

CONSULTATIVE COMMITTEE FOR LENGTH – CCL
Discussion Group On Coordinate Metrology – DG6

A. Balsamo, 2018-06-15

Guidance document GD-06

Version 1

CMCs of category “Standards of 1D point-to-point dimensions” – Guidelines

1 Scope

This document provides guidance on CMCs of category “Standards of 1D point-to-point dimensions”. This is intended for:

- NIMs with this CMC already approved, to decide whether it covers a requested calibration;
- applicant NIMs, to help in their applications;
- CMC reviewers, to promote approval/disapproval in a consistent way worldwide.

This document also defines the changes to the DimVIM (rev. 10) necessary to accommodate such CMCs (Annex B).

2 Background

CMCs specific to an instrument or standard type are unable to capture situations where possibly different standards can be calibrated in similar conditions, with similar measuring systems and with similar calibration uncertainty. As long as the measurand and the equipment stay the same, calibrations of even diverse standards may be encompassed by a same flexible capability.

For this reason, a new entry is introduced in the DimVIM (rev. 10, see Annex B) to accommodate artefacts of diverse nature and shape in a single CMC, all sharing the same prominent feature, which is in fact the measurand under calibration.

For instance, a CMM coupled with an interferometer is expected to achieve similar uncertainties when calibrating different 1D standards, no matter whether they are gauge blocks, step gauges, or rings for point-to-point diameters.

Flexible CMCs covering such situations overcome the requirement for separate and redundant CMCs. To avoid possible ambiguity, misinterpretation and misuse, a precise definition of their applicability bounds is required.

3 Requirements for the standards

For a standard to fall within a CMC based on this DimVIM category, the type of measurand(s) is restricted rather than the type of standard, which can be of any nature and shape.

The measurand(s) shall be defined as distances of *individual points* along a *single straight reference line*. In the simplest case, the measurand is the distance of two points; in a more complex case, the measurands are the individual distances of a sequence of points all coincident with a straight reference line, to a common point (origin) on the same reference line. Examples of such an origin are the first point in the sequence, or the centroid of the point set.

Artefacts with measurands that rely on points not aligned on a single straight line do not fall in this category. Examples of such measurands are the least-squares diameter of a circle (the points are not aligned to a single line), or the coaxiality of two cylinders (the measurand is not a point-to-point distance).

In practice, the measured points in the sequence may be not perfectly aligned: some operator is usually defined to align the points mathematically, e.g. a projection onto the reference line.

The reference line is established by measurement of the standard under calibration, effectively setting up a workpiece coordinate system. This reference line is based on either

- the point sequence only (e.g. the least-squares line, or the line through the first and the last points), or

- other datum features as well (e.g. a step gauge aligned to its lateral faces, or a gauge block aligned to a wrung platen).

Establishing the reference line requires measurements in addition to the point sequence. This is evident in the second case above, where independent features are probed and evaluated as datums; it is also true in the first case, because the points of interest must be localised in reference to some additional geometrical feature of the standard. For example, the point-to-point diameter of a sphere requires prior measurement to localise the sphere and orient the reference line; the abscissae of the faces of a step gauge requires the prior localisation and orientation of a reference line (the gauge axis).

Let us define *auxiliary measurements* to be those that are instrumental in defining the reference line, but which exclude points that are part of the measurand(s). If the auxiliary measurements have small impact on the measurand(s) then the measurement of the standard falls within the scope of the category of CMCs under discussion. The effect of the auxiliary measurements is regarded as small if reducing their uncertainty to nought would result in a reduction of the combined uncertainty of no more than 10 %. Even if the impact is small, it shall not be neglected in the evaluation of the uncertainty reported for a specific calibration.

In summary, the following specific requirements for eligibility in this category are set on the measurand(s):

1. An individual measurand shall be a distance of individual points.
2. In case of multiple measurands, all points defining a distance shall lay on the same straight reference line, and all measurands shall be taken from a common point (effectively, they shall be abscissae on a common axis from a common origin).
3. Any auxiliary measurements used to establish the straight reference line and its origin, shall contribute small uncertainty¹, that is, reducing their uncertainties to nought would reduce² the combined calibration uncertainty no more than 10 %.

Associating an instrument or standard to an approved CMC category is usually easy. However, boundary cases exist where some expert judgement is required, for example when a novel instrument or artefact is considered. In this regard, a CMC of category *Standards of 1D point-to-point dimensions* is not different: ascertaining whether a measurand meets the requirements above is left to expert judgment. Particularly the third requirement may ask for specific expertise and competence. Great care is recommended for this evaluation, which should be reserved to competent experts. See Appendix A for examples.

4 Requirements for CMC approval

In addition to specific requirements in § 3, the ordinary rules for CMC approval apply. In particular, a calibration procedure and a detailed uncertainty budget are required, which also provide the elements for assessing the fulfilment of the specific requirements.

In view of the flexible nature of these CMCs, calibration procedures are allowed to cover only the parts common to the virtually infinite number of possible calibrations, and the uncertainty budget to include only the uncertainty components therein. Whenever an actual calibration is performed, the procedure and the uncertainty budget shall be complemented

¹ This uncertainty propagates from the auxiliary measurements to the coordinate system and eventually to the measurand(s).

² The numerical computation of this reduction can be easily done by e.g. spreadsheet simulation. Alternatively, an analytical approach is also possible. If the standard uncertainty of the auxiliary measurement, u_{aux} , and the combined standard uncertainty of all contributors but the auxiliary measurement, u_{rest} , are not correlated, then

$$u_c = \sqrt{u_{\text{aux}}^2 + u_{\text{rest}}^2}$$

The requirement 3 is met when the reduced combined uncertainty, \tilde{u}_c , resulting from bringing u_{aux} to nought, is no lower than 90 % of the combined uncertainty:

$$\tilde{u}_c = u_{\text{rest}} \geq 0.9 u_c$$

By introducing this inequality into the previous equation, it holds

$$u_c \geq \sqrt{u_{\text{aux}}^2 + (0.9 u_c)^2}$$

which leads to

$$u_{\text{aux}} \leq \sqrt{1 - 0.9^2} u_c \approx 0.44 u_c, \quad \text{or} \quad u_c \geq \frac{u_{\text{aux}}}{\sqrt{1 - 0.9^2}} \approx 2.3 u_{\text{aux}}$$

with the specifics of such calibration. The complemented procedure may or may not undergo official internal approval, according to the NMI Quality System provisions. This may be decided based on a trade-off between the need for controlled documentation and economic viability (how expensive the approval process is and how frequent the calibration). In all cases, the complemented documents shall be recorded; in particular, the uncertainty budget shall provide supporting evidence that the auxiliary measurements introduce negligible uncertainty (requirement 3).

5 Applicability of alternative CMCs

Artefacts which fulfil the requirements in § 3 and are covered by the flexible CMC *Standards of 1D point-to-point dimensions* are likely covered by another CMC too, possibly with different claimed uncertainties. For instance, a NMI may have a conventional CMC for step gauges (DimVIM 2.2.4) approved as well as this flexible CMC (DimVIM 2.5.1), which also covers the step gauges. The NMI would then have the option to decide which one to use based on the accuracy requested by the customer, or the cost, or the availability and readiness of the equipment involved. As registered documentation at the NMI is required for any approved CMC, and includes the calibration procedure and the uncertainty budget, there is no confusion between the two, e.g. as to the resulting calibration uncertainty.

The overlap is not harmful for the commercial aspects of the calibration either. As long as the reported calibration uncertainty is consistent with an approved CMC, and it matches the contracted one, the calibration certificate is valid and the contractual commitments are satisfied, no matter which CMC was used.

6 Supporting the CMC

This flexible CMC must be supported by experimental evidence, just as any other CMC. Due to its flexible nature, it covers numerous (virtually infinite) standard types, and an exhaustive experimental validation is impossible. Successful results of a comparison on one specific standard is regarded as sufficient evidence to support this CMC, provided that the standard is suitable in length (compared with the CMC range) and that the uncertainty of the comparison reference value is sufficiently small (compared with that of the CMC).

No specific international comparisons are run to support flexible CMCs. When the standard used in an international comparison is covered by an NMI's flexible CMC, the NMI may decide to participate in the intercomparison. In this case, in a same comparison data submitted by participants according to conventional CMCs would coexist with data submitted by other participants according to flexible CMCs.

When an NMI's flexible CMC overlaps (as illustrated in § 5) with one or more already approved conventional CMCs, a successful internal comparison under the NMI's Quality System supervision is regarded as sufficient experimental evidence, provided that the range of the approved CMC is no less than that of the flexible CMC under approval, and that the uncertainty is no greater.

When an NMI participates in an international comparison on a standard covered by a conventional CMC as well as by a flexible CMC, the NMI may decide to participate to support both. In this case, the NMI shall provide two independent sets of results and disclose their provenance honestly. The comparison reference value must be based on no more than one set of results per participant NMI (to be fair to other participants); if an NMI provides two independent ones, it shall indicate which one to use for determining the reference value.

Annex A Examples

This Annex gives examples of evaluations whether individual standards are covered in the CMC of category *Standards of 1D point-to-point dimensions*. The examples are worked out in full for illustration; in practical cases, simplified evaluations may be sufficient.

Example 1: Point-to-point diameter of a sphere

Essential description of the calibration

The point-to-point diameter of a sphere is calibrated with a CMM. The (physical) sphere is probed in four nominal points³:

- two poles defining the measurand,
- two points on the equator at 90° to each other, i.e. in quadrature.

The calibrated point-to-point diameter is derived as the diameter of the least-squares associated sphere.

Fulfilment of requirements

The requirement 1 (§ 2) is met (point-to-point distance), and the requirement 2 does not apply (single measurand).

A sensitivity analysis of the least-squares associated sphere parameters (centre and radius) to the probed points reveals that those contributing to the radius are the two at the poles only: the other two are in orthogonal directions and contribute to the centre localisation in the equatorial plane only⁴. As four points are probed, an associated sphere exists

³ This probing strategy is recognised in the ISO 10360-2 [5] § B.3.2, as an alternative to measuring a gauge block.

⁴ Let

$$d(\mathbf{P}, \mathbf{x}_C, r) = \|\mathbf{P} - \mathbf{x}_C\| - r$$

be the orthogonal distance of a point \mathbf{P} to a sphere with radius r centred in \mathbf{x}_C . The sphere parameters are derived by solving the simultaneous equations

$$\begin{cases} \|\mathbf{P}_1 - \mathbf{x}_C\| - r = 0 \\ \dots \\ \|\mathbf{P}_4 - \mathbf{x}_C\| - r = 0 \end{cases}$$

For the purpose of this sensitivity analysis, we are interested in the first order approximation

$$\mathbf{d}_0 + \mathbf{J}_0 \Delta \mathbf{u} = \mathbf{0}, \quad \text{with } \mathbf{d}_0 = \begin{pmatrix} \|\mathbf{P}_1 - \mathbf{x}_{C0}\| - r_0 \\ \dots \\ \|\mathbf{P}_4 - \mathbf{x}_{C0}\| - r_0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} \mathbf{x}_C \\ r \end{pmatrix}, \quad \mathbf{J}_0 = \begin{pmatrix} \text{grad}_{\mathbf{u}}^T(\|\mathbf{P}_1 - \mathbf{x}_C\| - r) \\ \dots \\ \text{grad}_{\mathbf{u}}^T(\|\mathbf{P}_4 - \mathbf{x}_C\| - r) \end{pmatrix}_0$$

where the subscript 0 indicates the current approximation, \mathbf{d}_0 is the vector of the residual distances, \mathbf{u} the vector of the unknown sphere parameters, and \mathbf{J}_0 the Jacobian matrix at the current approximation. This leads to the solution

$$\Delta \mathbf{u} = -\mathbf{J}_0^{-1} \mathbf{d}_0$$

It holds

$$\mathbf{J}_0 = - \begin{pmatrix} \mathbf{n}_{10}^T & 1 \\ \dots & \dots \\ \mathbf{n}_{40}^T & 1 \end{pmatrix}, \quad \text{with } \mathbf{n}_{i0} = \frac{\mathbf{P}_i - \mathbf{x}_{C0}}{\|\mathbf{P}_i - \mathbf{x}_{C0}\|}$$

where the \mathbf{n}_{i0} 's are the radial unit vectors to the probed points.

With no loss of generality, let us take a coordinate system centred in the sphere centre \mathbf{x}_{C0} , with the z-axis through the poles $\{\mathbf{P}_1, \mathbf{P}_2\}$, and the x- and y-axes through the equatorial points $\{\mathbf{P}_3, \mathbf{P}_4\}$. Then

$$\mathbf{J}_0 = - \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \quad \mathbf{J}_0^{-1} = - \begin{pmatrix} -1/2 & -1/2 & 1 & 0 \\ -1/2 & -1/2 & 0 & 1 \\ 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \end{pmatrix}$$

that fits them all *exactly*, i.e. such that all points lay on it. If two points were probed *exactly* at the poles, then the associated sphere diameter and the point-to-point diameter through the poles would coincide. As the associated sphere diameter is sensitive only to the points probed on the poles, the point-to-point diameter through the nominal poles is achieved even in the presence of a slight deviation of the probed points from the nominal poles and of sphericity error.

The radius sensitivity to the equatorial points is null only with nominal probing distribution. In practice, the inevitable deviations from this distribution results in additional uncertainty, expected to be certainly below the threshold of 10 % of requirement 3, and very likely to be entirely negligible (second order).

In conclusion, the sphere calibrated for a point-to-point diameter with suitable point sampling falls in the CMC of category *Standards of 1D point-to-point dimensions*; no specific uncertainty component is expected in addition to the calibration uncertainty stated in the CMC.

Example 2: Abscissae of the faces of a step gauge

Essential description of the calibration

The measurands are the abscissae of the intersection points of the step gauge measuring faces with a reference straight line (gauge axis), taken along such axis, with reference to the first face elected as origin.

The plane associated with the first lateral side (primary datum) sets the xz coordinate plane orientation. The plane associated with the second lateral side (constrained to be orthogonal, secondary datum) sets the xy coordinate plane orientation. The line resulting from the intersection of these two planes is shifted of a predefined quantity to the nominal centres of the measuring faces; it constitutes the x axis. Its intersection with the first measuring face sets the origin.

The measurement is performed with a CMM.

Fulfilment of requirements

The requirements 1 (§ 2) (point-to-point distance) and 2 (common line and origin) are obviously met. Let us consider the requirement 3.

The auxiliary measurement occurs at the lateral sides, while the nominally axial points on the measuring faces – including the first face (tertiary datum) – are not part of it being part of the measurand. The quantitative assessment of requirement 3 depends on the actual step gauge geometry and instrument accuracy. What follows assumes typical situations.

In general, a straight line in space possesses four degrees of freedom (DoF), two of translation along, and two of rotation about, two axes orthogonal to the straight line. This is the case of the gauge axis, subject to two translational and two rotational errors.

A translational error, \mathbf{t} , would not be relevant if the measuring faces were all perfectly parallel. Let D be the lateral size of the faces, T_p their perpendicularity tolerance⁵, and let us assume that the faces are well-behaved, i.e. nearly planar; then the measurement error of the i -th face is bounded by

$$|\varepsilon_{ti}| \leq 2T_p \frac{\|\mathbf{t}\|}{D}$$

Such error is second order and is expected to be well below the 10 % threshold set by the requirement 3.

A rotational error⁶, $\boldsymbol{\alpha}$, results in a cosine error. Because it also shifts the probed points laterally on the measuring faces, it combines with the face perpendicularity error, similarly to the translational error. The rotational error is bounded by

This shows that the radius (last row of \mathbf{J}_0^{-1}) is fully controlled by the poles (first two columns), while the equatorial points (last two columns) contribute to the x and y centre coordinates only (first two rows): the sensitivity of the diameter to the equatorial points is null.

⁵ This means that each face is contained in a tolerance zone in between two planes orthogonal to the gauge axis, separated by T_p (see ISO 1101 [3] § 17.11).

⁶ The components of the vector $\boldsymbol{\alpha}$ are three rotation angles. As the one about the gauge axis, x , is irrelevant, it is assumed null.

$$|\varepsilon_{ri}| \leq \frac{\|\alpha\|^2}{2} L + T_p \frac{L\|\alpha\|}{D}$$

where L is the largest measurand abscissa. With proper systematic correction and under the assumption of normally distributed angles with variance σ_α^2 , the resulting uncertainty is (see GUM [2] § F2.4.4)

$$u(\varepsilon_{ri}) \leq \sqrt{\sigma_\alpha^4 L^2 + \left(\frac{T_p L}{D}\right)^2 \sigma_\alpha^2} = \sigma_\alpha L \sqrt{\sigma_\alpha^2 + \left(\frac{T_p}{D}\right)^2}$$

The error α is generated by two causes: the probing errors and the deviation from axial straightness⁷ of the lateral sides. The geometrical errors of the CMM are not expected to contribute instead, as the gauge axis is very close to the lateral sides, and the geometrical errors are expected to be nearly identical and to compensate.

As to the probing errors, the datums are typically associated with the lateral sides by least-squares⁸, so that the angular uncertainty results in

$$\sigma_{\alpha,p} = \frac{u(p)}{s}, \quad \text{with } s = \sqrt{\sum_i (x_i - \bar{x})^2} \geq \frac{x_{\max} - x_{\min}}{\sqrt{2}}$$

where $u(p)$ is the probing unidirectional repeatability, \bar{x} is the abscissa of the centroid of the probed points⁹, $x_{\max} - x_{\min}$ is the axial span of the probed area, and the inequality is derived in the worst case of two probed points only. If the lateral sides are probed over a distance comparable with the largest measurand abscissa, $x_{\max} - x_{\min} \approx L$, then the angular uncertainty due to probing is bounded by

$$\sigma_{\alpha,p} \leq \sqrt{2} \frac{u(p)}{L}$$

The deviation from axial straightness of the lateral sides contributes no uncertainty when the details of the lateral side sampling are included in the definition of the gauge axis, i.e. the sampling is repeated always the same in spite of possibly different CMMs and part programmes. When this is not the case, the datum is the plane associated with the entire continuous surface (integral feature, ISO 17450-1:2011 [7] § 3.3.5). The misalignment introduced by discrete probing depends on where the probed points are taken onto each (non-straight) lateral side. We can assume that this error is bounded by¹⁰

$$\|\alpha_s\| \leq \frac{T_s}{L}, \quad \sigma_{\alpha,s} \leq \frac{T_s}{\sqrt{3}L}$$

where T_s (see ISO 1101 [3] § 17.2) is the axial straightness tolerance of the lateral side and an overall uniform distribution is assumed for the angular error.

By combining the two components, we get

$$\sigma_\alpha = \sqrt{\sigma_{\alpha,p}^2 + \sigma_{\alpha,s}^2} \leq \sqrt{2 \left(\frac{u(p)}{L}\right)^2 + \frac{1}{3} \left(\frac{T_s}{L}\right)^2} = \frac{1}{L} \sqrt{2u^2(p) + \frac{T_s^2}{3}} \approx \frac{T_s}{\sqrt{3}L}$$

as the probing repeatability, $u(p)$, is usually much smaller than the straightness tolerance of the lateral sides. By substituting this value into the equation for $u(\varepsilon_{ri})$, we get

⁷ The lateral sides are nominally planar. The axial straightness is considered hereafter instead of the planarity because the direction orthogonal to the gauge axis is irrelevant for the measurand(s).

⁸ This is in contrast with the ISO 5459 [4] § A.2.2.3, which requires the minimax as the association criterion for planar datum features. The least-squares criterion is assumed here instead because the uncertainty of the derived parameters is much easier to evaluate, and because it is often the default option in many CMM software, and hence widely used.

⁹ It is assumed that the probing strategy is equal for the two lateral sides, at least as far as the abscissae are concerned, i.e. the set of $\{x_i\}$ is equal for the two.

¹⁰ The following inequality holds as an exact equation when the axial profile is a third order polynomial symmetrical at the side centre, two points only are probed at the extremes, and the actual straightness value is equal to its full tolerance, T_s . In practice, the actual straightness may be smaller, and the points probed more cleverly (e.g. several points along the profile; or two points only, taken symmetrically and separated by $\sqrt{\frac{3}{5}}(x_{\max} - x_{\min})$, which contribute no angular error at all for profiles up to the third order).

$$u(\varepsilon_{ri}) = \sigma_a L \sqrt{\sigma_a^2 + \left(\frac{T_p}{D}\right)^2} = \frac{T_s}{\sqrt{3}} \sqrt{\frac{T_s^2}{3L^2} + \left(\frac{T_p}{D}\right)^2}$$

Finally, by combining the translational and rotational components, the uncertainty due to the auxiliary measurement is

$$u_{\text{aux}} = \sqrt{u^2(\varepsilon_{ti}) + u^2(\varepsilon_{ri})} \approx u(\varepsilon_{ri}) = \frac{T_s}{\sqrt{3}} \sqrt{\frac{T_s^2}{3L^2} + \left(\frac{T_p}{D}\right)^2}$$

As an example, if

$$\begin{cases} T_s = 50 \mu\text{m} \\ T_p = 5 \mu\text{m} \\ L = 1 \text{ m} \\ D = 10 \text{ mm} \end{cases}$$

then

$$u_{\text{aux}} = \frac{50 \mu\text{m}}{\sqrt{3}} \sqrt{\frac{1}{3} \left(\frac{50 \mu\text{m}}{1 \text{ m}}\right)^2 + \left(\frac{5 \mu\text{m}}{10 \text{ mm}}\right)^2} = 14 \text{ nm}$$

This value sets a lower bound to the combined calibration uncertainty (see footnote 2):

$$u_c \geq 2.3 u_{\text{aux}} = 2.3 \cdot 14 \text{ nm} = 32 \text{ nm}$$

which is likely to be the case in most NMI calibration facilities.

In conclusion, step gauges are usually eligible as CMC of the category *Standards of 1D point-to-point dimensions*. However, individual scrutiny is required, taking account of the gauge size and geometrical quality, particularly for very high accuracy calibrations.

Annex B

Change to the DimVIM

1 Background

The DimVIM [1] is a categorised and ordered list of calibration services, introduced by the CCL-WGDM (*Working Group on Dimensional Metrology*) in 1999 and maintained by the CCL-WG MRA, which provides a unified list of possible CMCs in the area of length. All approved CMCs are named using entries from that list.

The categories of the DimVIM are based on *instruments* or *standards* under calibration. They also define the measurand(s) that those instruments or standards realise.

Flexible CMCs challenge this formulation, as they intend to cover instruments or standards of diverse nature and shape. The following change to the DimVIM introduces a new CMC category that preserves the general formulation while opening to flexibility.

2 Addition to the DimVIM

The following category is added to the DimVIM:

2 Linear Dimensions		
2.5 Standards of 1D dimensions		
2.5.1	Standard of 1D point-to-point dimensions	Sizes, distances.

This new CMC category is open to any measurement system used for calibration.

The terms *size* and *feature of linear size* are defined in the ISO 14405-2 [6] § 3.2 and ISO 17450-1 [7] § 3.3.1.5.1.

References

- [1] CCL Length Services Classification ([DimVIM](#))
- [2] [JCGM 100:2008](#) Evaluation of measurement data – Guide to the expression of uncertainty in measurement
- [3] [ISO 1101:2017](#) Geometrical product specifications (GPS) – Geometrical tolerancing – Tolerances of form, orientation, location and run-out
- [4] [ISO 5459:2011](#) Geometrical product specifications (GPS) – Geometrical tolerancing – Datums and datum systems
- [5] [ISO 10360-2:2009](#) Geometrical product specifications (GPS) – Acceptance and reverification tests for coordinate measuring machines (CMM) – Part 2: CMMs used for measuring linear dimensions
- [6] [ISO 14405-2:2011](#) Geometrical product specifications (GPS) – Dimensional tolerancing – Part 2: Dimensions other than linear sizes
- [7] [ISO 17450-1:2011](#) Geometrical product specifications (GPS) – General concepts – Part 1: Model for geometrical specification and verification