Guide to preparation of Key Comparison Reports in Dimensional Metrology

**Document history**

V1.0 This document was prepared in late 2011 and early 2012.

V1.1 Revision by R. Thalmann Feb 2012.

V1.2 Revision by A. Lewis Nov 2013.

V1.4 Revision by A. Lewis Nov 2014.

V1.5 Revision by A. Lewis May 2015 – adding Metrologia Tech. Suppl. Template.

V1.6 Revision by A. Lewis, T. Coveney Nov 2020 – updated for KCDB2.0 and Inter-RMO report approval.

V1.7 Revision and additional material by I. Leroux, Jul 2024.

# Rationale

At the 2011 meeting of the CCL’s WG-MRA, it was decided that pilots of future key comparisons in length metrology would benefit from access to template documents and advice on how to prepare the protocol and reports of a key comparison including information on how to process the data. Three templates have been set up: WG-MRA/GD-3.1 is a template for comparison protocols, WG-MRA/GD3.2 is a template for the Final Report and WG-MRA/GD3.3 is a template for the Executive Report. This document (WG-MRA/GD-3) discusses how to prepare Final and Executive Reports including use of the templates.

# Document summary

This is a CCL-WG-MRA Guidance Document on how to prepare and submit the Draft A, Draft B, Final Report and subsequent Executive Report at the end of a key comparison in dimensional metrology. The same process may be followed for preparing the report on a supplementary comparison, but with the exception that a supplementary comparison does not require preparation of Degrees of Equivalence graphs or tables for the KCDB – only the Final Report is published.

This document presents a summary of the requirements for these MRA comparison reports, referencing published documents and decisions of various committees, followed by relevant references to documents which give further details.

A template Final Report WG-MRA/GD3.2 [14] is available on the BIPM server which includes full instructions for data processing, preparing graphs and accompanying text. The template can be modified appropriately to become a report for a specific comparison. A template Executive Report WG-MRA/GD3.3 [16] is also available on the BIPM server.

# Summary of requirements from the MRA, JCRB, CCL, WGDM, WG-MRA

The requirements for a comparison report are already detailed mostly in document ~~CIPM-MRA-D-05~~ [4], plus a few other ancillary documents and minutes from WGDM and WG-MRA meetings. References to these documents can be found in section 6.

# Outline of the Key Comparison reporting processes

## Before the Draft A report

Whilst the artefact circulation is still in progress, it may be useful for the pilot to enter all results, as they are received, into a spreadsheet, to enable simple on-going analysis, in order to detect any ‘blunders’. This spreadsheet can be used later as the basis for the calculations of the KCRV(s) and DoE(s). If a result appears to be an outlier, before the Draft A report is prepared, the pilot should ask the participant to check their result (and reply within 4 weeks), but not inform them of the magnitude or sign of the discrepancy. The intention is to correct blunders or identify that the participant has a problem with their service (which could affect measurements for customers if un-reported). Multiple attempts at correcting results nominated by the pilot as outliers are not allowed.

In larger comparisons it may be helpful to run a draft A.0 stage. This involves sending each participant a copy of their results only, in the format which will be used for the comparison analysis. This allows participants to check their own data entry into the comparison analysis process directly and avoids transcribing errors by the pilot. This can be done ahead of draft A.1 for all participants other than the final participant, hopefully catching any errors ahead of the general circulation of results and speeding up the draft A.1 stage.

## The first Draft A report (Draft A.1)

As soon as the last results are received from the participants, the pilot should inform the participants, the KCDB coordinator and WG-MRA chair that the artefact circulation is completed. They should also update the KCDB record of the comparison. The option to withdraw or modify results expires when the pilot has received all results and the final reporting deadline of the last participant has expired – participants at the end of the circulation loop should be aware that they may have little time to make any corrections/withdrawals after result submission, and before the Draft A.1 report is prepared.

The CIPM guide (~~CIPM MRA-D-05~~ [4]) requests a KCRV to be calculated at Draft A stage, however this WG-MRA Guidance Document recommends that KCRV calculation is delayed until after the results of the participants are confirmed in the first Draft A report, to prevent unnecessary work for the pilot.

The first Draft A report (Draft A.1) should be a concise summary of the results received by the pilot. No analysis is attempted at this stage. It is perfectly acceptable, and recommended, to use just a simple spreadsheet containing all the received data, properly and consistently formatted. The Draft A.1 report should include the name and number of the comparison (KCDB designator), the measurands, results and the accompanying standard uncertainties and their degrees of freedom, as submitted by the participants, but formatted into a consistent style and convenient multiple/sub-multiple SI units.

The exact layout of the spreadsheet will depend on the number and complexity of the measurands and the preference of the pilot. As a general guide, it is useful to include one sheet with all the results as reported by the participants and then perform data analysis on additional sheets, linking to the original data. Complex measurands may require one sheet per participant, for the raw data.

In the case where an NMI has submitted results from several instruments (and this was allowed in the protocol), it should be made clear which result is deemed to be the primary result and only this result should be used in the later calculation of the KCRV (*i.e.* one result per participant). This also applies to repeat measurements by the pilot or other laboratories used as stability checks. Only one result (usually the first obtained) should be used in the KCRV calculation.

The Draft A.1 ‘report’ is sent to all the participants, inviting them to check the transcription of the submitted data. Obvious blunders in copying data into the sheet are corrected after the pilot is notified.

As soon as Draft A.1 is sent out, no NMI may modify its results (excepting obvious typographical errors, or use of the wrong *k* value) and withdrawal of results is no longer permitted – results must appear in the report, but outliers will be excluded from contributing to the KCRV.

The only exception to the above rules is covered by a decision of the WGDM (see CCL/WGDM-02-44 §8.1 [9], or WGDM-02-55 §8.1 [10]) that allows salvaging of numerical blunders after Draft A.1. This is at the discretion of the pilot in conversation with the participant concerned and the corrections should be clearly described in the Final report.

Individual values and uncertainties may be changed or removed or the complete comparison abandoned, only with the agreement of all participants and on the basis of a clear failure of the traveling standard or some other phenomenon that renders the comparison or part of it invalid.

**Any modifications to results after distribution of Draft A.1 must be described in the report.**

Draft A.1 and all subsequent Draft A reports are confidential to the participants.

## Subsequent Draft A reports (Draft A.2, A.3, …)

After receiving any corrections from participants, the pilot should prepare the main report on the comparison. This report starts as a Draft A document and eventually changes to become the Final Report.

The exact format of the Draft A Report is at the discretion of the pilot. The template in Guidance Document CCL-WG-MRA-GD/3.2 [14] is recommended as a suitable basis for the document. The following content is considered mandatory:

* title page, containing comparison name, number (KCDB designator), author list, version, date;
* information on the measurand(s);
* participant list;
* artefact details including stability measurements of artefacts (if performed);
* planned timetable and actual timetable;
* raw data submitted by participants;
* any corrections to data after Draft A.1;
* any initial analysis or processing;
* calculation of the KCRV (& KCRV uncertainty) including iterative outlier rejection (as necessary);
* calculation of the Degrees of Equivalence (& uncertainties);
* suitable graphs;
* summary & conclusions.

For more detailed information on these items and the formulae used for performing the calculations, please see the template [14]. In general limit the reporting of uncertainties to 2 significant figures.

It is suggested that any graphs that are inserted into the Report are inserted as links. Then, any changes to the analysis spreadsheet will automatically propagate to the graphs in the report. However, care is needed if a graph shows only one participant’s results at a time as leaving such graphs linked to the source sheet will risk them updating to another participant’s results whenever the source file changes. When the Report is sent to participants, it can either be sent as a PDF or (at the pilot’s discretion) as a copy of the Word file, with the Links converted to *in situ* graphics.

The Draft A report is iterated until the participants have all agreed on the report and all changes and corrections have been made and there are no outstanding discussions. Each major re-issue of the report is allocated a new number: A.2, A.3, *etc.*

According to a recommendation (WGMRA.2011.R6) at the 2011 meeting of the WG-MRA (see [19]), pilots of comparisons should request from participants, a list of CMCs which the comparison is intended to support, along with the current CMC values in these services, when submitting results to the pilot. This list of CMCs should be included in either the Final Report or, if issued, the Executive Report.

Sometimes, the pilot and/or participants desire external opinions on the report whilst at Draft A stage, for example seeking opinion of the TC-L community or discussion of some problem of the analysis during a meeting of the WG-MRA or local RMO. This is permitted, provided that:

* prior permission is given by all participants (4 week deadline to object);
* discussions and any tabled documents are considered confidential;
* the report remains as a Draft A report.

## Final Draft A report becomes Draft B

As soon as a final version of the Draft A report is ready and agreed by all participants, it is renamed as the Draft B report (remembering to change the front page name and any headers/footers *etc*.).

If not done so already, the front-page author list should now be finalised to be the pilot, followed by the NMI participants.

The KCDB coordinator, WG-MRA chair and the participants should be informed that the report is now at Draft B status. The pilot should update the KCDB entry status to Draft B.

The Draft B report is no longer considered confidential. If, as is usual, KCRV and Degrees of Equivalence values are included along with their uncertainties, they must be labelled as ‘Provisional’ since the KCRV is not valid until confirmed by the WG-MRA.

The Draft B report should be studied and endorsed by the relevant committee:

* RMO key comparison reports are endorsed by the relevant RMO TC-L;
* inter-RMO key comparison reports are endorsed by the relevant RMO TC-L, (remembering to copy any discussions/outcomes to extra-RMO participants) then must be endorsed by all RMO TC-Ls with participants in the comparison;
* RMO supplementary comparison reports are approved by the relevant RMO TC-L (but require WG-MRA to examine them before entering the KCDB);
* CCL key and supplementary comparison reports are approved by the WG-MRA.

Ideally, there should be only a single version of the Draft B report unless the TC-L or WG-MRA requests specific changes to be made. In any case, a final version of the Draft B report should be prepared.

## Final approval process

The CIPM guide (~~CIPM MRA-D-05~~ [4]) states that the WG-MRA (having this authority delegated to it by CCL [16]) has to approve all comparison reports, except for RMO supplementary comparison reports, which may be approved within the RMO, usually by the relevant TC-L.

However, the CIPM guide [4] also states that, for supplementary comparisons to be used to support CMCs, their final reports must be approved by the WG-MRA (on behalf of the CCL) and published in the KCDB (this is a requirement in the technical Annex of the MRA itself). The review of the Final Report is not supported by the KCDB platform.

Therefore, in summary, all length comparison reports must be approved by the WG-MRA if they are to be used to support CMC claims. The approval route is *via* submission to the WG-MRA’s sWG-KC, which will nominate two reviewers to examine the report in detail before sending it to the full WG-MRA for approval. The WG-MRA informs the CCL at each CCL meeting on the reports it has approved on behalf of the CCL.

The pilot of a comparison should therefore submit the Draft B report (or Final report of an RMO supplementary comparison) to the chair of sWG-KC. As a courtesy, this is often done *via* the local TC-L chair (for RMO comparisons).

## Entry into the KCDB

After final approval by WG-MRA, the pilot and KCDB Coordinator will be informed of this by sWG-KC chair and the pilot should then do the following:

* edit the report, changing it to be Final Report (title, headers, footers *etc*.);
* update the date of the report;
* prepare an unprotected PDF copy of the finalised report;
* (only for key comparisons): prepare Degree of Equivalence (DoE) tables (spreadsheet) and graphs in KCDB format;
* submit the PDF Final Report (and for key comparisons the DoE tables, graphs) directly to the KCDB via the KCDB web platform tab ‘FINAL REPORT’.

The KCDB web platform will then automatically notify the TC chairs, WG chairs, CCL Executive Secretary and KCDB office.

DoE tables and graph templates can be obtained from the KCDB coordinator or previous comparison pilots. The KCDB coordinator will normally supply the template when the Final Report is submitted.

Once the Final Report of a key or supplementary comparison has been approved and posted in the KCDB, it is automatically published in the *Metrologia* *Technical Supplement* (unless the authors do not wish it).

A short abstract in Word format is therefore required to accompany the PDF Final Report. A template for the abstract is available from the CCL-WG [website](http://www.bipm.org/utils/common/documents/CIPM-MRA/MET-Technical-Supplement.docx). This form requires the names, contact addresses and ORCID IDs (where available) of all the co-authors. It is recommended that the pilot gathers these during the report process. The KCDB Office handles the publication process.

The author list for the *Metrologia* abstract should be the same as that of the Final Report, *i.e.* pilot, followed by the NMI participants.

# Executive Report

Executive Reports are (currently) unique to CCL. The purpose of the Executive Report is to summarise the critical data from the comparison that will be needed to validate CMC claims, and to report on any corrective actions including those undertaken or still outstanding. The Executive Report should therefore be much shorter than the Final Report. The intended audience for the Executive Report is sWG-CMC, which will use the report to examine if there is any impact on CMC claims of the participants.

An Executive Report should ideally be prepared for every comparison, because the analysis of CMCs is not normally the subject of a Final Report.

An Executive Report should be as short as possible, but should contain the following:

* name, number (KCDB designation) of comparison;
* summary of measurands, ideally with list of corresponding DimVIM category(s);
* list of participant NMIs/DIs and the dates when they made the measurements (month-year);
* KCRV(s);
* DoE of NMIs with respect to KCRV(s) computed at 95 % confidence level and any existing KCDB CMC claims for a matching service, ideally combined into a single table;
* discussion of whether or not an NMI’s CMC claims are supported by the comparison;
* discussion of any agreed corrective actions and progress on completing these.

A template Executive Report is available from the BIPM server [16].

The Executive Report is normally prepared approximately 6 months after the Final Report enters the KCDB. The deliberate delay is to allow any short corrective actions to be performed and reported to the pilot. However, if there are no corrective actions and the effect on CMC claims is simple to interpret, there is no need to delay the Executive Report.

If the participants did not supply a list of CMCs supported by the comparison at the time of submitting results, the pilot should now request that data.

### Discussion on CMC claims

The recommendation on whether or not the results for a participant support their CMC claims is at the discretion of the pilot, subject to certain guidance.

Usually, it is common to include a statement such as ‘*unless discussed otherwise below, any existing CMC claims of the participants are supported by the results of this comparison*’.

This is then followed by a discussion of any borderline or unsupported CMC claims on a case-by-case basis. There are several factors to consider:

* the achieved DoE and its (95 %) uncertainty, in the comparison;
* the CMC claim (at 95 %);
* the reported measurement uncertainty in the comparison;
* issues regarding non-standard procedures, difficult artefacts, *etc*. which lead to increased uncertainty.

Example CMC scenarios

The following are a few examples of situations which might occur to the pilot.

En values for DoE are well controlled (<≈ 1):

No CMC claim in the KCDB

No problem. NMI has no CMCs but, in future, could use performance in this comparison to support CMCs with an uncertainty similar to that reported in their comparison results.

CMC claim in the KCDB ≈ DoE

No problem. The existing CMC claim is supported.

CMC claim in the KCDB << DoE

Possible problem because the uncertainty reported for the comparison measurements is much larger than the CMC claim. The NMI should explain why they had to use a larger uncertainty than their CMC claim. Possible explanations could be: a non-standard procedure was required; the artefact was damaged but measured anyway.

CMC claim in the KCDB >> DoE

No problem. The existing CMC claim is supported. The NMI performance is better than anticipated. They may want to study their uncertainty budget and consider a future revision of the CMC with a smaller uncertainty (if supported by other evidence and a re-calculated uncertainty budget).

En values for DoE are excessive (>>1):

No CMC claim in the KCDB

No problem regarding CMCs as there are none yet, but the NMI would need to increase the uncertainty value or revise equipment/procedures before a future CMC claim would be supported by this comparison.

CMC claim in the KCDB ≈ DoE

Problem. The excessive En values for the DoE indicate that the claimed uncertainty in the comparison is too small. Because the CMC claim is a similar value, it may not be supported by the comparison. The DoEs should be carefully compared against the CMC claim. If a significant number of DoEs exceed the claimed CMC uncertainty by more than a reasonable margin, the CMC claim is unsupported.

CMC claim in the KCDB < DoE

Problem. The excessive En values for the DoE indicate that the claimed uncertainty in the comparison is too small. Because the CMC claim is a smaller value, it is not supported by the comparison. If a significant number of DoEs exceed the claimed CMC uncertainty by more than a reasonable margin, the CMC claim is unsupported.

CMC claim in the KCDB >> DoE.

Possible problem. The excessive En values for the DoE indicate that the claimed uncertainty in the comparison is too small. However, the CMC claim is much larger, therefore is might still be supported. The DoEs should be carefully compared against the CMC claim. If a significant number of DoEs exceed the claimed CMC uncertainty by more than a reasonable margin, the CMC claim is unsupported.

***Significant* and *reasonable***

In the above discussion, the terms ‘reasonable’ and ‘significant’ were used, as well as the symbols ‘≈’ and ‘>>’ to indicate a necessary degree of flexibility. The pilot should use discretion when interpreting these terms bearing in mind that:

* at 95 % confidence level, 1 in 20 results may be expected to have En > 1.00. It might not be significant if one or two results in a group of 10 or 20 have En value slightly larger than 1. In a comparison with 100 measurands (*e.g.* ball plate centre locations), achieving ‘poor’ results on one or two balls may also not be significant. Guidance for the quantitative assessment of participant results in comparisons with multiple measurands is provided in section 5.2.
* the DoEs have an uncertainty and there is an uncertainty of the uncertainty. It might be reasonable to allow some results with En = 1.1, especially if the artefacts were damaged during the comparison and no additional artefact uncertainty was used.

The choice of requesting the pilot to prepare these discussions is quite deliberate – it is the pilot who has the closest understanding of the performance of the NMIs in the comparison and, in EURAMET, it is the pilots of the comparisons which are the initial (intra-RMO) CMC reviewers.

Pilots should not be afraid to state that a CMC claim is not supported by a comparison result, where this is clearly shown by the data. All the participants and then the relevant committee will discuss the report and final discussion will be within the WG-MRA – there will be plenty of opportunities for the affected NMI to argue its case.

### Discussion on Corrective Actions

When a participant’s results do not support their CMC claim, the NMI concerned has primary responsibility to solve the problem either by fixing any technical problem, or by increasing the CMC uncertainty, or requesting suspension of the CMC.

An NMI which has poor results in a comparison should explain to the pilot what their corrective actions will be and the anticipated likely timescale for these actions. The pilot should include these in the Executive Report. In some cases, the 6 month delay before submitting the Executive Report is sufficient time to complete the actions, and then the pilot can report that these were concluded, when the Executive Report is finalised. Any outstanding actions, not completed before the Executive Report is issued, should be listed in the report as being outstanding. These will be reviewed at a later date by the relevant committee (RMO TC-L, WG-MRA, CCL, *etc.*).

If possible, where completion of a corrective action has fixed any problem, to the satisfaction of the pilot, the pilot should state this in the Executive Report, indicating that the comparison results plus the Corrective Action are sufficient to support a CMC claim.

### Submitting an Executive Report

The Executive Report is discussed ‘locally’ (RMO TC-L) as necessary and sent to the WG-MRA for final discussion and approval.

After discussion and approval of the Executive Report by WG-MRA, it is sent to the CCL Executive Secretary, for storing on the website, and copied to chair of sWG-CMC.

The Executive Report is normally considered confidential to the participants, pilot and the WG-MRA. It is stored on the private area of the BIPM server.

### Including Executive Report in the Final Report

At the pilot’s discretion, and for simple comparisons where the conclusions are clear, *e.g*. all results have good En values and all CMC claims are supported, the Executive Report may be omitted, discussion on CMC claims might be omitted from the Executive Report and included, instead, in the Final Report. However, the pilot should ensure that the relevant recipients who would normally receive the Executive Report receive a copy of the Final Report – particularly important is that the chair of sWG-CMC receives a copy. Also, the participants should be aware that the discussion of CMCs, if included in the Final Report, will be public because the Final Report is made available on the public KCDB website.

## Assessment of KC results with several measurands

Judging whether a participant’s results support their uncertainty claims is more complicated when there are several measurands in the comparison, increasing the risk that even a participant with correctly evaluated uncertainties will see a few large normalized errors. While no rigid decision rule can replace well-informed scientific judgment, the following procedure can systematize the acceptance of straightforwardly reasonable uncertainty claims.

Consider a single comparison participant reporting results for different measurands, labelled , relating to a particular uncertainty claim or CMC. Denote the DoEs as , the difference between the value reported by the participant and the corresponding KCRV, together with its uncertainty . The uncertainty includes the uncertainty of the KCRV, the uncertainty claimed by the participant, and the correlation between the two when the KCRV was a consensus value to which the participant contributed [see Beissner, Metrologia 39, p.59 (2002)].

1. Compute normalized errors as with the coverage factor for approximately 95% confidence. If , that is if the difference between the participant’s result and the KCRV is less than the expanded uncertainty of this difference for all of the participant’s measurements, immediately *accept* the uncertainty claim.
2. Otherwise, compute the chi-squared test statistic . Compare to , the 95th percentile of the chi square distribution with degrees of freedom. This threshold value can be obtained from statistical tables as the upper-tail critical value of the chi-square distribution at a significance of , or from software implementations such as CHISQ.INV(0.95, ) in Excel, scipy.stats.chi2.ppf(0.95, ) in Python with the SciPy libraries, or chi2inv(0.95, ) in Matlab. If , *accept* the uncertainty claim.
3. Otherwise, exceeds the threshold and for at least one measurement. There may be a problem, so *investigate* the participant’s results and uncertainty claim.

The rationale for this procedure is discussed in the appendix.

# References to applicable documents

## CIPM/MRA documents

~~[1] The MRA [CIPM MRA]~~

[~~http://www.bipm.org/en/cipm-mra/mra\_online.html~~](http://www.bipm.org/en/cipm-mra/mra_online.html)

~~[2] MRA Technical supplement [CIPM MRA-T]~~

[~~http://www.bipm.org/utils/en/pdf/mra\_techsuppl2003.pdf~~](http://www.bipm.org/utils/en/pdf/mra_techsuppl2003.pdf)

~~[3] Calibration and Measurement Capabilities in the context of the CIPM MRA [CIPM MRA-D-04]~~

[~~http://www.bipm.org/utils/common/CIPM\_MRA/CIPM\_MRA-D-04.pdf~~](http://www.bipm.org/utils/common/CIPM_MRA/CIPM_MRA-D-04.pdf)

~~[4] Measurement comparisons in the CIPM MRA [CIPM MRA-D-05]~~

[~~http://www.bipm.org/utils/common/CIPM\_MRA/CIPM\_MRA-D-05.pdf~~](http://www.bipm.org/utils/common/CIPM_MRA/CIPM_MRA-D-05.pdf)

~~[5] Guidelines for CIPM Key Comparisons [CIPM KC guidelines]~~

[~~http://www.bipm.org/utils/en/pdf/guidelines.pdf~~](http://www.bipm.org/utils/en/pdf/guidelines.pdf)

~~[6] CIPM MRA Guidelines for Authorship of […] Comparison Reports [CIPM MRA-G-04]~~[~~http://www.bipm.org/utils/common/CIPM\_MRA/CIPM\_MRA-G-04.pdf~~](http://www.bipm.org/utils/common/CIPM_MRA/CIPM_MRA-G-04.pdf)

## JCRB documents

~~[7] Key & Supplementary Comparison registration form [JCRB-9/9(1)]~~

[~~http://www.bipm.org/utils/common/documents/jcrb/registration\_form.doc~~](http://www.bipm.org/utils/common/documents/jcrb/registration_form.doc)
[No longer used with KCDB2]

~~[8] Monitoring the impact of key and supplementary comparison results on CMC claims [JCRB-11/7(a)]~~

[~~http://www.bipm.org/utils/common/documents/jcrb/impact\_comparisons.pdf~~](http://www.bipm.org/utils/common/documents/jcrb/impact_comparisons.pdf)

## CCL/WGDM documents

[9] WGDM decision on blunders after Draft A.1 [WGDM-02-44-§8.1]
<http://www.bipm.org/wg/CCL/CCL-WG/Restricted/2002/WGDM-2002.zip>

[10] WGDM decision on blunders after Draft A.1 [WGDM-02-55-§8.1]
[http://www.bipm.org/wg/CCL/CCL-WG/Restricted/Minutes/WGDM\_7th\_(2002)\_min.pdf](http://www.bipm.org/wg/CCL/CCL-WG/Restricted/Minutes/WGDM_7th_%282002%29_min.pdf)

[11] WGDM decision on discrepant results during a comparison circulation [WGDM-03-55-§8.3]
<http://www.bipm.org/wg/CCL/CCL-WG/Restricted/Minutes/WGDM-03-55-Minutes-Issue1.pdf>

[12] WGDM guidelines on key comparison analysis [WGDM-05-80, CCL12-§9]

<https://www.bipm.org/utils/common/pdf/CC/CCL/CCL12.pdf>

## CCL/WG-MRA documents

[13] WG-MRA decision on reviewing reports [WG-MRA-13-99]
<http://www.bipm.org/wg/CCL/CCL-WG/Restricted/Minutes/>

[14] WG-MRA Final Report template(s) [WG-MRA-GD-3.2]
<https://www.bipm.org/utils/common/pdf/CC/CCL/CCL-GD-3.2.doc>
<https://www.bipm.org/utils/common/pdf/CC/CCL/CCL-GD-3.2b.doc>

[16] WG-MRA Executive Report template
<https://www.bipm.org/utils/common/pdf/CC/CCL/CCL-GD-3.3.doc>

## CCL documents

[17] Minutes of 14th meeting of the CCL [CCL 2009]
<https://www.bipm.org/utils/common/pdf/CC/CCL/CCL14.pdf>

[18] Minutes of 15th meeting of the CCL [CCL 2012]

<https://www.bipm.org/utils/common/pdf/CC/CCL/CCL15.pdf>

[19] CCL Guidance document GD-1 – Running of MRA comparisons in length metrology and monitoring their impact on CMCs [CCL-GD-1]
~~<https://www.bipm.org/utils/common/pdf/CC/CCL/CCL-GD-1.doc>~~

## General MRA reference

Many of the above documents can be accessed via the CIPM MRA documents web page:

[~~http://www.bipm.org/en/cipm-mra/documents/~~](http://www.bipm.org/en/cipm-mra/documents/)

# Appendix: Discussion of the assessment procedure for multi-measurand KCs

If the errors in the measurement results and in the KCRV are normally distributed and if the uncertainties are correctly estimated, then the quantity will follow a standard normal distribution. If the errors for the different measurands are also independent, then the test statistic computed in section 5.2 is the sum of the squares of independent standard normal random variables, and hence it follows a chi-square distribution with degrees of freedom. Under these idealized conditions, the second step of the proposed procedure in section 5.2 accepts the participant’s claim with 95% probability.

Because the different results all come from a single participant, and generally a single measurement campaign lasting a few days or weeks, the assumption of independence is unlikely to hold in practice. Any systematic effects, or even fluctuating effects that fluctuate more slowly than the time between measurements, will introduce correlations between the values . Ideally, we would like to maintain the same acceptance probability no matter how the errors are correlated. The procedure of section 5.2 approximates this by combining two tests, one suitable for independent errors () and one for perfectly-correlated ones (). In the intermediate case of partial correlations, the approximation is too strict: for instance the acceptance probability drops to 87% in a scenario with M = 10 and a systematic uncertainty that contributes three-quarters of the total variance. The slightly increased risk of investigating valid claims, and the need for expert judgment in such cases, is the price paid for the relative simplicity of the procedure.

The assumption of normally-distributed errors is common in dimensional measurements, but always open to question. An unexpected fat-tailed error distribution will lead to a reduction of the acceptance probability. This may not be a drawback in practice, since such an error distribution, with more large outliers than expected, would be worth investigating in any case. Known deviations from normality, expressed as small degree-of-freedom estimates, are included in the test by the use of appropriate coverage factors . This allows the procedure to handle correlated or systematic errors with a known heavy-tailed distribution. Correcting the chi-squared test for known heavy-tailed distributions would be more complicated and we do not attempt it, but note that the chi-squared test is primarily aimed at uncertainty contributions that fluctuate independently from measurement to measurement, and high-degree-of freedom type-A uncertainty estimates are likely to be available for those contributions.