***MeP*-K relative primary thermometry (radiometry)**

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Version 7.0 (revision of the 07 August 2017 document)

Date: Final revision 21 May 2024. Temperature values of HTFP Fe-C, Pd-C, Ru-C and WC-C inserted 20 March 2024, alongside those already in the table of Co-C, Pt-C and Re-C. Approved CCT WG NCTh 12 April 2024. Uncertainties of Fe-C, Pd-C, Ru-C and WC-C updated 21 May 2024.

This is the second revision of the temperature values of HTFPs. The first version was issued on 24th July 2012, the second version 07 August 2017. The document is unaltered except for:

1) Section 1: Low uncertainty temperatures for high temperature fixed points of Fe-C, Ru-C and WC-C
2) Section 1: An updated value for the Pd-C point – superseding the entry in the 2017 document.
3) References: The replacement of the original reference [3] in Section 1with an updated reference.

Section 1.0: Definitive values for a selected set of high-temperature fixed points (HTFPs)

The below tabulated point of inflection (poi) and equilibrium liquidus (elq) thermodynamic temperatures were agreed for Fe-C, Co-C, Pt-C, Ru-C, Re-C and WC-C as definitive values by the CCT WG NCTh meeting 14 May 2024, the uncertainties for the Fe-C, Pd-C, Ru-C and WC-C were revised after that meeting. The revised uncertainty values are included in the table below. The values are taken from [1], [2] and [3] respectively.

HTFP	Thermodynamic temperature (poi) /K	Uncertainty (poi) (*k*=2) /K	Thermodynamic temperature (elq) /K	Uncertainty (elq) (*k*=2)/K
Fe-C	1426.92	0.14	1427.02	0.16
Co-C	1597.39	0.13	1597.48	0.14
Pd-C	1765.05	0.16	1765.18	0.18
Pt-C	2011.43	0.18	2011.50	0.22
Ru-C	2226.99	0.24	2227.08	0.24
Re-C	2747.84	0.35	2747.91	0.44
WC-C	3020.85	0.40	3020.92	0.40

This table will be reviewed and updated periodically.

References for Section 1.0.

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Section 2.0: Relative primary spectral-band radiation thermometry

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Introduction

This section of the *MeP*-K outlines the relative primary thermometry that could be used to realise thermodynamic temperature based on HTFPs. It is anticipated that this approach will give uncertainties similar to (though slightly higher than) absolute primary spectral-band radiometry, but will be considerably easier to implement. Absolute primary radiometry (n = 0), and ITS-90 (n = 1) are described in different sections of the *MeP*-K. The relative primary thermometry methods fall into three approaches:

- Extrapolation based on one fixed point.
- Interpolation between two or three fixed points.
- Least-squares fitting if more than three fixed points are used.

It is assumed that the fixed points used for each of the approaches will have assigned thermodynamic temperatures and associated uncertainties.

Mathematical Model of the Measurement

The measurement equation for the radiometer signal, $S(T)$, is the following [1]:

$$S(T) = K\int_0^\infty \varepsilon\left(\lambda,T\right) s(\lambda) L_\mathrm{b}\left(\lambda,T\right) d\lambda \, . \qquad (1)$$

The terms in the equation are: K is a geometric factor associated with the actual form of primary radiometry, $\varepsilon\left(\lambda,T\right)$ the emissivity of the blackbody radiator, $s\left(\lambda\right)$ the spectral responsivity, $L_\mathrm{b}\left(\lambda,T\right)$ the Planck function.

Since equation (1) is not well suited to interpolation or least-squares fitting when n > 1, because of the integral over the spectral responsivity, the radiometer signal must be approximated by an analytic equation with a number of adjustable parameters. The Planck version of the Sakuma–Hattori equation [2] is recommended:

$$S(T) = \frac{C}{\exp\left(\dfrac{c_2'}{AT+B}\right)-1} \, , \qquad (2)$$

where $c_2' = c_2/n = hc/kn$, where k is the Boltzmann constant, h is the Planck constant, c the speed of light in a vacuum, n is the refractive index of the gas in the optical path and A, B and C are related to the radiometer's spectral responsivity, $s(\lambda)$. Measured (T, S) pairs form the raw data for the extrapolation, interpolation and least-squares fitting. One important condition is that the bandwidth of the radiometer must be narrow compared to the centre wavelength, $\Delta\lambda/\lambda << 1$.

More details of these approaches can be found in [3], particularly Annex 2, and in [1].

<u>Extrapolation using one fixed point</u>
This n = 1 scheme is equivalent to the ITS-90 formalism, but uses the current thermodynamic temperature of the fixed point (as well as knowledge of the relative spectral responsivity) to extrapolate to other thermodynamic temperatures. However, the choice of the fixed point is not restricted to the Ag, Au or Cu point, as in the ITS-90. The uncertainty associated with the thermodynamic temperature of the fixed point must be taken into account in the uncertainty analysis (this uncertainty is zero for the ITS-90 approach).

<u>Interpolation between two fixed points</u>
Two fixed points (n = 2) are the minimum required for interpolation [4, 5, 6]. If the two fixed points are as far from each other as practically feasible (e.g., the Ag point and the highest available HTFP), the scheme combines extension in range with a very low uncertainty. Some extrapolation beyond the calibration points may also be acceptable. Thus, the scheme can be successfully implemented over the range from about 1000 K to 3300 K. Only a crude knowledge of the relative spectral responsivity function is required for successful implementation. The uncertainties associated with interpolation and extrapolation will clearly depend on the temperatures of the calibration points used and their associated uncertainties.

<u>Interpolation between three fixed-points</u>
An alternative option is a three-point interpolation scheme (n = 3) using one pure metal fixed point and two HTFPs, or alternatively three HTFPs. One advantage of this approach is that knowledge of the spectral responsivity function is no longer needed. A disadvantage is that the uncertainty tends to have slight oscillations between the interpolating points. In this case extrapolation beyond the interpolating end points is not recommended because of the rapid increase in uncertainty.

<u>Least squares approach</u>
Using more than three fixed points allows T to be realised by least-squares fitting of equation (2) to the measured (T, S) pairs. Because the uncertainty components associated with the calibration of the radiometer are generally reduced approximately by a factor of $\sqrt{n/3}$, where n is the number of fixed points, the total uncertainty is reduced as the number of fixed points increases. However, the uncertainty associated with measuring the unknown temperature is the same as for the three-point interpolation scheme, so the total uncertainty may not be significantly lower than the three-point scheme. A particular advantage is that the redundancy of HTFPs in a least-squares scheme provides additional security in realisation of temperatures. Extrapolation beyond the extreme end points is again not recommended.

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