Updated Estimates of the Differences between Thermodynamic Temperature T and the ITS-90 Temperature T_{90}

In 2010, at the CCT's request, Working Group 4 (WG4) critically reviewed all available measurements of T-T_{90} including constant-volume gas thermometry, acoustic gas thermometry, spectral radiation thermometry, total radiation thermometry, noise thermometry, and dielectric-constant gas thermometry. These were published in 2011 [1]. Consensus estimates were provided for $T - T_{90}$, deduced from selected measurements from 4.2 K to 1358 K, as well as a recommendation for analytic approximations to T-T_{90} for the range 0.65 K to 1358 K [1]. Due to significant progress in the field of contact thermometry below 335 K since that time, CCT working group on contact thermometry (WG-CTh) according to its terms of reference reviewed all new available T-T_{90} data sets since 2011 leading to updated values for T-T_{90} [2] with in part a reduction of one order of magnitude in uncertainty compared to the 2011 estimates. Note that the best estimate of the thermodynamic temperature of the triple point of water remains 273.1600(1) K with the standard uncertainty given in brackets, and readers who wish to use the triple point of water as a thermodynamic reference point should continue to use this value and uncertainty.

Table of Differences

Above 335 K, Table 1 summarizes the best estimates of T-T_{90} in 2011. These are still considered the best estimates in this temperature region due to the paucity of new values between 2011 and 2022. In this case a weighted average was formed using the uncertainties identified by WG4 (details see [1]), and the values in Table 1 were, therefore, the direct representation of the best estimates of T-T_{90} and their respective uncertainties.

In the range below 335 K, the primary representation of the updated values is given by a 12th-order polynomial (coefficients given in Table 2). These were determined as a result of a weighted least squares fit to all the relevant data [2]. For convenience, below 335 K, the calculated T-T_{90} estimates together with the uncertainties derived at these temperatures (a combination of the fit uncertainty and further components (for details see [2])) are given in Table 1. The uncertainties below 335 K, given in Table 1, have further been fitted for convenience using a 6th order polynomial obtained by a least-squares fit to the data and the coefficients are given in Table 2.

Table 1 Estimates of T-T_{90} between 4.2 K and 1358 K. The substances of the defining fixed points and secondary reference points of the ITS-90 are marked in the 2nd and 6th columns. All uncertainties are standard uncertainties (k = 1). Values below 335 K have been updated in 2022, the values above 335 K are identical with the 2011 estimates.

T_{90} K		T-T_{90} mK	u mK	T_{90} K		T-T_{90} mK	u mK
4.2		0.00	0.13	161.405	Xe	-7.34	0.27
5		0.07	0.13	195		-4.73	0.23
6		0.16	0.12	234.3156	Hg	-2.89	0.13
7		0.22	0.12	255		-1.97	0.15
8		0.27	0.13	273.16	TPW	-0.07	0.12
9.288	Nb	0.32	0.13	290		2.29	0.23
11		0.36	0.15	302.9146	Ga	3.84	0.34
13.8033	e-H_2	0.36	0.19	335		7.09	0.60
17.035	e-H_2	0.29	0.19	373.124	H_2O	9.74	0.6
20.27	e-H_2	0.16	0.19	429.7485	In	10.1	0.8
22.5		0.05	0.19	505.078	Sn	11.5	1.3
24.5561	Ne	-0.06	0.20	600.612	Pb	9.21	6.1
35		-0.76	0.26	692.677	Zn	13.8	6.9
45		-1.51	0.17	800		22.4	6.4
54.3584	O_2	-2.21	0.14	903.778	Sb	27.6	7.6
70		-3.30	0.15	933.473	Al	28.7	6.6
77.657		-3.80	0.15	1052.78	Cu/Ag	40.9	26
83.8058	Ar	-4.21	0.15	1150		46.3	20
90		-4.62	0.16	1234.93	Ag	46.2	14
100		-5.32	0.20	1337.33	Au	39.9	20
130		-7.30	0.27	1357.77	Cu	52.1	20

Interpolation Functions

From 0.65 K to 2 K, the polynomial for the temperature scale PTB-2006 (based on the ^{3}Helium vapor-pressure) should be used [3] with

$$T - T_{90} \equiv T_{2006} - T_{90.}$$

The recommendation is supported by a recently performed direct comparison of the melting and vapor pressures of helium-3 at the LNE [4].

Below 1 K, T_{2006} is identical to $T_{\text{PLTS-2000}}$. From 2 K to 4 K,

$$T - T_{90} = 0.$$

From 4 K to 335 K,

$$D(T_{90})\,/\,\text{mK} = (T - T_{90})\,/\,\text{mK} = \sum_{i=0}^{n} a_i (T_{90}/\text{K})^i \qquad (1)$$

with the coefficients given in Table 2:

Table 2 (Published as Table 3 in [2] Copyright 2022 Author(s), licensed under a Creative Commons Attribution 4.0 IGO (CC BY 4.0 IGO) license.) Fitting coefficients for the power series approximating the $(T - T_{90})$ input data in the range from 4 K to 335 K ($D(T_{90})$: a_i) and its combined standard uncertainty estimates ($u(D(T_{90}))$: b_i) resulting from a least-squares fit to u listed in Table 1 in the specific temperature range.

i	a_i	b_i
0	-6.393509785E-01	6.362639E-02
1	2.044362025E-01	1.251359E-02
2	-1.453482491E-02	-3.880108E-04
3	4.860355653E-04	4.878407E-06
4	-1.152913045E-05	-2.789077E-08
5	1.932372065E-07	7.268939E-11
6	-2.222708123E-09	-6.999818E-14
7	1.722390583E-11	
8	-8.878574513E-14	
9	2.985516966E-16	
10	-6.273436285E-19	
11	7.467125710E-22	
12	-3.840581614E-25	

From 335 K to 1357.77 K (copper point):

If it is not convenient to use Table 1, the differences T-T_{90} may be approximated by the following expressions. The relative differences of the interpolation functions (with respect to the values of Table 1) are less than 15 %, except at 600 K and the gold point.

$$D_{2011}(T_{90}) / \mathrm{mK} = (T - T_{90}) / \mathrm{mK} = (T_{90} / \mathrm{K}) \sum_{i=0}^{4} c_i (273.16\ \mathrm{K} / T_{90})^{2i} \quad (2)$$

with the coefficients:

$$c_0 = 0.0497 \quad c_1 = \text{-}0.3032 \quad c_2 = 1.0254 \quad c_3 = \text{-}1.2895 \quad c_4 = 0.5176$$

Users working in a broader temperature range above and below 335 K might be interested in a smooth correction. A smooth transition from the new function $D(T_{90})$ to the old WG4 fitting function for temperatures from the TPW up to the copper point, $D_{2011}(T_{90})$, is the point of intersection at T_{90} = 288.418 K. In addition, the change of slope at the transition point is very small. If the user is only interested in the temperature range below 335 K, the new function $D(T_{90})$ is recommended.

References

[1] J. Fischer, M. de Podesta, K. D. Hill, M. Moldover, L. Pitre, R. Rusby, P. Steur, O. Tamura, R. White, L. Wolber, *Int. J. Thermophys.* **32,** 12-25 (2011).

[2] C. Gaiser, B. Fellmuth, R. M. Gavioso, M. Kalemci, V. Kytin, T. Nakano, A. Pokhodun, P. M. C. Rourke, R. Rusby, F. Sparasci, P. P.M. Steur, W. L. Tew, R. Underwood, R. White, I. Yang, J. Zhang, *J. Phys. Chem. Ref. Data* **51**, 043105 (2022)

[3] J. Engert, B. Fellmuth, K. Jousten, *Metrologia* **44**, 40-52 (2007).

[4] C. Pan, F. Sparasci, M. Plimmer, L. Risegari, J.-M. Daugas, G. Rouille, B. Gao, and L. Pitre, *Metrologia* **58**, 025005 (2021)

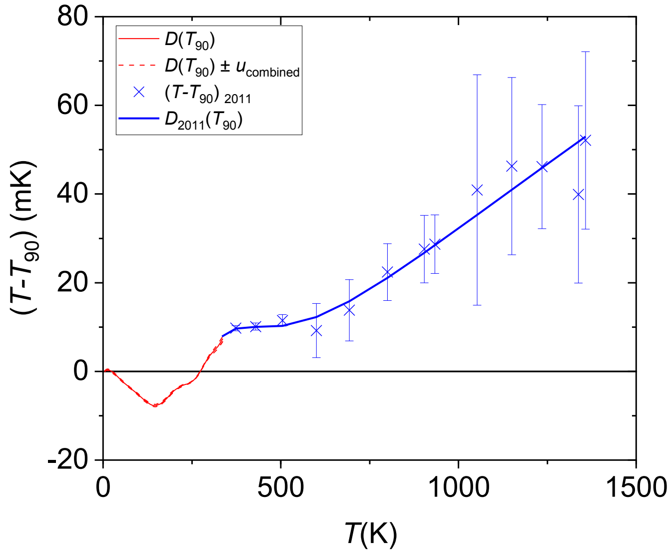

Fig. 1 Overview of consensus estimates for T-T_{90}. The red line shows the updated estimates $D(T_{90})$ and the red dashed lines envelop the range $\pm\, u(D(T_{90}))$, with the combined standard uncertainty $u(D(T_{90}))$. The blue crosses show the WG4 data contained in Table 1 as published in [1] and the corresponding error bars. The blue line shows $D_{2011}(T_{90})$.

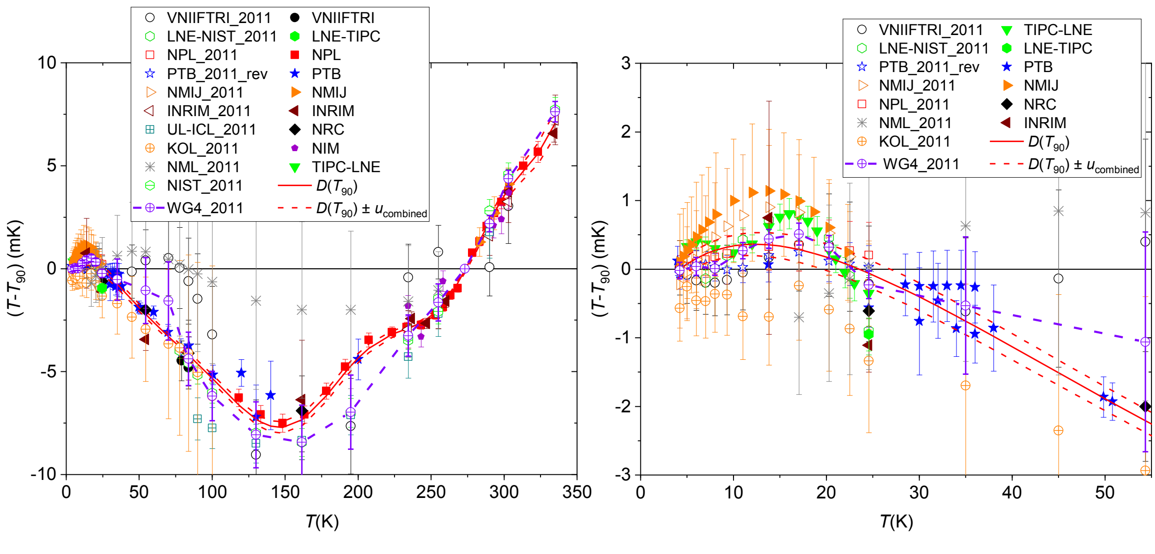

Fig. 2 (published as Fig 5 in [2] Copyright 2022 Author(s), licensed under a Creative Commons Attribution 4.0 IGO (CC BY 4.0 IGO) license.) Left: $(T - T_{90})$ data versus temperature T_{90}. The institutes at which the data has been obtained are listed in the legend. WG4 is the data contained in Table 2 of Ref. [1] and shown for comparison purposes. The red full line shows the fitting function $D(T_{90})$, and the dashed lines envelop the range $D(T_{90}) \pm u(D(T_{90}))$, with the combined standard uncertainty $u(D(T_{90}))$. Right: The same as on the left but in a restricted temperature range.