

## **Guidelines for RMO PR Key Comparisons**

CCPR-G6 Version 1.0, 10 October 2014

These guidelines are prepared by CCPR WG-KC and RMO P&R representatives, and approved by CCPR to ensure that RMO Key Comparisons are prepared and performed in a uniform manner and that the results be linked to the CCPR KCs appropriately. This document is to supplement the CIPM guidelines on measurement comparisons [1]. This document does not cover RMO bilateral KCs, for which another Guideline document, CCPR-G5, is available.

### **1. Preparation for the comparison**

#### **1.1 Appointment of a pilot laboratory**

- 1.1.1 The pilot laboratory is appointed by the RMO.
- 1.1.2 The pilot laboratory is responsible for developing the comparison protocol, in consultation with the comparison's Task Group, preparing and transfer standard artifacts (artifacts may also be prepared by participants), conducting measurements of all transfer standards to provide linkage of all participants' results, and preparing the comparison report.
- 1.1.3 It is preferred that the pilot laboratory of an RMO KC should be a participant of the previous or current CCPR KC of that quantity, but it is not a requirement as long as other participants can serve as link laboratories.
- 1.1.4 It is important for the pilot laboratory to have small uncertainty associated with random effects (not necessarily a small uncertainty associated with systematic effects) in the calibration of the comparison quantity so that a small uncertainty can be obtained in linking the results to CCPR KC.

#### **1.2 Appointment of link laboratories**

- 1.2.1 A link laboratory is a participant of the RMO KC that has been a participant of the previous CCPR KC of the same quantity and provides the link of results between the RMO KC and CCPR KC of the quantity.
- 1.2.2 Generally there must be two link laboratories in an RMO KC. It is important that link laboratories have small uncertainties associated with random effects and a scale that is stable between the CCPR and RMO KCs. When there are only two participants to be linked a single link laboratory is acceptable. In this case it should be treated as two simultaneous bilateral comparisons with the analysis following guidelines CCPR-G5. There can be a single protocol and report.
- 1.2.3 All participants of the previous or current CCPR comparison of the quantity who are members of the RMO have an obligation to serve as a link laboratory at least once in the current round, if requested.

#### **1.3 Call for participants**

- 1.3.1 The pilot laboratory sends out a call for participants to all member NMIs of the Group of RMOs as defined in the *G4-Guidelines for preparing CCPR Key comparisons* (plus any other invited NMIs) with the information of the comparison quantity (and wavelength range, if it is a spectral quantity).

1.3.2 The RMO PR TC chair informs the chairs of all other RMOs that a comparison is planned so that bilateral comparisons in other RMOs can be avoided.

#### 1.4 Task Group

1.4.1 A Task Group for the comparison is formed, if necessary, to assist the pilot laboratory in developing the comparison protocol. The Task Group consists of several participants of the RMO comparison and the link laboratories.

## 2. Development of Technical Protocol

2.1 The technical protocol is developed by the pilot laboratory and the Task Group of comparison.

2.2 The technical protocol should basically follow that of CCPR KC of the quantity. The same, or similar, types of transfer standard artifacts should be used.

2.3 The technical protocol should describe a sequence of measurements which allows the stability of the transfer standards to be verified. The measurement sequence should use at least one of the following elementary patterns:

A) Transfer standards are first measured by the pilot laboratory, sent to each participant for their measurement, then sent back to the pilot laboratory for the second measurement. (Pilot – Participant – Pilot)

B) The transfer standards are first measured by each participant, sent to the pilot laboratory for their measurements, then sent back to each participant for the second measurement. (Participant – Pilot – Participant)

2.4 The technical protocol should include the following information.

2.4.1 Complete specification of the comparison quantity, wavelength range and geometrical conditions, as applicable.

2.4.2 The pilot and link laboratories and the list of participants (NMIs) and their contact information.

2.4.3 Time table (month, year) including

- delivery of transfer artifacts to each participant (if the pilot laboratory prepares transfer artifacts), or submission of transfer artifacts and results by each participant (if each participant prepares the artifacts). Time tables should explicitly include sufficient time for transportation
- measurements by each participant
- distribution of Pre-Draft A data (stability of artifacts and internal consistency), following the *G2: Guidelines for CCPR comparison report preparation*.
- distribution of Draft A
- a statement that participants will be given a deadline date for submitting the results, and if they do not meet the deadline, they will be disqualified.

2.4.4 Description of transfer artifacts. If the participants are to procure the artifacts, the detailed product information of the artifacts.

2.4.5 Advice on handling the travelling standards, including unpacking and subsequent packing and shipping for return. Detailed operating conditions of the transfer artifacts including electrical parameters and method for alignment.

- 2.4.6 Instructions to perform measurements using the facilities and procedures that the participant laboratory normally uses for their calibration services, while meeting the conditions of measurement specified by the technical protocol.
- 2.4.7 Instructions for submitting the measurement results including uncertainty (attach a form) for each transfer standard. If the participants perform measurements in two rounds (before and after measurement by the pilot laboratory), they must send the results after each of 1<sup>st</sup> and 2<sup>nd</sup> measurement, but the results of the 1<sup>st</sup> measurement can be treated as preliminary and these can be revised if necessary when the 2<sup>nd</sup> measurement results are submitted.
- 2.4.8 Instructions to submit the information below. These should be submitted before or, at the latest, at the time of submitting final measurement results.
- information on the traceability of the scale of the comparison quantity of the laboratory
  - detailed uncertainty budget, including the list of uncertainty contributions identifying uncertainty components related to correlated and uncorrelated effects between measurements rounds, for the laboratory's measurement of the comparison quantity (general one for the quantity, not for each individual transfer artifact). The protocol shall include a list of the typical uncertainty contributions to include in the budget.
  - description of the laboratory facility for the quantity
- 2.4.9 Method of data analysis, which should follow Appendix A below. Any deviations from the recommended methods should be justified and explained in detail.
- 2.5 The technical protocol must be distributed to and approved by all participants. It is highly recommended that prior to approval the protocol is clearly labeled 'DRAFT' on all pages. Only once it has been approved by both the participants and the WG-KC (Section 3.3) should a final version be distributed as PDF and labeled as 'Final Approved Protocol'.

### **3. Registration to KCDB and approval of the protocol**

- 3.1 Upon completion of the technical protocol following review by the participants, the pilot laboratory sends the protocol document to the RMO PR TC Chair.
- 3.2 The RMO PR TC Chair will submit the protocol document to WG-KC Secretary (copy to WG-KC Chair) to request approval or comments by WG-KC.
- 3.3 After the technical protocol is approved, the RMO PR TC Chair sends the KCDB entry form and the Final Approved Protocol to the CCPR Executive Secretary, who will confirm reception and pass it on to the KCDB manager.
- 3.4 Upon the registration of the comparison in the KCDB, the comparison is given a designation as RMO(.PR)-Kx.YEAR. RMO is the name of the RMO of the pilot laboratory, .PR is an optional addition to the RMO name, YEAR is the year of registration in the KCDB.

### **4. Monitoring the comparison**

- 4.1 After the comparison is registered in the KCDB and during the process of the comparison (until the report is published), pilot laboratories are required to send a

progress report to the participants and the RMO Chair every six months (January and July as default). The RMO Chair will include this in the report to the WG-KC.

- 4.2 If significant delay occurs, the pilot laboratory is required to notify the participants and the RMO Chair promptly at any time between the periodic reports.
- 4.3 The RMO Chair should remind the pilot laboratory to submit the status report as required above.

## **5. Measurement**

- 5.1 Measurement can start upon the approval of the protocol by WG-KC and should follow the time scale in the technical protocol.
- 5.2 The participants and pilot laboratory should inform each other upon receipt of the transfer standards.
- 5.3 If a participant fails to submit the results by the deadline (except for reasons such as failure of artifacts), the participant will be disqualified. This decision, proposed by the pilot laboratory, is to be agreed by all other participants.

## **6. Pre-Draft A process**

- 6.1 After completion of all measurements, sections 1, 2, and 3 of the *Guidelines for CCPR Comparison Report Preparation* (CCPR G2) must be followed by the pilot laboratory, which describe the Pre-Draft A process. Section 4 is not required for RMO comparisons linking to the KCRV of the CCPR comparison.

## **7. Preparation and Distribution of Draft A**

- 7.1 After the Pre-Draft A processes are complete, the pilot laboratory prepares and distributes Draft A to all participants, which discloses the absolute results of the comparison. The Draft A should tabulate all the results as well as present them in graphical form as necessary. It is recommended that the pilot laboratory also distribute the data of the analyses in a spreadsheet file. The Draft A should be distributed within six months after completion of all the measurements of the comparison.
- 7.2 Draft A should give the designation of the comparison, as described in 3.4.
- 7.3 The results of the RMO KC are linked to the results of the most recent (past) CCPR KC of the same quantity. The unilateral Degree of Equivalence (DoE) of the participating laboratories should be calculated using all appropriate information available from the CCPR KC and the RMO KC, following the approach described in Appendix A.
- 7.4 The Draft A report, and subsequently the Draft B and final reports, must contain unilateral DoEs for the participating laboratories but not for the link laboratories, including the deviation and associated expanded uncertainty of each participant to the KCRV of the CCPR comparison.
- 7.5 Bilateral DoEs are not required.

## **8. Review of Draft A by participants**

- 8.1 The Draft A report should be reviewed by the participants following all parts of the procedure in section 6 of the *Guidelines for CCPR Comparison Report Preparation* (CCPR G2).

## **9. Submission of Draft B (for Key Comparisons)**

- 9.1 When the final version of Draft A has been agreed by all participants, it becomes Draft B.
- 9.2 The Pilot laboratory submits Draft B to the RMO chair, who forwards it to the WG-KC for approval, within four months from distribution of Draft A (if no further version of Draft A needs to be prepared).
- 9.3 Draft B (the final version of Draft A-x) must include tables of unilateral Degrees of Equivalence. Tables of Bilateral DoE are not required. The tables can be in the main body or an Appendix of the report.
- 9.4 Draft B will be reviewed by WG-KC (and no longer by participants). As the result of this review, changes in Draft B may be requested and such changes will be sent to the Pilot laboratory. If a revision is produced, it is called Draft B-2 (B-3, ... if repeated) and reviewed again by WGKC. Participants do not participate in this process unless some major revision is proposed by WG-KC.
- 9.5 When Draft B-x is approved by WG-KC, it will be submitted to CCPR. When it is approved by CCPR, the approved version of Draft B becomes the Final Report.
- 9.6 Any versions of Draft B are not considered confidential, and may be the subject of a publication with the exception of the proposals for the reference value and degrees of equivalence.
- 9.7 After the Draft B has been submitted, the Pilot laboratory sends to all participants, with copies to the RMO P&R TC chairs, a reminder to check the consistency of their CMCs with the KC results and to report to the participant's RMO TC chair (with a copy to the pilot) about their evaluation and any proposed actions in case of inconsistency, within two months from the reminder.

## **10. Publication of Final Report**

- 10.1 The final reports of Key Comparisons will be published in the *Technical Supplement of Metrologia* (electronic media on the website). If the Pilot lab chooses to do so, the reports can also be published in a printed journal.

## **References**

1. CIPM MRA-D-05, Measurement comparisons in the CIPM MRA, Version 1.5, available at <http://www.bipm.org/utis/common/documents/CIPM-MRA/CIPM-MRA-D-05.pdf>

## Appendix A: Recommended analysis approach

This example analysis is an expansion of the concepts developed for G5: Guidelines for bilateral KCs. It assumes that there are two link laboratories for the comparison.

The approach uses a single artifact for each pair of measurements (e.g. pilot to participant). The ‘effective single artifact’ can be determined using a simple mean of different artifact values of  $(y_p - y_i)$  in e.g. Eq. (1) or a simple mean of different artifact values of  $(y_p/y_i - 1)$  in Eq. (2). The uncertainty associated with this effective artifact is given by the uncertainty declared by the participant for a single artifact. This ensures that the uncertainties associated with random effects are not reduced depending on the number of measurements made by the participant – in this way the participants are treated more equitably.

The abbreviation KC relates here to the CCPR Key Comparison that the RMO comparison is linked to. The abbreviation RMO relates to the RMO comparison.

### A.1 Relative and absolute models

It is important to understand whether the comparison model is based on absolute differences or relative differences. In an absolute-difference model:

- The KCRV has units of the key comparison quantity.
- All uncertainty components have the same units.
- The DoE of the RMO participant will be the best estimate of the systematic offset of that participant’s measurements. So had that participant measured the CCPR KC ‘virtual artifact’, this would be the offset in that participant’s measured value from the KCRV in the same units.

In a relative-difference model:

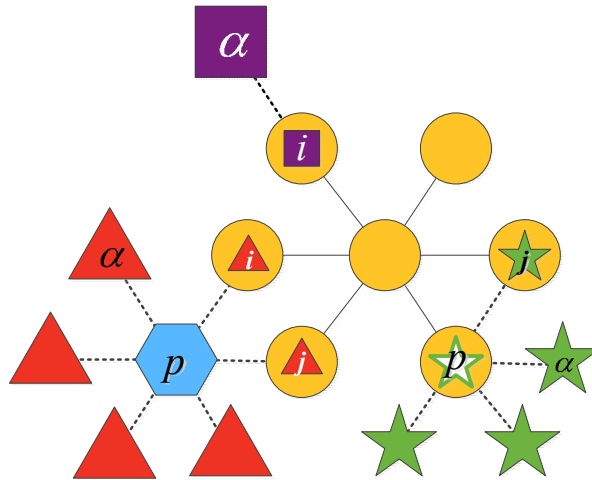
- The KCRV is the value 1. It is the ‘average’ ratio of participant’s measurements of the ‘virtual artifact’ to the KC value of the ‘virtual artifact’.
- All uncertainty components are expressed relatively, often as percentages.
- The DoE of the RMO participant will be the best estimate of the systematic ratio (minus one) of that participant’s measurements to the KCRV. So had that participant measured the CCPR KC ‘virtual artifact’, this would be the ratio of that participants’ measured value from the KCRV minus one.

The uncertainty equations below can be used for both relative difference and absolute difference models. In relative difference models, the uncertainty components are expressed in percent, in absolute difference models, in the unit of measurement.

### A.2 The different types of RMO participant

The comparison analysis will depend on whether the pilot of the RMO comparison was a link laboratory or not. Figure 1 shows the form of an original CCPR comparison (yellow circles), a linked bilateral (purple squares) and two linked RMO comparisons. In one (red

triangles) the pilot (blue hexagon) is not one of the original CCPR comparison participants. In the other (green stars) the pilot is one of the two link laboratories. The same notation is used for all examples,  $\alpha$  describes a participant to be linked in the RMO or Bilateral Comparison, the link laboratories have the notation  $i$  and  $j$  and the pilot has the specific notation  $p$ . Each link (solid lines for the CCPR comparison, dotted lines for linked comparisons) corresponds to the measurement by the two NMIs of the same artefact or set of artifacts.



**Figure 1** The original CCPR comparison (yellow circles), with subsequent linked comparisons. The bilateral comparison (considered in Guideline G5) is shown by the purple squares. There are two different RMO comparisons. One (red triangles and blue hexagon) has a pilot who was not one of the original CCPR comparison participants. The other (green stars) has a pilot who is one of the CCPR comparison link laboratories.

The unilateral DoE and its associated uncertainty are different for these three cases.

### A.2.1 For the link of the pilot who is not a link laboratory (blue)

The DoE of the pilot, through each of the two links independently is given (for an absolute difference model) by:

$$\begin{aligned}
 D_{p(i)} &= D_i + \underbrace{(y_p - y_i)}_{\substack{\text{difference pilot to link,} \\ \text{can be average of multiple artifacts}}} \\
 D_{p(j)} &= D_j + \underbrace{(\tilde{y}_p - \tilde{y}_j)}_{\substack{\text{difference pilot to link,} \\ \text{can be average of multiple artifacts}}} .
 \end{aligned} \tag{1}$$

Where

- $D_{p(i)}$  is the unilateral DoE for the pilot, calculated via link  $i$  and  $D_{p(j)}$  is the unilateral DoE for the pilot calculated via link  $j$ .
- $D_i$  and  $D_j$  are the unilateral DoEs for the link laboratories  $i$  and  $j$  respectively calculated during the KC.

- $y_p - y_i$  is the “average value” (of multiple artifacts) of the difference between the pilot’s measurement result and the link  $i$ ’s measurement result.
- $\tilde{y}_p - \tilde{y}_j$  is the “average value” (of multiple artifacts) of the difference between the pilot’s measurement result and the link  $j$ ’s measurement result.

For a relative-difference model:

$$\begin{aligned}
 D_{p(i)} &= D_i + \underbrace{\left( y_p / y_i - 1 \right)}_{\substack{\text{ratio pilot to link,} \\ \text{can be average of multiple artifacts}}} \\
 D_{p(j)} &= D_j + \underbrace{\left( \tilde{y}_p / \tilde{y}_j - 1 \right)}_{\substack{\text{ratio pilot to link,} \\ \text{can be average of multiple artifacts}}} .
 \end{aligned} \tag{2}$$

Where:

- $D_{p(i)}$  is the unilateral DoE for the pilot, calculated via link  $i$  and  $D_{p(j)}$  is the unilateral DoE for the pilot calculated via link  $j$ .
- $D_i$  and  $D_j$  are the unilateral DoEs for the link laboratories  $i$  and  $j$  respectively calculated during the KC.
- $y_p / y_i$  is the “average value” (of multiple artifacts) of the ratio between the pilot’s measurement result and the link  $i$ ’s measurement result.
- $\tilde{y}_p / \tilde{y}_j$  is the “average value” (of multiple artifacts) of the ratio between the pilot’s measurement result and the link  $j$ ’s measurement result.

The DoE of the pilot is then calculated as

$$D_p = W_i D_{p(i)} + W_j D_{p(j)}; \quad W_i + W_j = 1. \tag{3}$$

Optimal weights for this case are:

$$\begin{aligned}
 W_i &= \frac{\hat{W}}{\sigma_i^2}, & W_j &= \frac{\hat{W}}{\sigma_j^2} \\
 \hat{W} &= \frac{\sigma_i^2 \sigma_j^2}{\sigma_i^2 + \sigma_j^2} \\
 \sigma_i^2 &= s_{\text{KC}}^2 + s_{\text{RMO}}^2 + u_{i,\text{st}}^2 + u_{i,\text{r,KC}}^2 + u_{i,\text{r,RMO}}^2 \\
 \sigma_j^2 &= s_{\text{KC}}^2 + s_{\text{RMO}}^2 + u_{j,\text{st}}^2 + u_{j,\text{r,KC}}^2 + u_{j,\text{r,RMO}}^2
 \end{aligned} \tag{4}$$

(For the meaning of these terms, see Section A.2.4 below).

The uncertainty component of the unilateral DoE is given as an expanded uncertainty

$$U(D_p) = 2u(D_p) \tag{5}$$

where the standard uncertainty is calculated using



$$\begin{aligned}
u^2(D_p) = & \underbrace{u_p^2}_{p\text{'s measurement}} + \underbrace{(W_i^2 - 2W_iW_j + W_j^2 - 2W_jW_i)S_{KC}^2 + u^2(x_{ref})}_{KC \text{ effects}} + \underbrace{W_i^2(u_{i,st}^2 + u_{i,t,KC}^2 + u_{i,t,RMO}^2)}_{i \text{ link quality}} + \\
& \underbrace{W_j^2(u_{j,st}^2 + u_{j,t,KC}^2 + u_{j,t,RMO}^2)}_{j \text{ link quality}} + \underbrace{(W_i^2 + W_j^2)S_{RMO}^2}_{RMO \text{ effects}}.
\end{aligned} \tag{6}$$

This equation is described and the terms are defined in section A.2.4 below.

### **A.2.2 For the link of the non-pilot when the pilot is not a link laboratory (red)**

The DoE of the non-link laboratory, through the pilot and each of the two links independently is given (for an absolute difference model) by:

$$\begin{aligned}
D_{\alpha(i)} = D_i + & \underbrace{(y_p - y_i)}_{\substack{\text{difference pilot to link,} \\ \text{can be average of multiple artifacts}}} + \underbrace{(z_\alpha - z_p)}_{\substack{\text{difference participant to pilot,} \\ \text{can be average of multiple artifacts}}} \\
D_{\alpha(j)} = D_j + & \underbrace{(\tilde{y}_p - \tilde{y}_j)}_{\substack{\text{difference pilot to link,} \\ \text{can be average of multiple artifacts}}} + \underbrace{(z_\alpha - z_p)}_{\substack{\text{difference participant to pilot,} \\ \text{can be average of multiple artifacts}}}.
\end{aligned} \tag{7}$$

Where

- $D_{\alpha(i)}$  is the unilateral DoE for the laboratory  $\alpha$ , calculated via link  $i$  and pilot  $p$  and  $D_{\alpha(j)}$  is the unilateral DoE for the laboratory  $\alpha$ , calculated via link  $j$  and pilot  $p$ .
- $D_i$  and  $D_j$  are the unilateral DoEs for the link laboratories  $i$  and  $j$  respectively calculated during the KC.
- $y_p - y_i$  is the ‘‘average value’’ (of multiple artifacts) of the difference between the pilot’s measurement result and the link  $i$ ’s measurement result.
- $\tilde{y}_p - \tilde{y}_j$  is the ‘‘average value’’ (of multiple artifacts) of the difference between the pilot’s measurement result and the link  $j$ ’s measurement result.
- $z_\alpha - z_p$  is the ‘‘average value’’ (of multiple artifacts) of the difference between the laboratory  $\alpha$ ’s measurement result and the pilot’s measurement result.

For a relative-difference model:

$$\begin{aligned}
D_{\alpha(i)} = D_i + & \underbrace{(y_p/y_i - 1)}_{\substack{\text{ratio pilot to link,} \\ \text{can be average of multiple artifacts}}} + \underbrace{(z_\alpha/z_p - 1)}_{\substack{\text{ratio pilot to new participant,} \\ \text{can be average of multiple artifacts}}} \\
D_{\alpha(j)} = D_j + & \underbrace{(\tilde{y}_p/\tilde{y}_j - 1)}_{\substack{\text{ratio pilot to link,} \\ \text{can be average of multiple artifacts}}} + \underbrace{(z_\alpha/z_p - 1)}_{\substack{\text{ratio pilot to new participant,} \\ \text{can be average of multiple artifacts}}}.
\end{aligned} \tag{8}$$

Where:

- $D_{\alpha(i)}$  is the unilateral DoE for the laboratory  $\alpha$ , calculated via link  $i$  and pilot  $p$  and  $D_{\alpha(j)}$  is the unilateral DoE for the laboratory  $\alpha$ , calculated via link  $j$  and pilot  $p$ .

- $D_i$  and  $D_j$  are the unilateral DoEs for the link laboratories  $i$  and  $j$  respectively calculated during the KC.
- $y_p/y_i$  is the “average value” (of multiple artifacts) of the ratio between the pilot’s measurement result and the link  $i$ ’s measurement result.
- $\tilde{y}_p/\tilde{y}_j$  is the “average value” (of multiple artifacts) of the ratio between the pilot’s measurement result and the link  $j$ ’s measurement result.
- $z_\alpha/z_p$  is the “average value” (of multiple artifacts) of the ratio between  $\alpha$ ’s measurement result and the pilot’s measurement result.

The DoE of the laboratory  $\alpha$  is then calculated as

$$D_\alpha = W_i D_{\alpha(i)} + W_j D_{\alpha(j)}; \quad W_i + W_j = 1. \quad (9)$$

Optimal weights for this case are:

$$\begin{aligned} W_i &= \frac{\tilde{W}}{\tilde{\sigma}_i^2}, & W_j &= \frac{\tilde{W}}{\tilde{\sigma}_j^2} \\ \tilde{W} &= \frac{\tilde{\sigma}_i^2 \tilde{\sigma}_j^2}{\tilde{\sigma}_i^2 + \tilde{\sigma}_j^2} \\ \tilde{\sigma}_i^2 &= s_{\text{KC}}^2 + s_{\text{RMO}}^2 + u_{i,\text{st}}^2 + u_{i,\text{r,KC}}^2 + u_{i,\text{r,RMO}}^2 + u_{p,\text{r,RMO}}^2 \\ \tilde{\sigma}_j^2 &= s_{\text{KC}}^2 + s_{\text{RMO}}^2 + u_{j,\text{st}}^2 + u_{j,\text{r,KC}}^2 + u_{j,\text{r,RMO}}^2 + u_{p,\text{r,RMO}}^2 \end{aligned} \quad (10)$$

This has an extra term compared to Eq. (4). The terms are defined in Section A.2.4.

The uncertainty component of the unilateral DoE is given as an expanded uncertainty

$$U(D_\alpha) = 2u(D_\alpha) \quad (11)$$

where the standard uncertainty is calculated using

$$\begin{aligned} u^2(D_\alpha) &= \underbrace{u_\alpha^2}_{\alpha\text{'s measurement}} + \underbrace{\left( W_i^2 - 2W_i W_j + W_j^2 - 2W_j W_i \right) s_{\text{KC}}^2 + u^2(x_{\text{ref}})}_{\text{KC effects}} + \underbrace{W_i^2 \left( u_{i,\text{st}}^2 + u_{i,\text{r,KC}}^2 + u_{i,\text{r,RMO}}^2 \right)}_{i \text{ link quality}} + \\ &\quad \underbrace{W_j^2 \left( u_{j,\text{st}}^2 + u_{j,\text{r,KC}}^2 + u_{j,\text{r,RMO}}^2 \right)}_{j \text{ link quality}} + \underbrace{\left( W_i^2 + W_j^2 + 1 \right) \left( u_{p,\text{r,RMO}}^2 + s_{\text{RMO}}^2 \right)}_{\text{RMO effects}}. \end{aligned} \quad (12)$$

This equation is described and the terms are defined in section A.2.4 below.

### **A.2.3 For the link of non-link laboratories when the pilot is a link laboratory (green)**

In this case, there is an asymmetry. A participant’s link through the pilot is direct (like section A.2.1) and their link through the other link laboratory includes a ‘via the pilot’

step (like section A.2.2). We will call the pilot, link laboratory  $p$  and the non-pilot, link laboratory  $j$ .

The DoE of the non-link laboratory, through the pilot and the other link is given (for an absolute difference model) by:

$$\begin{aligned}
 D_{\alpha(p)} &= D_p + \underbrace{(y_\alpha - y_p)}_{\substack{\text{difference to link pilot } p \\ \text{can be average of multiple artifacts}}} \\
 D_{\alpha(j)} &= D_j + \underbrace{(\tilde{y}_p - \tilde{y}_j)}_{\substack{\text{difference pilot to link,} \\ \text{can be average of multiple artifacts}}} + \underbrace{(y_\alpha - y_p)}_{\substack{\text{difference participant to pilot,} \\ \text{can be average of multiple artifacts}}} .
 \end{aligned} \tag{13}$$

Where

- $D_{\alpha(p)}$  is the unilateral DoE for the laboratory  $\alpha$ , calculated via pilot  $p$  and  $D_{\alpha(j)}$  is the unilateral DoE for the laboratory  $\alpha$ , calculated via link  $j$  and pilot  $p$ .
- $D_p$  and  $D_j$  are the unilateral DoEs for the link laboratories  $p$  and  $j$  respectively calculated during the KC.
- $y_\alpha - y_p$  is the “average value” (of multiple artifacts) of the difference between participant  $\alpha$ ’s measurement result and the pilot  $p$ ’s measurement result.
- $\tilde{y}_p - \tilde{y}_j$  is the “average value” (of multiple artifacts) of the difference between the pilot’s measurement result and the link  $j$ ’s measurement result.

For a relative-difference model:

$$\begin{aligned}
 D_{\alpha(p)} &= D_p + \underbrace{(y_\alpha/y_p - 1)}_{\substack{\text{ratio new participant to pilot,} \\ \text{can be average of multiple artifacts}}} \\
 D_{\alpha(j)} &= D_j + \underbrace{(\tilde{y}_p/\tilde{y}_j - 1)}_{\substack{\text{ratio pilot to link,} \\ \text{can be average of multiple artifacts}}} + \underbrace{(y_\alpha/y_p - 1)}_{\substack{\text{ratio new participant to pilot,} \\ \text{can be average of multiple artifacts}}} .
 \end{aligned} \tag{14}$$

Where:

- $D_{\alpha(p)}$  is the unilateral DoE for the laboratory  $\alpha$ , calculated via pilot  $p$  and  $D_{\alpha(j)}$  is the unilateral DoE for the laboratory  $\alpha$ , calculated via link  $j$  and pilot  $p$ .
- $D_p$  and  $D_j$  are the unilateral DoEs for the link laboratories  $p$  and  $j$  respectively calculated during the KC.
- $y_\alpha/y_p$  is the average ratio (from multiple artifacts) between participant  $\alpha$ ’s measurement result and the link-pilot  $p$ ’s measurement result.
- $\tilde{y}_p/\tilde{y}_j$  is the average ratio (of multiple artifacts) between the pilot’s measurement result and the link  $j$ ’s measurement result.

The DoE of the laboratory  $\alpha$  is then calculated as

$$D_\alpha = W_p D_{\alpha(p)} + W_j D_{\alpha(j)}; \quad W_p + W_j = 1. \tag{15}$$

Optimal weights in this case are:

$$\begin{aligned}
W_p &= \frac{\bar{W}}{\bar{\sigma}_p^2 - u_{p,r,RMO}^2 - s_{RMO}^2}, & W_j &= \frac{\bar{W}}{\bar{\sigma}_j^2 + u_{p,r,RMO}^2} \\
\bar{W} &= \frac{(\bar{\sigma}_j^2 + u_{p,r,RMO}^2)(\bar{\sigma}_p^2 - u_{p,r,RMO}^2 - s_{RMO}^2)}{\bar{\sigma}_p^2 + \bar{\sigma}_j^2 - s_{RMO}^2} \\
\bar{\sigma}_p^2 &= s_{KC}^2 + s_{RMO}^2 + u_{p,st}^2 + u_{p,r,KC}^2 + u_{p,r,RMO}^2 \\
\bar{\sigma}_j^2 &= s_{KC}^2 + s_{RMO}^2 + u_{j,st}^2 + u_{j,r,KC}^2 + u_{j,r,RMO}^2
\end{aligned} \tag{16}$$

Note that this has an asymmetry. Terms are described in Section A.2.4 below.

The uncertainty component of the unilateral DoE is given as an expanded uncertainty

$$U(D_\alpha) = 2u(D_\alpha) \tag{17}$$

where the standard uncertainty is calculated using

$$\begin{aligned}
u^2(D_\alpha) &= \underbrace{u_\alpha^2}_{\alpha\text{'s measurement}} + \underbrace{(W_p^2 - 2W_p w_p + W_j^2 - 2W_j w_j) s_{KC}^2 + u^2(x_{ref})}_{\text{KC effects}} + \underbrace{W_p^2 (u_{p,st}^2 + u_{p,r,KC}^2 + u_{p,r,RMO}^2)}_{i \text{ link quality}} + \\
&\quad \underbrace{W_j^2 (u_{j,st}^2 + u_{j,r,KC}^2 + u_{j,r,RMO}^2)}_{j \text{ link quality}} + \underbrace{(W_j^2 + 1) s_{RMO}^2 + 2W_j u_{p,r,RMO}^2}_{\text{RMO effects}}.
\end{aligned} \tag{18}$$

This equation is described and the terms are defined in section A.2.4 below.

### **A.2.4 Terms in the equations**

The three uncertainty equations (5), (10), and (15) all take the same form. The first four terms are identical and have the following components:

#### **Uncertainty associated with the participant to be linked**

$u_\alpha^2$  in (12) and (18),  $u_p^2$  in (6).

- This is the full uncertainty (from both systematic and random effects) for the participant to be linked.

#### **KC effects**

$(W_i^2 - 2W_i w_i + W_j^2 - 2W_j w_j) s_{KC}^2 + u^2(x_{ref})$  in (6) and (12) are equivalent using the index  $p$  instead of  $i$  in (18).

- Here the weights  $W_i$  and  $W_j$  are as in Eqs. (4), (10) or (16).
- $u(x_{ref})$  is the standard uncertainty associated with the Key Comparison Reference Value. This value is available from the KC report.

- $s_{\text{KC}}$  is the transfer uncertainty for the KC. This may be an artifact instability factor calculated from known effects, or it may be the additional  $s$  term added during a Mandel-Paule approach in obtaining consistency of the KC results. In either case this term is available from the KC report. If it is not used in the KC report,  $s_{\text{KC}} = 0$ .
- $w_i$  and  $w_j$  are the weights for the two link laboratories in the calculation of the KCRV. This value is provided in the KC report; however, where it is not available, then  $w_i = w_j = 0$  will give a conservative estimate of the uncertainties.

### Link qualities of the two link laboratories

$\underbrace{W_i^2 (u_{i,\text{st}}^2 + u_{i,\text{r,KC}}^2 + u_{i,\text{r,RMO}}^2)}_{i \text{ link quality}} + \underbrace{W_j^2 (u_{j,\text{st}}^2 + u_{j,\text{r,KC}}^2 + u_{j,\text{r,RMO}}^2)}_{j \text{ link quality}}$  in (6) and (12) are equivalent using the index  $p$  instead of  $i$  in (18).

- These terms are weighted by the weights from Eqs. (4), (10) or (16).
- $u_{i,\text{st}}, u_{j,\text{st}}$  are the standard uncertainties associated with reproducibility of the two link laboratory's scales between the KC and the RMO-KC. If the time interval between the KC and the RMO-KC is short, or if the link laboratory can provide additional evidence to confirm the stability of its scale, then this can be assumed to be zero.
- $u_{i,\text{r,KC}}, u_{j,\text{r,KC}}$  are the standard uncertainties associated with uncorrelated (random) effects of the link laboratories during the KC. This should be available from the KC report. If it is not available, for older comparisons, it may be appropriate to assume that  $u_{i,\text{r,KC}}^2 + u_{i,\text{st}}^2 = u_i^2$  (scale stability unknown), where  $u_i$  is the declared total standard uncertainty of the link laboratory during the KC.
- $u_{i,\text{r,RMO}}, u_{j,\text{r,RMO}}$  are the standard uncertainties associated with uncorrelated (random) effects of the link laboratories during the RMO comparison.

### RMO effects

The RMO effects term is different for each of the three cases.

- For the non-link pilot it is:  $(W_i^2 + W_j^2) s_{\text{RMO}}^2$  (Eq. (6)).
- For the participant where the pilot is not a link, it is:  $(W_i^2 + W_j^2 + 1)(u_{p,\text{r,RMO}}^2 + s_{\text{RMO}}^2)$  (Eq. (12))
- For the participant where the pilot is a link, it is:  $(W_j^2 + 1)s_{\text{RMO}}^2 + 2W_j u_{p,\text{r,RMO}}^2$  (Eq. (18)).

Here,

- These terms are weighted by the weights from Eqs. (4), (10) or (16).

- $s_{\text{RMO}}$  is the standard transfer uncertainty of the RMO comparison. The transfer uncertainty may come from known effects (e.g. known artifact instability), or from the similar term during the key comparison (to account for artifact instability that is not ‘visible’ in the RMO-KC). Alternatively it may be appropriate to consider  $s_{\text{RMO}} = 0$ .
- For a participant who is not the pilot of the comparison, there is a term involving  $u_{p,r,\text{RMO}}$ , which is the standard uncertainty associated with uncorrelated (random) effects at the pilot laboratory. This is because the pilot provides the link between laboratories.