MeP-K LOW-TEMPERATURE JOHNSON NOISE THERMOMETRY

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Jost Engert and Alexander Kirste

Physikalisch-Technische Bundesanstalt, Berlin, Germany

1. Introduction

The determination of thermodynamic temperatures by low-temperature Johnson noise thermometry (JNT) is based on the Nyquist formula:

$$\langle V(f,T)^2\rangle = 4kTR\Delta f, \qquad (1)$$

stating that the mean square of the noise voltage $V(f,T)$ across an unbiased resistor in a frequency band Δf is proportional to temperature T and independent of any material properties except the resistance of the conductor. R is the resistance and k is the Boltzmann constant. The Nyquist formula is the limiting case of the fluctuation-dissipation theorem [Callen 1951], when quantum corrections are neglected, which is applicable in nearly all practical situations for temperatures from 1 K down to 1 mK and for frequencies up to a few hundred MHz or a few hundred kHz, respectively. The general approach and basic principles of JNT and its application are described in [White 1996]. JNT can be implemented in absolute primary or relative primary form, whereas the second requires the reference to a temperature of known thermodynamic value.

2. Traceability to SI units

The implementation of primary Johnson noise thermometry requires that the quantities involved are traceable to the units of the SI. Therefore, the measurement of the voltage and the resistance must be traceable to the definition of electrical units and the measurement of frequencies to the definition of the unit time.

3. Absolute primary low-temperature JNT

For JNT very small noise voltages must be measured (1.3×10^{-9} $\mathrm{V}/\sqrt{\mathrm{Hz}}$ for a 100 Ω resistor at 293 K and 3.3×10^{-13} $\mathrm{V}/\sqrt{\mathrm{Hz}}$ for a 2 mΩ resistor at 1 K), and the gain as well as the bandwidth of the measurement must be stable and known with high accuracy. Contributions to the measurement signal from non-thermal noise sources must be negligible. For low-temperature JNT, superconducting quantum interference devices (SQUIDs) are the method of choice for the measurement of the tiny noise voltages. A review on current SQUID-based low-temperature thermometers is given in [Beyer 2016].

Three main variants of SQUID-based low-temperature Johnson noise thermometers are mainly implemented. The first one is historically called the resistive SQUID (RSQUID) setup and uses as the main component a superconducting loop containing a Josephson tunnel junction and a resistive shunt closing the loop. The second one is the so-called current sensing noise thermometer (CSNT), where the current through the noise resistor is read out by a SQUID current sensor. The third one is the magnetic field fluctuation thermometer (MFFT), in which the fluctuating magnetic field caused by the noise resistor is read out by a SQUID magnetometer or gradiometer.

3.1. JNT with the RSQUID setup

In the RSQUID setup, a direct bias current develops a voltage V across a resistor R connected in parallel to the Josephson junction of the RSQUID. This voltage is converted by the junction into ac oscillations at the frequency ν given by the Josephson equation

$$\nu = \frac{2\,\mathrm{e}\,V}{h}, \tag{2}$$

where e is the elementary charge and h is the Planck constant. The voltage fluctuations due to thermal noise in the resistor cause frequency fluctuations with the variance

$$\sigma_\nu = \frac{2\,k\,R\,T}{\phi_0\,\tau}, \tag{3}$$

where k is the Boltzmann constant, $\phi_0 = h/(2\,e)$ is the magnetic flux quantum and τ is the gate time of the frequency measurement. The frequency fluctuations are measured by coupling a tank circuit to the RSQUID excited at the resonance frequency. From the filtered sideband of the resonance, the variance of the Josephson oscillations is determined. The value of the resistor R can be measured independently as well as the gate time τ. Then, using equation (3) the thermodynamic temperature T is directly calculated from the variance σ_ν of the Josephson frequency fluctuations.

An overview of the details of thermodynamic temperature measurements with rf RSQUIDs is given in [Soulen 1994]. JNT realizations based on RSQUIDs have been used, for example, for the establishment of the PLTS-2000 [Rusby 2002]. In [Menkel 2000], the theory of temperature measurement with rf RSQUIDs was extended to the application of modern thin-film dc RSQUIDs.

3.2. Current sensing noise thermometer, CSNT

Current sensing noise thermometers utilizing rf or dc SQUIDs resolve the thermal noise signal in time domain [Giffard 1972, Webb 1973, Lusher 2001, Shibahara 2016]. The thermal voltage noise across the resistor causes fluctuating currents in the input coil that are detected as thermal magnetic flux noise by the SQUID, which is operated in flux-locked loop (FLL) mode and read out by the SQUID electronics. In the FLL mode, the SQUID is used as a null detector with negative feedback to maintain linear operation. The noise current I_N through the noise resistor is converted to a noise voltage $V_\mathrm{N} = I_\mathrm{N}\,(M_\mathrm{in}\,R_\mathrm{f})/M_\mathrm{f}$ at the output of the SQUID electronics, where M_in is the mutual inductance between the SQUID loop and the input coil, M_f is the mutual inductance between the SQUID loop and the feedback coil applied for the FLL mode, and R_f is the resistance of the feedback resistor. Then, the noise voltage V_N is digitized, filtered in the time domain for glitches, Fourier transformed and averaged to obtain the power spectral density (PSD) of the noise spectrum. Before analysis, the PSD is subjected to a peak removal algorithm to remove non-thermal noise pickup. Finally, the noise temperature T_{CSNT} is computed from the PSD using the equation:

$$T_{\mathrm{CSNT}} = \frac{R}{4\,k}\left(\langle S_{V,\mathrm{N}}\rangle - \langle S_{\mathrm{V,SQUID}}\rangle\right)\left(\frac{M_{\mathrm{f}}}{M_{\mathrm{in}}\,R_{\mathrm{f}}}\right)^{2}, \tag{4}$$

where $\langle S_{V,\mathrm{N}}\rangle$ is the averaged PSD of the measured noise voltage in the limit of zero frequency obtained from a fit to the PSD and $\langle S_{V,\mathrm{SQUID}}\rangle$ is the averaged PSD of the noise contribution from the SQUID and the SQUID electronics. $\langle S_{V,\mathrm{SQUID}}\rangle$ is a system parameter, which does not depend on the temperature to be measured, provided the SQUID is operated at constant temperature. Typically, $\langle S_{V,\mathrm{N}}\rangle \gg \langle S_{V,\mathrm{SQUID}}\rangle$. The uncertainty of the measured noise temperature T_{CSNT} (see Table 2.) is determined by system parameters, which must be measured independently [Shibahara 2016]: the value of the noise resistor R, the value of the feedback resistor R_{f}, the mutual inductances between the SQUID and the input coil M_{in} as well as the feedback coil M_{f}, and the non-thermal noise components from the SQUID and the SQUID electronics.

3.3. Primary magnetic field fluctuation thermometer, pMFFT

The primary magnetic field fluctuation thermometer (pMFFT) [Kirste 2016] utilizes the fact that thermally activated noise currents in the noise resistor cause fluctuating magnetic fields, which are accessible above the surface of the resistor. These magnetic field fluctuations are measured as thermal magnetic flux noise using a SQUID-based gradiometer. The pMFFT employs two SQUID channels (labeled 1 and 2) and a cross-correlation technique for the thermal magnetic flux noise measurement to minimize uncorrelated non-thermal noise contributions. Each channel is connected to an independent, planar thin-film detection coil. The layout of the detection coils enables analytical electromagnetic modelling and an easy determination of the geometric dimensions of the gradiometers and their distances to the noise resistor necessary for the calibration of the SQUID channels. The pMFFT has a separate calibration coil, which is used to determine the transfer functions (flux-to-voltage calibration coefficients) of the two SQUID gradiometer channels and the electrical conductivity σ_{el} of the noise resistor. The final equation for calculating the noise temperature T_{pMFFT} is:

$$T_{\mathrm{pMFFT}} = \frac{\hat{V}_{R\mathrm{cal},1}\cdot\hat{V}_{R\mathrm{cal},2}\cdot M_1\cdot M_2}{\hat{V}_{\mathrm{cal},1}\cdot\hat{V}_{\mathrm{cal},2}\cdot R_{\mathrm{cal}}^{2}}\cdot\frac{1}{N_f}\sum_{k=1}^{N_f}\frac{\mathrm{Re}\left[S_{V,12}(f_k,V_1,V_2)\right]}{\mathrm{Re}\left[S_{\Phi,12}(f_k,\sigma_{\mathrm{el}},T_{\mathrm{Ref}},\ldots)\right]/T_{\mathrm{Ref}}}, \tag{5}$$

where Re is the real part of the complex cross-power spectral densities (CPSDs), N_f denotes the number of frequency bins considered in the CPSD of the noise $S_{V,12}$. The first part of Equation (5) is related to the calibration of the SQUID channels 1 and 2. R_{cal} is the value of the resistor used to measure the current through the calibration coil, M_1 and M_2 are calculated values (based on the given geometry of the pMFFT setup) of the mutual induction between calibration coil and the respective detection coil. The four voltage amplitudes are determined during calibration using quasi-dc currents: $\hat{V}_{\mathrm{cal},i}$ is the voltage of the signal channel i (1, 2), while $\hat{V}_{R\mathrm{cal},i}$ is the corresponding voltage drop at the calibration resistor. The second part of Equation (5) provides the actual temperature information as obtained from the measured CPSD $S_{V,12}$, normalized by the CPSD of the thermal magnetic flux noise $S_{\Phi,12}$ calculated for the given geometrical setup and averaged over suitable frequency bins f_k [Varpula 1984, Kirste 2008]. $S_{\Phi,12}$ originates from a forward calculation and is proportional to the arbitrary reference temperature T_{Ref} chosen for this calculation. Since $S_{\Phi,12} \propto T_{\mathrm{Ref}}$, T_{Ref} effectively cancels out in the ratio of both quantities. Hence T_{Ref} is only formally introduced in (5) so that T_{pMFFT} can be represented as a ratio of common physical quantities [Kirste 2016]. The pMFFT setup allows to check in situ the electrical conductivity of the noise resistor, which turns out to be independent of temperature. Hence, the pMFFT can be also operated in relative primary mode as a MFFT.

4. Relative primary low-temperature JNT

In relative primary low-temperature JNT, ratios of temperatures are determined from the ratio of the measured noise PSD $S(f, T)$ to the noise PSD $S(f, T_{\mathrm{Ref}})$ measured at a reference temperature T_{Ref} for which the thermodynamic value is known. An essential requirement for this operational mode of low-temperature JNT is the independence of the value of the noise resistor of temperature in the regime of residual resistance. Therefore, the noise resistor usually is made of a high-purity metal containing negligible amount of magnetic impurities to ensure a constant electrical conductivity over the temperature range of operation. Then, the thermodynamic noise temperature T_{JNT} is calculated according:

$$T_{\mathrm{JNT}} = T_{\mathrm{Ref}} \left\langle \frac{S(f,T)}{S(f,T_{\mathrm{Ref}})} \right\rangle. \quad (6)$$

The brackets $\langle\cdot\rangle$ stand for averaging over a suitable frequency range. Even though relative primary low-temperature JNT avoids the elaborate determination and calibration of the system parameters of the noise thermometer setups, for reaching a desired uncertainty level, it may be necessary to subtract from the corresponding PSDs the contributions of non-thermal noise sources. In case of the pMFFT, the PSDs in Equation (6) correspond to the real part of the CPSD $\mathrm{Re}[S_{V,12}]$ measured for the considered frequency bins at T and T_{Ref}, respectively.

5. Attainable uncertainties with low-temperature JNT

5.1. JNT with the RSQUID setup

The relative combined standard uncertainties of the thermodynamic temperature values determined with rf RSQUIDs were estimated in [Soulen 1994] to range from 0.13% to 0.38% based on comparisons with reference temperatures according to the EPT-76 scale [BIPM 1979] and with ^{60}Co nuclear orientation thermometry, another variant of primary low-temperature thermometry [Hudson 1975]. The lowest relative standard uncertainty of about 0.07% was achieved by another rf RSQUID-based JNT realization [Fellmuth 2003]. As an example, the uncertainty budget for such noise measurements is given in Table 1.

5.2. Current sensing noise thermometer, CSNT

For the CSNT operated in absolute primary mode, relative combined standard uncertainties of about 1.5% are reached, which are expected to be lowered to sub-percent level with an improved setup. The corresponding uncertainty budget is shown in Table 2. For further details see [Shibahara 2016] and references therein. International comparison measurements within the European EURAMET Project 'Implementing the new kelvin' have shown that thermodynamic temperature values and their uncertainties determined with the CSNT agree with other determinations of thermodynamic temperature as well as with copies of the PLTS-2000 better than 1% [Engert 2016].

Table 1. Relative standard uncertainty estimates u_{rel} for the PTB noise thermometer (coverage factor k = 1) [Fellmuth 2003].

Uncertainty component	u_{rel} %
Measurement of R	0.020
Gate time τ	0.003
Filter correction	0.010
Interference by external magnetic fields with the mains frequency	0.020
Statistics (5 measurements over 11 hours with τ = 20 s)	0.060
Relative combined standard uncertainty	0.067

Table 2. Uncertainty budget for a temperature measurement at 20 mK using a CSNT in absolute primary mode (noise resistor of approximately 2 mΩ, u_{rel} – relative standard uncertainty, u – standard uncertainty, last column is the relative contribution to the combined standard uncertainty, coverage factor k = 1) [Shibahara 2016].

Uncertainty component	unit	value	u_{rel} %	u mK	rel. contribution %
Noise resistor R	mΩ	1.86	0.68	0.137	20.92
PSD of thermal noise, $\langle S_{V,\mathrm{N}} \rangle$	μV Hz$^{-1/2}$	35.06	0.10	0.040	1.79
PSD of SQUID noise, $\langle S_{V,\mathrm{SQ}} \rangle$	nV Hz$^{-1/2}$	388.0	0.30	0.004	0.02
Mutual inductance of input coil M_{in}	nA/Φ_0	306.2	0.30	0.118	15.55
Mutual inductance of feedback coil M_f	μA/Φ_0	43.59	0.58	0.234	61.14
Feedback resistor R_{f}	kΩ	10	0.06	0.023	0.60
Temperature gradients	μK	16.34	11.55	0.003	0.01
T_{CSNT} / mK	combined standard uncertainty / mK			relative combined standard uncertainty / %	
20.000	0.299			1.53	

5.3. Primary magnetic field fluctuation thermometer, pMFFT

For the pMFFT, the relative combined standard uncertainty of temperature measurements is currently 0.6% and is expected to be reduced further with the improvement of the geometric setup [Kirste 2016]. For the operation in absolute primary mode, the uncertainty budget for the pMFFT is shown in Table 3. Here, the first 5 uncertainty components are related to the determination of the relevant geometric parameters of the pMFFT necessary for the analytical modelling and the calculation of T_{pMFFT}. The following uncertainty contributions result from the determination of the electrical conductivity of the noise sensor, from the measurement equipment, from estimates of influence of noise sources other than the noise senor as well as from estimates of deviations of the analytical model from the practical implementation of the pMFFT. For more details and further explanation see [Kirste 2016]. The uncertainty budget for the operation in relative primary mode can be found in [Engert

2016]. As for the CSNT, comparison measurements within the European EURAMET Project 'Implementing the new kelvin' have shown that thermodynamic temperature values and their uncertainties determined with the pMFFT agree with other determinations of thermodynamic temperature as well as with copies of the PLTS-2000 better than 1% [Engert 2016]. The relative combined standard uncertainty of T determined by relative primary low-temperature JNT is about 0.14% for the pMFFT [Kirste 2014].

Table 3. Uncertainty budget for a temperature measurement at 16 mK using a pMFFT in absolute primary mode (u – standard uncertainty, c_i - sensitivity coefficients, last column is the relative contribution to the combined standard uncertainty, coverage factor k = 1) [Kirste 2016].

Uncertainty component x_i	unit	value	$u(x_i)$	$c_i \cdot u(x_i)$ K	rel. contribution %
type B					
z	m	$1.038\cdot10^{-4}$	$2.74\cdot10^{-7}$	$2.92\cdot10^{-5}$	9.30
d_{31}	m	$2.016\cdot10^{-3}$	$1.45\cdot10^{-6}$	$-8.75\cdot10^{-5}$	83.68
t	m	$1.857\cdot10^{-3}$	$1.57\cdot10^{-6}$	$-4.30\cdot10^{-12}$	0.00
$r_{1,1} \ldots r_{9,1}, r_{1,2} \ldots r_{9,2}$	m	$6.485\cdot10^{-4}$	$1.15\cdot10^{-7}$	$-3.53\cdot10^{-6}$	0.14
$r_{1,3} \ldots r_{93}$	m	$6.425\cdot10^{-4}$	$1.15\cdot10^{-7}$	$1.14\cdot10^{-5}$	1.43
σ	$(\Omega\cdot m)^{-1}$	$5.915\cdot10^{9}$	$3.59\cdot10^{7}$	$1.43\cdot10^{-5}$	2.23
μ_r		1.00	$2.80\cdot10^{-6}$	$-9.06\cdot10^{-8}$	0.00
Flatness of ICL transfer function depending on bandwidth $f_{3\,dB,k}$	Hz	∞	$6.20\cdot10^{5}$	$-4.18\cdot10^{-7}$	0.00
Stability of feedback resistor $R_{f,k}$ of the SQUID electronics	Ω	$3.00\cdot10^{4}$	$8.66\cdot10^{-1}$	$9.34\cdot10^{-7}$	0.01
AC voltage meas. (ADC gain, flatness)		1.00	$1.73\cdot10^{-4}$	$1.68\cdot10^{-5}$	3.09
Timing accuracy of the ADC (relative)		1.00	$1.16\cdot10^{-5}$	$1.87\cdot10^{-7}$	0.00
Correlation limit $S_{\Phi,12,min}$	Φ^2/Hz	(0)	$1.07\cdot10^{-15}$	$2.63\cdot10^{-6}$	0.08
Edge effect M_k/M_k(unshielded)		1.00	$6.06\cdot10^{-6}$	$-1.96\cdot10^{-7}$	0.00
Edge effect $S_\Phi/S_\Phi(\infty)$		1.00	$3.18\cdot10^{-7}$	$5.14\cdot10^{-9}$	0.00
Edge effect S_Φ/S_Φ(unshielded)		1.00	$3.18\cdot10^{-7}$	$5.14\cdot10^{-9}$	0.00
Parasitic coil areas, S_Φ/S_Φ(model)		1.00	$4.45\cdot10^{-5}$	$-7.19\cdot10^{-7}$	0.01
R_{cal}	Ω	$9.988\cdot10^{2}$	$2.01\cdot10^{-2}$	$-6.51\cdot10^{-7}$	0.00
type A					
Effect of signal-to-noise ratio, N_f and N_{avg} on CPSD in $T_{12}(f)$		$1.617\cdot10^{-2}$	$1.62\cdot10^{-6}$	$1.62\cdot10^{-6}$	0.03

T_{pMFFT} / mK	combined standard uncertainty / mK	relative combined standard uncertainty / %
16.175	0.096	0.59

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