Recent Progress in Determining Gravitational Constant $G$ at HUST

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Outline

- Brief Review of $G$
- HUST-09 experiment
- Recent progress
Newtonian law of universal gravitation

\[ F_1 = F_2 = G \frac{m_1 \times m_2}{r^2} \]
The first $G$ value

Cavendish’ experiment

\[
G = \frac{2\pi^2 LR^2}{MT^2} \theta
\]

1798, $G = (6.67 \pm 0.07) \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$ 1%
During past 200 years, there are more than 300 results but its accuracy was improved by only two orders!

<table>
<thead>
<tr>
<th>Year</th>
<th>G value/ $\times 10^{-11}$ m$^3$kg$^{-1}$s$^{-2}$</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1798, Cavendish</td>
<td>6.67 ± 0.07</td>
<td>1%</td>
</tr>
<tr>
<td>1973 CODATA</td>
<td>6.6720 ± 0.0041</td>
<td>0.061%</td>
</tr>
<tr>
<td>1986 CODATA</td>
<td>6.67259 ± 0.00085</td>
<td>0.013%</td>
</tr>
<tr>
<td>1998 CODATA</td>
<td>6.673 ± 0.010</td>
<td>0.15%</td>
</tr>
<tr>
<td>2002 CODATA</td>
<td>6.6742 ± 0.0010</td>
<td>0.015%</td>
</tr>
<tr>
<td>2006 CODATA</td>
<td>6.67428 ± 0.00067</td>
<td>0.01%</td>
</tr>
<tr>
<td>2010 CODATA</td>
<td>6.67384 ± 0.00080</td>
<td>0.012%</td>
</tr>
</tbody>
</table>

$G$ was known earliest, but with worst precision.
Uncertainty < 50 ppm

consistent with each other at only ~500 ppm
Outline

♦ Brief Review of G
♦ HUST-09 experiment
♦ Recent progress
Time-of-swing method

\[ I \ddot{\theta} + \gamma \dot{\theta} + K \theta = \tau_G(\theta) \]

\[ \tau_G(\theta) \approx \tau_0 - K_G \theta \]

\[ \omega_n^2 = \frac{K_n + GC_{gn}}{I} \quad \text{near} \]

\[ \omega_f^2 = \frac{K_f + GC_{gf}}{I} \quad \text{far} \]

\[ G = \frac{I \left( \omega_n^2 - \omega_f^2 \right) - \left( K_n - K_f \right)}{C_{gn} - C_{gf}} \]

\[ = \frac{I \Delta \omega^2 - \Delta K}{\Delta C_g} \]

1. Determination of \( P_g = \Delta C_g/I \)
2. \( \Delta \omega^2 \) between near and far positions
3. Anelasticity of torsion fiber \( \Delta K \)
HUST-99 G measurement

Merits:

The period change: 27%
High Q = 36000

\[ T_1 = 3483.79 \text{ s} \quad T_2 = 4439.15 \text{ s} \]

HUST-99 = \((6.6699 \pm 0.0007) \times 10^{-11}\) m³kg⁻¹s⁻²

105 ppm
HUST-09 experimental design

Compare with HUST-05

**Merits:**

1. Spherical source mass
2. Simple pendulum
3. All in vacuum
4. Direct Measurement of anelasticity
Experimental environment

Daily fluctuations: 5/1000 °C
Determination of $\Delta \omega^2_{nf}$

10 sets of data (6 days per set) with the spheres in near and far positions alternately

Period drift:
1. Aging
2. Background gravitational change caused by zero drift effect
3. A-B-A
Determination of $\Delta \omega^2_{\text{back}}$

4 sets of background data (without the spheres) in near and far positions alternately

\[ \Delta(\omega^2) = \Delta \omega^2_{nf} - \Delta \omega^2_{back} = 1.682245(31) \times 10^{-6} \text{ s}^{-2} \]

18.428 ppm
Fiber anelasticity

\[ K = K_0 + \Delta K(\omega) \]

Correction for \( G \) value due to the fiber’s anelasticity

Kuroda: \[ \frac{\Delta G}{G} = \frac{1}{\pi Q} \]  
(PRL 75 (1995) 2796)

Newman: \[ 0 < \frac{\Delta G}{G} < \frac{1}{2Q} \]  
(MST 10 (1999) 445)

If \( Q = 1000 \), \[ \frac{\Delta G}{G} = \frac{1}{\pi Q} \approx 320 \text{ ppm} \]
Direct measurement of anelasticity

Pendulum 1  Pendulum 2

Tungsten fiber \( \omega_1^2, I_1 \) \( \omega_2^2, I_2 \)

Silica fiber \( \Omega_1^2, I_1 \) \( \Omega_2^2, I_2 \)

TABLE IV. One \( \sigma \) uncertainty budget of the anelasticity to \( \Delta G/G \).

<table>
<thead>
<tr>
<th>Source</th>
<th>Value (error)</th>
<th>ppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_1 )</td>
<td>( 5.3318(8) \times 10^{-5} \text{ kgm}^2 )</td>
<td>0.03</td>
</tr>
<tr>
<td>( I_G )</td>
<td>( 4.505679(35) \times 10^{-5} \text{ kgm}^2 )</td>
<td>0.00</td>
</tr>
<tr>
<td>Magnetic damper</td>
<td></td>
<td>4.78</td>
</tr>
<tr>
<td>Statistical ( (\omega_2/\omega_1)^2 )</td>
<td>( 1.4171723(92) )</td>
<td>18.41</td>
</tr>
<tr>
<td>Statistical ( (\Omega_1/\Omega_2)^2 )</td>
<td>( 0.7056861(5) )</td>
<td>2.01</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>( -4.14(89) \times 10^{-6} )</td>
<td>2.52</td>
</tr>
<tr>
<td>( \frac{L}{L_1} = \left( \frac{\Omega_1}{\Omega_2} \right)^2 (1 + \varepsilon) )</td>
<td>( 0.7056832(8) )</td>
<td>3.22</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>18.69</td>
</tr>
</tbody>
</table>

\[
\frac{K(\omega_n) - K(\omega_f)}{I_G(\omega_n^2 - \omega_f^2)} = \frac{I_1}{I_G(\omega_2^2 / \omega_1^2 - 1)} \left( \frac{\Omega_1^2 \omega_2^2}{\Omega_2^2 \omega_1^2} - 1 \right) = (211.80 \pm 18.69) \text{ ppm}
\]

PRD 80 (2009) 122005
Final $G$ value: HUST-09

Result of $G$ ($10^{-11}$ m$^3$kg$^{-1}$s$^{-2}$)

$G_1 = (6.67352\pm0.00019)$

$G_2 = (6.67346\pm0.00021)$

After change the position and orientation of spheres:

The difference: 9 ppm

Combined result:

$G = (6.67349\pm0.00018)$

26.3 ppm

PRL 102 (2009) 240801
PRD 82 (2010) 022001
Uncertainty < 50 ppm

consistent with each other at only ~500 ppm
Are there any method-dependent systematic errors?

- Same laboratory
- Different methods
- Same $G$ value?

- Time-of-Swing method
- Angular Acceleration Feedback method

A large discrepancy?
Brief Review of $G$

HUST-09 experiment

Recent progress
Two methods

- **Time-of-swing**
  \[ I \ddot{\theta} + \gamma \dot{\theta} + K \theta = \tau_g \]

- **Angular Acceleration**
  \[ I \ddot{\theta} + \gamma \dot{\theta} + K \theta = \tau_g - I \alpha_t \]

**G**:
\[ G = \frac{I \left( \omega_n^2 - \omega_f^2 \right) - (K_n - K_f)}{C_{gn} - C_{gf}} \]

\[ G = \frac{\alpha_t(\omega)}{P_g} \]

- **Simple device**
  - Differential measurement
- **Independent of anelasticity**
- **Dependence on fiber properties**
  - Environmental influences
- **Complicated device**
  - High-performance turntable
1. Improved time-of-swing method

To reduce the large corrections in HUST-09:

1. Aluminum layer
2. Three-point mount
3. Copper tube
4. High-Q Silica fiber \( Q > 5 \times 10^4 \)
5. Thick torsion fiber
6. Gravity compensation

<table>
<thead>
<tr>
<th>Error sources</th>
<th>Corrections (ppm)</th>
<th>( \delta G/G ) (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torsion pendulum</td>
<td>5.05 [5.05]</td>
<td></td>
</tr>
<tr>
<td>Dimension</td>
<td>1.95</td>
<td></td>
</tr>
<tr>
<td>Attitude</td>
<td>0.13 [0.07]</td>
<td></td>
</tr>
<tr>
<td>Density inhomogeneity</td>
<td>&lt;0.21</td>
<td></td>
</tr>
<tr>
<td>Chamfer property</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>Three chips</td>
<td>-0.12</td>
<td>0.17</td>
</tr>
<tr>
<td>Coating layer</td>
<td>-24.28</td>
<td>4.33</td>
</tr>
<tr>
<td>Clamp and ferrule</td>
<td>1.65</td>
<td></td>
</tr>
<tr>
<td>Reflecting mirror</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>Source masses</td>
<td>10.66 [10.64]</td>
<td></td>
</tr>
<tr>
<td>Masses</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>Distance of GCs</td>
<td>9.64 [9.61]</td>
<td></td>
</tr>
<tr>
<td>Density inhomogeneity</td>
<td>4.50</td>
<td></td>
</tr>
<tr>
<td>Relative positions</td>
<td>1.10 [1.31]</td>
<td></td>
</tr>
<tr>
<td>Height of pendulum</td>
<td>0.76 [0.40]</td>
<td></td>
</tr>
<tr>
<td>Height of spheres</td>
<td>0.48 [0.27]</td>
<td></td>
</tr>
<tr>
<td>Position of torsion fiber</td>
<td>0.63 [1.22]</td>
<td></td>
</tr>
<tr>
<td>Position of turntable</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>( \theta_0 )</td>
<td>0.06 [0.01]</td>
<td></td>
</tr>
<tr>
<td>Fiber</td>
<td>18.76</td>
<td></td>
</tr>
<tr>
<td>Nonlinearity</td>
<td>&lt;0.70</td>
<td></td>
</tr>
<tr>
<td>Thermoelasticity</td>
<td>-39.83 [8.37]</td>
<td>1.52 [0.82]</td>
</tr>
<tr>
<td>Anelasticity</td>
<td>-211.80</td>
<td>18.69</td>
</tr>
<tr>
<td>Aging</td>
<td>&lt;0.01</td>
<td></td>
</tr>
<tr>
<td>Gravitational nonlinearity</td>
<td>7.73 [4.79]</td>
<td>0.30 [0.20]</td>
</tr>
<tr>
<td>Magnetic damper</td>
<td>17.54</td>
<td>0.31</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>Electrostatic field</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Statistical ( \Delta \omega^2 )</td>
<td>18.43 [23.31]</td>
<td>28.86 [32.17]</td>
</tr>
<tr>
<td>Total</td>
<td>26.33</td>
<td></td>
</tr>
<tr>
<td>Combined</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Improvement I: coating layer**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Material</th>
<th>Density (g/cm³)</th>
<th>Correction to G (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HUST-09</td>
<td>Cu</td>
<td>8.96</td>
<td>-24.28(4.33)</td>
</tr>
<tr>
<td></td>
<td>Au</td>
<td>19.26</td>
<td></td>
</tr>
<tr>
<td>Present</td>
<td>Al</td>
<td>2.70</td>
<td>-1.7(8)</td>
</tr>
</tbody>
</table>
Improvement II: three-point mount

The repeatability of mounting is better than 0.25 μm

Supporting ring in HUST-09 → Three-point mount

Zerodur ring
Invar steel cylinder
Zerodur ring

0.35 μm → 9.0 ppm
Improvement III: copper tube

Thermal conductivity of copper: 401 W/mK

HUST-09: <0.05 °C

Temperature difference between sensors: <0.02 °C

<5 ppm
Improvement IV: silica fiber

Trade name: **SuprasilR 311**
Length: 900 mm
Diameter: 44 and 50 μm
Germanium: 8-nm-thickness
Bismuth: 11-nm-thickness
Anelasticity: ~6.4 ppm

- **Resistance:** 2.3 MΩ

**Q=5.0(4)×10⁴**
**Improvement V: magnetic damper**

\[ \frac{\Delta G}{G} = \frac{I_1 K^2}{I K_1^2} \propto \frac{1}{K_1^2} \]

\[ K_1 = \frac{\pi D^4 S}{32l} \]

\[ \frac{\Delta G}{G} \propto \frac{l^2}{D^8} \]

**Correction to G:** 17.54(0.31) ppm → 0.4(0.1) ppm

Two-stage torsion balance

Diameter: 50 μm → 80 μm
Length \( l \): 9 cm → 5 cm
Improvement VI: gravity compensation

1. The background gravitational gradient is reduced to 1/5 of that before compensation

2. The equilibrium position of $G$ pendulum is located at the min-gradient position

For $G$: <0.4 ppm
The difference between them is: ~10 ppm
Repeat the measurements with different fibers to find the potential systematic errors, such as entangled dislocations:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Present</th>
<th>Next</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber material</td>
<td>Silica</td>
<td>Silica</td>
</tr>
<tr>
<td>Diameter(μm)</td>
<td>44</td>
<td>50</td>
</tr>
<tr>
<td>Natural period(s)</td>
<td>391</td>
<td>306</td>
</tr>
<tr>
<td>Period change(s)</td>
<td>1.26</td>
<td>0.61</td>
</tr>
<tr>
<td>Period relative change</td>
<td>0.32%</td>
<td>0.20%</td>
</tr>
<tr>
<td>Ratio of loading</td>
<td>40%</td>
<td>31%</td>
</tr>
<tr>
<td>Thickness of Ge and Bi(nm)</td>
<td>~8, ~11</td>
<td>~2, ~2</td>
</tr>
<tr>
<td>Resistance(Ω)</td>
<td>~2.3×10^6</td>
<td>(8-10)×10^6</td>
</tr>
<tr>
<td>Q</td>
<td>~5.0×10^4</td>
<td>~8×10^4</td>
</tr>
<tr>
<td>Anelasticity(ppm)</td>
<td>~6.4</td>
<td>~4.0</td>
</tr>
</tbody>
</table>
2. Angular acceleration feedback

Repeatability of $\alpha_t(\omega)$: $\sim 100$ppm

Preliminary result

Phil. Trans. R. Soc. A 372, 20140031(2014)
Major problems

1. C.M. of apparatus is too high: inverted pendulum
2. Temperature fluctuation: distance of sphere
3. Effect of shelves
New apparatus

Air Bearing Turntable
Vacuum Chamber
Torsion Balance (not visible)
SS316 Spheres
ULE Shelves
Gear Bearing Turntable
Shock-proof Platform
Sphere shelves

Ultra-low expansion coefficient 0.1×10^{-6}/°C

<table>
<thead>
<tr>
<th>Distance (mm)</th>
<th>Expansion Coefficient (/°C)</th>
<th>Temperature Fluctuation (°C)</th>
<th>Distance Variation (μm)</th>
<th>Relative Uncertainty (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>0.1×10^{-6}</td>
<td>0.7</td>
<td>0.02</td>
<td>0.10</td>
</tr>
<tr>
<td>342.319</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>0.1×10^{-6}</td>
<td></td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>139.751</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ULE Shelves

Ultra-low expansion coefficient 0.1×10^{-6}/°C
Distance measurement

Coordinate Measuring Machine

Distance vs. Temperature

± 2 μm
Effect of shelves

sphere + shelves

Angular Acceleration

(920±15)ppm

shelves

Angular Acceleration

Signal
Effect of shelves

Compensate with copper and aluminum cylinders

920(15) ppm $\Rightarrow$ 4(7) ppm
Result from raw data

Amplitude: $(462.016 \pm 0.005) \text{ nrad/s}^2$

11ppm
Error budget (not complete)

Error Sources | Correction (ppm) | $\delta G/G$
--- | --- | ---
Torsion pendulum | | 2.0
Pendulum | | 2.0
Coating layer | | 0.1
Clamp and ferrule | | 0.2
Glue | | 0.2
Source masses | 6.5
Masses ($\times 4$) | | 0.2
Distance of GCs ($\times 4$) | 6.5
Relative positions | | 0.9
Residual twist angle | | 0.9
Time base | | 0.1
Magnetic damper | 455.6 | 1.6
Air buoyancy | 149.9 | 1.5
Amplitude of $\alpha(\omega)$ | | 15.2
Total | | 16.8

$G = (6.674^{**\pm0.00011}) \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$

~16.8ppm
Present results of two methods

**TOS**: $G = (6.674^{**\pm 0.00010}) \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}$

< 20 ppm

**AAF**: $G = (6.674^{**\pm 0.00011}) \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}$

< 20 ppm

“Blind” experiments

We expect to get the updated $G$ value in 2015!
Thanks for Your Attention!
Next plan

Room 2068 Experimental site

~ 80 m

Directly measure the distance of spheres in the experimental site

Room 209 Measuring room