2023 CCU/CCQM workshop on: The metrology of quantities which can be counted

Stefan Kück, PTB
Report at the CCPR WG-SP
2023-09-09
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### Session 1, 28 March 2023, 12:00-14:00 UTC (14:00-16:00 CEST)
**Concepts and theoretical aspects of counting and the unit one**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Speaker</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welcome and background to the workshop</td>
<td>Pavel Nefedov (NSC-IIM)</td>
<td>15 min</td>
</tr>
<tr>
<td>What questions is the workshop addressing?</td>
<td>Bernd Götter (PTB)</td>
<td>15 min</td>
</tr>
<tr>
<td>Concepts of continuous quantities &amp; countable aggregates and nomenclature</td>
<td>Charles Ehrlich (NIST)</td>
<td>15 min</td>
</tr>
<tr>
<td>Quantities with the unit one</td>
<td>Peter Blattner (METAS)</td>
<td>15 min</td>
</tr>
<tr>
<td>Counting &amp; why it is different from amount of substance</td>
<td>Richard Brown (NPL)</td>
<td>15 min</td>
</tr>
</tbody>
</table>

| Panel Q&A / Discussion                                            | All                            | 45 min   |

### Session 2, 29 March 2023, 12:00-14:00 UTC (14:00-16:00 CEST)
**Counting entities (case studies from electricity, mass, chemistry and biology)**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Speaker</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction to the case studies</td>
<td>Richard Brown (NPL)</td>
<td>5 min</td>
</tr>
<tr>
<td>Counting electrons (CCEM)</td>
<td>Werner Schumacher (PTB)</td>
<td>15 min</td>
</tr>
<tr>
<td>Counting $^{232}$Si in a silicon sphere (CCQM, CCM)</td>
<td>Olaf Rionitz (PTB)</td>
<td>15 min</td>
</tr>
<tr>
<td>Digital PCR</td>
<td>Inchul Yang (KRISS)</td>
<td>15 min</td>
</tr>
<tr>
<td>Counting cells</td>
<td>Jonathan Campbell (LGC)</td>
<td>15 min</td>
</tr>
<tr>
<td>Counting particles in air</td>
<td>Konstantina Vasilatou (METAS)</td>
<td>15 min</td>
</tr>
</tbody>
</table>

| Panel Q&A / Discussion                                            | All                            | 40 min   |

### Session 3, 30 March 2023, 12:00-14:00 UTC (14:00-16:00 CEST)
**Counting processes & other phenomena (case studies from radioactivity to light)**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Speaker</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction to the case studies</td>
<td>Bernd Götter (PTB)</td>
<td>5 min</td>
</tr>
<tr>
<td>Counting in radionuclide metrology</td>
<td>Ryan Fitzgerald (NIST)</td>
<td>15 min</td>
</tr>
<tr>
<td>Counting not countable quantities – The CCL perspective</td>
<td>Alessandro Balsamo (INRIM)</td>
<td>15 min</td>
</tr>
<tr>
<td>The SI second as a count of oscillations and much more</td>
<td>Elizabeth Donley (NIST)</td>
<td>15 min</td>
</tr>
<tr>
<td>Candela - by counting photons?</td>
<td>Stefan Kück (PTB)</td>
<td>15 min</td>
</tr>
<tr>
<td>Discussion &amp; concluding remarks: how should the metrology community respond and next steps</td>
<td>Sang-Ryoun Park (KRISS) &amp; Joachim Ulrich (PTB)</td>
<td>55 min</td>
</tr>
</tbody>
</table>
2023 CCU/CCQM workshop on
The metrology of quantities which can be counted

Candela - by counting photons?

CCPR, Stefan Kück

In consultation with: Maria Luisa Rastello
Maria Nadal

Lecture, in excerpts
Overview

SI base unit: candela (cd)

Photometric units vs. Radiometric units

How to measure single photons

How to generate single photons

The candela in numbers

Photons and mole?

1 cd \( \triangleq 4.09 \ldots \times 10^{15} \) ph/(s sr)

152.83 \( \mu \text{mol/s/m}^2 \) (PPFD)
Photometric Units vs. Radiometric Units

To consider:
Measured quantities in photometry are spectrally integrated quantities!

\[ X_{v,x} = \frac{K_{cd}}{V_x(\lambda_a)} \int \lambda X_{e,\lambda}(\lambda)V_x(\lambda) \, d\lambda \]

The most important of these visual functions is the photopic luminous efficiency function for the light-adapted eye, \( V(\lambda) \), which is defined by the CIE over the wavelength range 360 nm to 830 nm at 1 nm intervals.
Nonetheless: the candela (cd) in numbers

A radiant intensity of $1/683$ W per steradian for photons with a frequency of $540 \times 10^{12}$ Hz corresponds to $1/683 \, W/(h\nu)$ photons per second per steradian:

$$\Rightarrow N/s = 1/683 \, W/(h\nu) = 1 \, Js^{-1} / (683 \times 6.626 \, 070 \, 15 \times 10^{-34} \, Js \times 540 \times 10^{12} \, s^{-1})$$

$$\Rightarrow N/s = 4.091942356... \times 10^{15} \, s^{-1}$$

I.e.,:

- the candela corresponds to $4.091942356... \times 10^{15}$ photons per second per steradian with photons at a frequency of $540 \times 10^{12}$.
- a nanocandela corresponds to $4.091942356... \times 10^{6}$ photons per second per steradian with photons at a frequency of $540 \times 10^{12}$.

Measurable (countable) with single-photon detectors!
How to measure single-photons?

Single Photon Avalanche Diode (SPAD)

$\eta \approx 80 \%$

Human Eye!

$\eta \text{ "significantly above chance"}$

$\eta > 95 \%$

Transition Edge Sensor (TES)

$\eta > 95 \%$

http://web.sensor-ic.com/


$\eta \approx 80 \%$

http://www.nist.gov

$\eta > 85 \%$

Superconducting Nanowire Single-Photon Detector (SNSPD)

http://www.nist.gov
Standard detector – Traceability

Classical Detector against (Cryogenic) Radiometer

Photon Counter by Spontaneous Parametric Down-Conversion

Validation

SNSPD, TES against Photon Counter

SNSPD, TES against Photon Counter

\[ u \approx 1 \times 10^{-5} \]

\[ u \approx 0.2\% \]

\[ \approx 100 \, \mu W \]

\[ \sim 10^6 \text{ attenuator} \]

\[ 1 \text{ nW} \]

\[ \sim 10^9 \text{ photons/s} \]

\[ 10^3 \text{ photons/s} \]

\[ 1 \text{ photon/s} \]

\[ u \approx 0.2\% \]
Standard detector – Traceability

Classical methods are much better than quantum methods for classical photometry and radiometry! At least so far...

\[ u \approx 1 \times 10^{-5} \]
\[ u \approx 0.2\% \]
What about sources?

Counting by generating!?
Single photon sources – how to?

Semiconductor quantum dots

Single molecules

Colour centres in diamond

I. Aharonovich et al., Rep. Prog. Phys. 74 076501 (2011)
My dream...

Radiant Flux Power

Number of photons

\[ \Phi = \frac{nhc}{t \lambda} \]

Photon energy
• Constants of nature
• Wavelength

Note:
Candela is the unit for luminous intensity, thus involving the steradian
My dream... comes true!

PULSED LASER \( f_{rep} \)

SPS

Single-Photon Detector

\[ n/t = f_{rep} \]

\[ \Phi = f_{rep} \frac{hc}{\lambda} \]

Absolute, predictable Single-Photon Source

\[ \Phi = \frac{n \cdot hc}{t \cdot \lambda} \]
Waking up is hard...!

PULSED LASER $f_{\text{rep}}$

SPS

Single-Photon Detector

$\Phi = f_{\text{rep}} \frac{hc}{\lambda}$

$\Phi = \frac{n}{t} \frac{hc}{\lambda}$

- Internal Quantum Efficiency < 100%
- Photon Collection Efficiency < 100%

Absolute, predictable Single-Photon Source
Motivation for single-photon sources in metrology

Quantum Radiometry

- Reduction of measurement uncertainty
- Standard source
- Realization of photon-number-based candela

\[
\Phi = f_{\text{rep}} \frac{hc}{\lambda}
\]

Sub-shot noise metrology

Ideal SPS has no noise!
Noise-reduced measurements:
- e.g. transmission measurement

\[
\frac{\Delta T^2_{\text{SP}}}{\Delta T^2_{\text{C}}} = 1 - 2\eta \frac{T}{1 + T}
\]

- \(\Delta T\): variance in transmission
- \(T\): transmission
- \(\eta\): total efficiency of setup

Applications, e.g.:
- quantum cryptography
- quantum repeater
- quantum computing

Photon statistics

Thermal light
Super-Poissonian-Distribution bunching effect

\[ p_{hv}^{\text{therm.}}(n) = \frac{\langle n \rangle^n}{(\langle n \rangle + 1)^{n+1}} \]

Coherent (Laser) light
Poissonian-Distribution

\[ p_{hv}^{\text{Laser}}(n) = \frac{\langle n \rangle^n e^{-\langle n \rangle}}{n!} \]

Nonclassical light
Sub-Poissonian-Distribution anti-bunching effect

\[ p_{hv}^{\text{Fock}}(n) = \delta_{n,m} \]

50 W
10^{19} Photonen/s

250 fW
10^6 Photonen/s
Photon statistics

With thermal or with laser light, there will always be - with a specific probability - more than one photon within a time slot!

Sub-Poissonian-Distribution anti-bunching effect

\[ P_{hv}^{\text{Fock}}(n) = \delta_{n,m} \]
Influence on measurement

Detected photons (1/s) vs. Photon rate at detector (1/s)

- Single-photon source
- Attenuated laser

Detection efficiency

miscounting!
Finally: the candela and the mole?

\[ 4.091942356 \times 10^{15} \text{ photons/(s sr)} \]

\[ = \]

\[ 6.794830142 \times 10^{-9} \text{ mol/(s sr)} \]

Note:
The mole is the unit of amount of substance
Photons sometimes are / behave like particles
Photons and mole?

**FULL SPECTRUM**
Sunlight for all stages of plant growth

- **Blue** (400-499nm): 21.99%
  - Blue rays help promote photosynthesis.
- **Green** (500-599nm): 36.87%
  - Green rays are meaningful for plant morphology.
- **Red** (600-699nm): 35.47%
  - Red rays are the most helpful for growth, bloom, and fruiting.
- **FR** (700-790nm): 5.78%
  - FR helps regulate physiological activities such as shading and flowering.

**PPFD**
Photosynthetic Photon Flux Density

**HIGH PPFD**
PPFD is measuring how much photons actually land on the canopy, the higher the better.

**SANSI 2er Pack LED Pflanzenlampe**
Vollspektrum E27 15W Weiß
Model: C23ZW004-V0015A27

152.83 µmol/s/m²

**PPFD: Photosynthetic Photon Flux Density**
Photons and mole?

FULL SPECTRUM
Sunlight for all stages of plant growth

**Action spectrum**

By Original: Daniele Pugliesi
Vector: M0tty - This file was derived from: Chlorophyll ab spectra2.png, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=20509583

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PPFD: Photosynthetic Photon Flux Density

200W Equivalent

By Original: Daniele Pugliesi
Vector: M0tty - This file was derived from: Chlorophyll ab spectra2.png, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=20509583
Take home messages

Candela – by counting photons?
  • No, at least not yet

Nonetheless, **counting photons** is useful for many applications, e.g.:
  • Quantum communication
  • Quantum computing
  • Low flux radiometry / Quantum radiometry

**Photons and mol:**
  • PPFD: µmol/s/m²
  • Are photons „entities“ or an „amount of substance“, i.e., are they like Ni-atoms or like fish?
Conference Report

Report of the CCU/CCQM Workshop on “The metrology of quantities which can be counted”

Richard J. C. Brown,*, Bernd Göttler,*, Pavel Neyezhmakov†, Michael Stock‡, Robert I. Wielgosz‡, Stefan Kück‡
and Konstantina Vasilatou

* National Physical Laboratory, Hampton Road, Teddington, TW11 0LW, UK
† University of Surrey, Guildford, GU2 7XH, UK
‡ Physikalisch-Technische Bundesanstalt, 38116 Braunschweig, Germany
†† National Scientific-Centre “Institute of Metrology”, UA-61002 Kharkiv, Ukraine
‡‡ Bureau International des Poids et Mesures, Pavillon de Breteuil, 92312 Sèvres CEDEX, France
§ Federal Institute of Metrology METAS, Lindenhof 50, 3033 Bern, Switzerland
* Correspondence: Corresponding authors. E-mail: richard.brown@npl.co.uk, bernd.gottler@ptb.de

Abstract: This article provides a report of the recent workshop on “The metrology of quantities which can be counted” organised jointly by the International Committee for Weights and Measures' Consultative Committees for Amount of Substance (CCQM) and for Units (CCU). The workshop aimed to trigger a discussion on counting and number quantities across the metrological community so that a common understanding of counting and a common nomenclature could be achieved and there was clarity on the differences between these increasingly important concepts. This article details the background to the workshop, provides a summary of the presentations given and the discussions on the topics raised. It also reports the conclusions, agreed actions and next steps resulting from the workshop.

Keywords: metrology; units; dimensionless quantities; one; counting
The presentation “Candela – by counting photons?” offered an overview of the SI unit candela and its measurement. The candela, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency $540 \times 10^{12}$ Hz, $K_{cd}$, to be 683 when expressed in the unit lm/W, which is equal to cd sr W$^{-1}$, or cd sr kg$^{-1}$ m$^{-2}$ s$^3$, where the kilogram, metre and second are defined in terms of h, c and $\Delta v_C$. This means, that the candela corresponds to a radiant intensity of 1/683 watt per steradian for monochromatic radiation of frequency $540 \times 10^{12}$ hertz. Measured quantities in photometry must be considered as spectrally integrated quantities, where the integration is carried out over the product of the radiometric quantity and a luminous efficiency function. The most important of these functions is the photopic luminous efficiency function for the light-adapted eye, $V(\lambda)$, which is defined by the CIE over the wavelength range 360 nm to 830 nm at 1 nm intervals.

Expressing the candela numerically, a radiant intensity of 1/683 W/sr corresponds to $4.091942356\ldots \times 10^{15}$ photons/(sr s) at a frequency of $540 \times 10^{12}$ Hz. Single photon detectors, like Si single-photon avalanche diode (SPAD) detectors, can measure lower radiant intensities, e.g., $4.091942356\ldots \times 10^6$ (photons/(sr s)), which corresponds to 1 nCd, however, traceability to classical radiometric methods is currently more accurate than to quantum-based approaches. Generating single photons is another promising method in the realm of photon techniques, utilizing sources like semiconductor quantum dots, single molecules, or colour centres in diamond. However, the accuracy of measurement is influenced by internal quantum efficiency and photon collection efficiencies. It is important to emphasize that the candela is a unit for luminous intensity, so it must include the steradian, which is sometimes omitted in these considerations. Realizing the candela by counting or producing single photons is currently not as accurate as the classical method of using a cryogenic radiometer.

Despite limitations, single-photon sources find uses in quantum metrology, in particular quantum radiometry and sub-shot noise metrology. They offer sub-Poissonian photon statistics and exhibit the anti-bunching effect, which classical light sources or lasers cannot achieve. Single photon sources are particularly valuable when paired with digital detectors like SPAD detectors. The presentation also explored the relation between the candela and the mole. In principle, the mole can replace the number of photons, expressing the candela as $6.794830142\ldots \times 10^9$ (mol/sr)/s at $540 \times 10^{12}$ Hz. Notably, the mole is the unit of amount of substance, and photons sometimes behave like particles. In horticulture, photons and the mole are combined in units like PPFD (photosynthetic photon flux density) with $(\mu$mol/m$^2$/s). However, merely knowing the number of photons is insufficient: understanding the spectrum of photons and the receiver's action spectrum is also essential. To summarize, although counting photons is valuable in various applications, realizing the candela through photon counting is currently suboptimal. Emerging fields like horticulture lighting emphasize the significance of combining photons and the mole. The question of whether photons numbers can be described as amount of substance remains open for discussion.
New task group: CCU TG-ADQSIB-FG:CNQ

CCU Task Group on angle and dimensionless quantities in the SI Brochure – Focus Group: Counting and number quantities (CCU TG-ADQSIB-FG:CNQ)

Introduction and draft proposals for comment

Richard Brown

Online meeting 2023-09-11