	1.0092	1.0085
0.10	1.0526	1.0344	1.0257	1.0212	1.0169	1.0127	1.0103	1.0098	1.0106	1.0110
0.15	1.0636	1.0412	1.0263	1.0164	1.0137	1.0160	1.0166	1.0147	1.0125	1.0107
0.20	1.0671	1.0462	1.0343	1.0250	1.0200	1.0167	1.0143	1.0124	1.0110	1.0098
0.25	1.0814	1.0506	1.0333	1.0250	1.0199	1.0166	1.0141	1.0121	1.0106	1.0093
0.30	1.0708	1.0257	1.0324	1.0299	1.0228	1.0180	1.0149	1.0129	1.0117	1.0107
0.35	1.0985	1.0491	1.0332	1.0255	1.0212	1.0186	1.0170	1.0163	1.0161	1.0162
0.40	1.0949	1.0527	1.0345	1.0274	1.0230	1.0218	1.0211	1.0213	1.0221	1.0238

Table d - As in Table 2a, for $\rho' = 10$ (10) 100

As an independent check, the densities $D(t)$ can also be obtained directly by a Monte Carlo simulation. This has been done for a number of cases and the agreement is very satisfactory (see Table 3). As an illustration, the probability densities $D(t)$ as obtained by simulation and by calculation are given in Fig. 2. Since the Monte Carlo approach is very time consuming, it can in no way replace the direct numerical evaluation of μ .

Measuring condition	Correction factor $\mu(\rho', \tau')$	
	by Monte Carlo	from Table 2c
$\rho' = 1, \tau' = 0.1$	1.101 \pm 0.003	1.100
0.2	1.201 0.002	1.200
0.3	1.300 0.002	1.300
0.4	1.402 0.002	1.403
0.5	1.632 0.001	1.632
$\rho' = 5, \tau' = 0.1$	1.071 \pm 0.003	1.071
0.2	1.112 0.001	1.111
0.3	1.148 0.001	1.147
0.4	1.114 0.002	1.115
0.5	1.199 0.002	1.199

Table 3 - Comparison of some numerical values of the correction factor μ obtained by simulation and by calculation. The result for each simulation is based on at least 10^6 intervals.

4. Final remarks

The inconveniences associated with the purely numerical approach we have chosen to solve some of the problems which arise in connection with the source-pulsar method are twofold. On the one hand, there is the trouble of calculating the tables for μ , requiring several nights of computer time. On the other hand, apart from some simple limiting values as

$$\mu = 1 \quad \text{for } \tau' = 0 \text{ or } 1,$$

$$\mu = \frac{1}{0.5 + 10.5 - \tau'} \quad , \text{ for } \rho' \rightarrow 0 \quad (12)$$

(and $0 \leq \tau' \leq 1$)

and $\mu = 1 + 1/\rho'$ for $\rho' \gg 1$,

no appropriate analytical form for the correction factor μ of sufficient precision is known for the moment. This drawback, we hope, is partly removed by a tabulation which should cover the whole region of practical interest. The computer programs for both the evaluation of μ and the Monte Carlo simulation are available upon request.

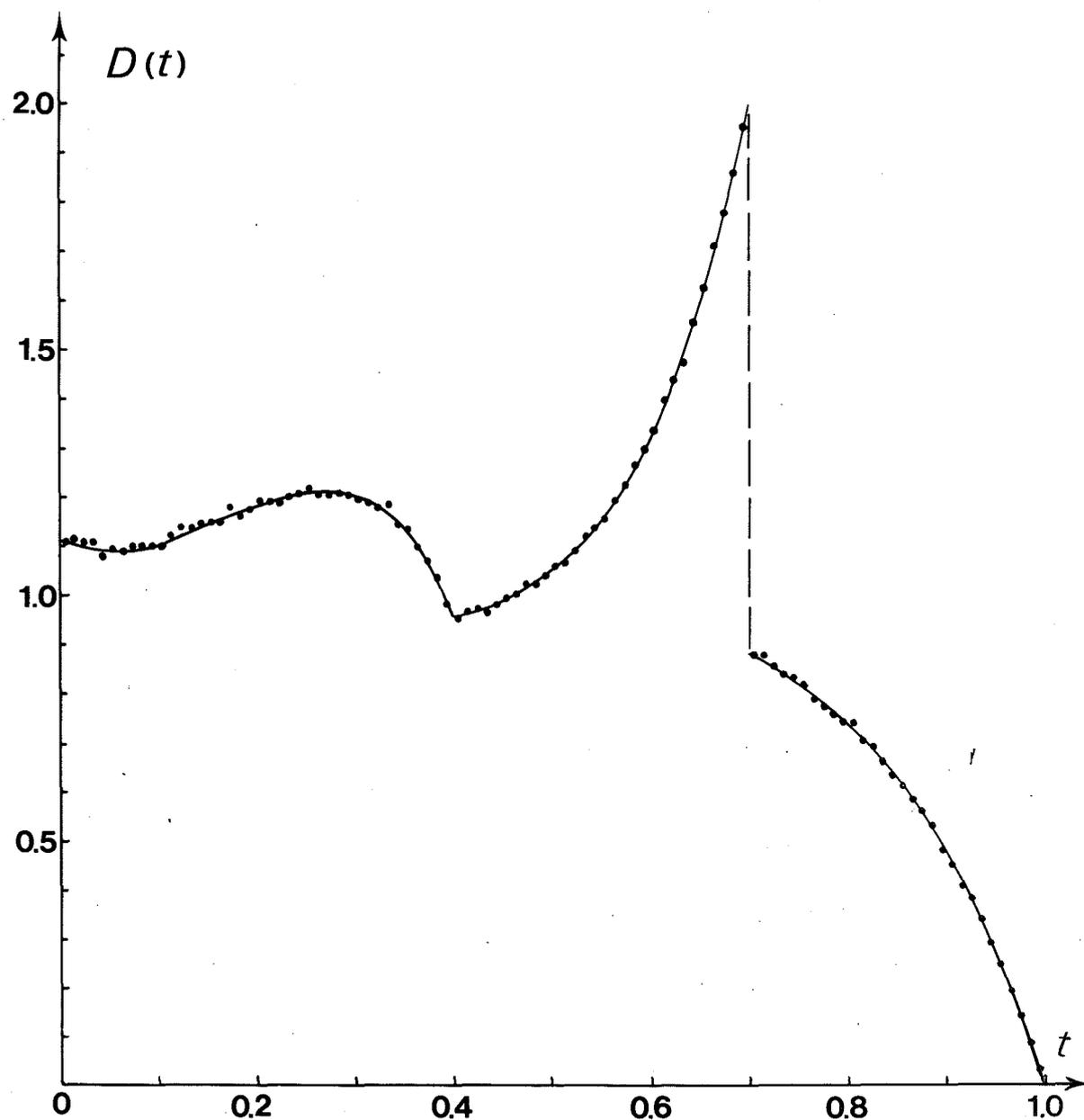


Fig. 2 - Comparison of the detailed shape of the interval density $D(t)$ as obtained by calculation and simulation. The parameters used are $\rho' = 5$ and $\tau' = 0.3$. The Monte Carlo simulation is based on 2×10^6 intervals.

For some applications it will be more convenient to use only quantities which can be measured directly. In this case, we need the correction factors μ as a function of $r' = r/\nu$ and $\tau' = \tau\nu$, where

$$r = \frac{\rho}{1 + \rho\tau} \quad (13)$$

is the measured count rate for a Poisson process of original rate ρ and a non-extended dead time τ . Such a tabulation of $\mu(r', \tau')$ is actually being calculated.

In view of the rather limited application of the source-pulser method one might wonder why a numerical method arising in this context should be described in so much detail. We should mention, therefore, that this was mainly done in the (perhaps vain) hope that one or the other of the techniques applied (e.g. renewal point or iterative procedure) might prove equally useful in some of the more important problems related to coincidences and inspire somebody to try them.

We realize that for those readers who are somewhat less familiar with this technique for measuring dead times, the relevance of the correction factor μ may be difficult to grasp. We are sure that the practical usefulness will become more evident as soon as a subsequent report on its application will be available which is now in preparation. In the meantime we cannot but hope those interested to be blessed with some patience.

Appendix

Reduction of the computing time

For the calculation of a whole table of correction factors μ according to the repetitive procedure sketched in sections 2 and 3, the computing time can become quite long. It will be obvious that the convergence may be markedly improved by starting with an approximately correct value $1/\tau\nu$ for the transmission and by using a realistic initial density $D_1(t)$. This is why the primitive guesses mentioned before are hardly ever used.

Let us assume that the values for a μ -table (as e.g. in Table 2) are calculated line by line. Since by a small increase of the source rate ρ'

(and for the same dead time τ') the corresponding distributions will not be drastically changed, but only somewhat modified, it seems reasonable to choose as a starting point the final values found for the previous ρ' rather than some trial function which has only the merit of looking "simple"*. Hence, the initial value μ_1 will be equal to the previous result μ , and this quantity may be readily inserted into (9) to give T_1 , avoiding thereby (8) where μ_1 was assumed to be 1. Likewise we proceed with respect to $D_1(t)$. Instead of using in (4) a constant value $D_1(t) = \mu_1/T$, we prefer to start with the empirical final density $D(t)$ corresponding to the previous value μ .

In addition, once a new value γT_i of the transmission factor has been derived, the convergence can in general be improved if instead a "smoothed" value, as for instance $(\gamma T_{i-1} + 2 \cdot \gamma T_i)/3$, is used for the next round, and in the same way a mean value is actually taken for the correction factor.

* Except, of course, for the very beginning of a "line" (with $\rho' = 0.01$, say). Here, however, the previously suggested initialization works well, especially so with $D_1(t) = \mu/T$, if for μ the limiting value for $\rho' \rightarrow 0$ from (12) is used.

References

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