Electron scattering from light nuclei

Ingo Sick

Interest

can calculate wave function exactly from $V_{NN}$
Faddeev, VMC, GFMC, Nocore SM, ...
(e,e) ± only probe sensitive to short range
rich set of form factors, including spin observables
→ excellent testing ground for understanding of nuclei

Questions

standard model of nuclear physics valid?
relativistic effects important?
role of mesonic degrees of freedom?
role of quark-structure of N?

Special emphasis of talk: radii

absolute radii, useful for isotope shifts from atomic physics
comparison with radii from $\mu$ atoms ($Z=1,2$)
reference for matter radii of unstable nuclei

radii ± only practical observable (scattering in inv. kin.)
parallel aspects to proton radius

presently not understood problems
Deuteron

fundamental system of nuclear physics
can be understood in terms of N+N?
rich set of observables: C0, M1, C2, M1 ΔT=1
± best neutron target
leading NR π-exchange absent (T=0)

Structure

loosely bound
for $r > 1.5 fm$ dominated by asympt. tail
only at short range interesting structure
$M=±1$: dumbbell-like
$M=0$: torus-like
Form factors

cross section complicated due to I=1 nature
3 independent form factors contribute
multipolarities C0, M1, C2

Equations in IA

\[
\frac{d\sigma(E, \theta)^{PWIA}}{d\Omega} = \sigma_{Mott}(E, \theta)[A(q) + B(q)tan(\theta/2)^2]
\]

\[A(q) = F_{C0}(q)^2 + (M_d^2 Q_d)^2 \frac{8}{9} \eta^2 F_{C2}^2(q) + \left(\frac{M_d}{M_p} \mu_d\right)^2 \frac{2}{3} \eta(1 + \eta) F_{M1}^2(q)\]

\[B(q) = \left(\frac{M_d}{M_p} \mu_d\right)^2 \frac{4}{3} \eta(1 + \eta)^2 F_{M1}^2(q) \quad \eta = q^2/(4M_d^2)\]

\[t_{20} F^2 = \frac{-1}{\sqrt{2}} \left(\frac{8}{3} \eta F_{C0} F_{C2} + \frac{8}{9} \eta^2 F_{C2}^2 + \frac{1}{3} \eta[1 + 2(1 + \eta)t g^2 \frac{\theta}{2}] F_{M1}^2\right)\]

To separate C0, C2 need polarization observables
Available data

Cross sections: many experiments, large $q, \theta$-range, very different accuracies
some 512 data points

Experimental form factors (much more sensitive than $\sigma$’s)

Usual determination

experiments measure some $\sigma, t_{2x}$
observable dominated by one of the form factors $F_i$
use other $F_j$’s from some other data to extract $F_i$ in PWIA
publish $F_i$
Non-optimal
involves inter/extrapolation of $F$’s
does not use all info on $F_i, F_j$ today available
ignores Coulomb distortion

Optimal determination
use *all* primary data $\sigma, t_{2x}, ...$
parameterize 3 $F$’s using flexible parameterization
apply Coulomb corrections
fit simultaneously to *all* data

Get
L/T-separation during fit
C0/C2-separation during fit
statistical errors (error matrix)
systematic errors (conservative estimation)
    change every data set by error
    refit
    add quadratically changes
total error: quadratic sum
Same procedure as in N-N scattering
use cross section data in (energy-dependent) phase-shift analysis
then discuss only phase shifts
see e.g. Stoks et al., PRC48(93)792

Main difference

do not ”prune” the data set
in N-N $\sim 30\%$ of data eliminated to get $\chi^2 \sim 1$
do not float normalization
largest effort of experimentalists has gone into normalization
take seriously
do not use theoretical NN potential as in phase-shift analysis
no bias from parametrization (energy dependence)

Result
form factors with reliable error bars

Note
resulting $F(q)$’s correlated over interval $\Delta q \sim 1/R_{max} \sim 0.25 \, fm^{-1}$
($R_{max} = \text{max. radius allowed for in r-space parametrization}$)
uncertainty given by $\delta F$, not by scatter of points
Results for deuteron
Find
good agreement with theory
substantial effect of MEC in C0, M1
C2 not sensitive (short-range suppressed)
C0 much more sensitive than $A(q)$
($A(q) = \text{sum of 2 terms}$)
Moments of interest

rms-radius $R$

$R^2$ defined as $\int r^4 \rho(r) 4\pi dr$

obtainable from $q=0$ slope of $G_e$: $G_e(q) = 1 - q^2 R^2 / 6 + q^4 \langle r^4 \rangle / 120 + ....$

Third Zemach moment

needed to get rms-radius from $\mu$ atoms data

$\langle r^3 \rangle_{(2)} = \int d^3r \ r^3 \rho_{(2)}(r)$ with $\rho_{(2)}(r) = \int d^3z \ \rho_{ch}(|z - r|) \ \rho_{ch}(z)$

measurable in (e,e) via

$\langle r^3 \rangle_{(2)} = \frac{48}{\pi} \int_0^\infty \frac{dq}{q^4} \left( G_e^2(q) - 1 + q^2 R^2 / 3 \right)$

First Zemach moment, needed to calculate HFS in atoms

$\langle r \rangle_{(2)} = \int d^3r \ r \int d^3r' \ \rho_{ch}(|r - r'|) \ \rho_{mag}(r')$

measurable in (e,e) via

$\langle r \rangle_{(2)} = -\frac{4}{\pi} \int_0^\infty \frac{dq}{q^2} \left( G_e(q) \ G_m(q) - 1 \right)$,
Important consideration
in which \( q \)-region are data sensitive to given moment?

Diffuse answer: ”small \( q \)”
when is \( q \)”small” enough? And not too small?

Quantitative answer: ±never studied

**Notch test:** change data in narrow interval around \( q_0 \) by 1%
refit, determine change of moment
plot this change as function of \( q_0 \)

**Example: for proton**

Note: data above \( q \sim 1.1 \text{fm}^{-1} \) not useful for \( R \)-determination
Note: peak occurs at lower \( q \) if \( R \) is bigger
Deuteron rms-radius: from slope of $A(q)$ at $q=0$?

Has been problem for long time
large scatter of results
disagreement with radius derived from n-p scattering length (Klarsfeld et al)
Part of problem: analysis of data in PWIA

Main difficulty: extremely long tail of $\rho(r)$
leads to structure of $A(q)$ at very low $q$
complicates (implicit) extrapolation to $q=0$

Demonstration: study $[\int_0^{R_{max}} \rho(r)r^4 \, dr]^{1/2}$ as function of $R_{max} \to \infty$

![Graph showing rms(r)/rms for deuteron vs. R (fm)](image1)

![Graph showing F(q) vs. q (fm⁻¹)](image2)
Consequence

- hopeless to get radius of %-type accuracy
- data at \(0.5 < q < 1\) have largest sensitivity to rms-radius
- extrapolation to \(q = 0\) dependent on model for \(A(q)\)
- strongly dependent on tail of corresponding density \(\text{FT}[A(q)]\)

Analogous to situation for proton, see PRC 89(14)012201
- for 98% of proton rms radius need \(\rho(r)\) out to \(r=3\text{fm}\)!
- effect of remaining 2% (at \(r > 3\text{fm}\)) on \(G(q)\) not measurable
- with \(q\)-space fit \((q_{\text{max}} = 2\text{fm}^{-1})\) can get rms-radii up to \(1.5\text{fm}\)

Solution for both deuteron and proton

- get away from \(q\)-space parametrization
- extrapolation to \(q=0\) too ambiguous
- use \(r\)-space parametrization
- with large-\(r\) tail constrained by physics
- helpful: fit of data up to maximal \(q\)
- such that also data constrain tail as much as possible

Shape of tail of deuteron density

- well known
- entirely given by \(\text{BE}=2.2\text{MeV}\) for \(r>1.6\text{fm}\)

Fit of data: see below
Floating vs. absolute $\sigma$

main purpose of floating: low $\chi^2$
→ taken as sign of ”successful” data fitting

Danger of floating

systematic errors increase toward edge of data set
particularly dangerous for low-q edge

consequence of sys.err. enhanced by extrapolation to q=0
this extrapolation determines overall normalization

Better: do not float, accept poorer $\chi^2$. Much safer!
Results for deuteron radius

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Value</th>
<th>Good Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron scattering</td>
<td>2.130 ±0.010 fm</td>
<td></td>
</tr>
<tr>
<td>Muonic $^2$H (prelim!)</td>
<td>2.1289±0.0012 fm</td>
<td>much better than for proton</td>
</tr>
<tr>
<td>N-p scattering length</td>
<td>2.131 fm</td>
<td></td>
</tr>
</tbody>
</table>

Tritium

(e,e) data more limited than for $^3$He

comparison to theory for $^3$He more instructive

isotope shifts for $^3$H poorly known; requires more attention!
Helium isotopes: interest
form factors
comparison to theory
isotope shifts (charge, matter)
comparison electron scattering – muonic $^4$He

$^3$He form factors
fairly compete set of $\sigma$ measured, $\sim 275$ data points
$F_{ch}$ and $F_m$ determined from global fit

quite good agreement theory-experiment
also for C0, despite importance of MEC
$^3$He rms-radius and Zemach moments

Zemach moments needed for analysis of muonic $^3$He (CREMA) recently determined (PRC90(14)064002) fit of world data including constraint on tail-shape

\[
\begin{array}{l}
\langle r \rangle \ (2) \quad 2.528 \pm 0.016 \, fm \\
\langle r^3 \rangle \ (2) \quad 28.15 \pm 0.70 \, fm^3 \\
\langle r_{ch}^2 \rangle^{1/2} \quad 1.973 \pm 0.014 \, fm \\
\langle r_m^2 \rangle^{1/2} \quad 1.976 \pm 0.047 \, fm \\
\langle r_{ch}^4 \rangle \quad 32.9 \pm 1.60 \, fm^4
\end{array}
\]

for Gauss (Exp) 26.68 (29.10) $fm^3$
$^4$He fairly complete data set, up to 8 $fm^{-1}$
recent high-q data from JLab
confirm 2. minimum
disagree somewhat with previous data
total of 192 data points
decent agreement with VMC V14 Schiavilla et al.
Moments

for rms-radius see below
Zemach moment and $\langle r^4 \rangle \rightarrow$ atomic data

Recent determination

analysis of world data
with constraint on tail of $\rho$ (as for d)
PRC 90(14)064002

Interesting question: which q-region important?

$$\langle r^3 \rangle_{(2)} = \frac{48}{\pi} \int_0^\infty \frac{dq}{q^4} \left( G_e^2(q) - 1 + q^2 R^2 / 3 \right)$$
dominated by extremely low q?

Sensitivity similar to the one of $R$

$\langle r^3 \rangle_{(2)}$ differs significantly from standard values for gaussian/exponential $\rho$’s

<table>
<thead>
<tr>
<th>$\langle r^3 \rangle_{(2)}$</th>
<th>$16.73 \pm 0.10 fm^3$</th>
<th>$16.50(17.99)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle r^2 \rangle^{1/2}$</td>
<td>$1.681 \pm 0.004 fm$</td>
<td></td>
</tr>
<tr>
<td>$\langle r^4 \rangle$</td>
<td>$14.35 \pm 0.11 fm^4$</td>
<td></td>
</tr>
</tbody>
</table>
Isotope shifts

3-4 measured via \((e,e)\n
4-6-8 measured via atomic transitions at ANL (PRL 99(2007)252501)

matter radii from scattering in inverse kinematics at GSI (EPJ A15(02)27)

3-4 shift from \((e,e)\) agrees with shifts from atomic helium (1.066, 1.028, 1.074 \(fm^2\))

cannot resolve discrepancies
RMS-radius: (e,e) $\leftrightarrow \mu X$

for $^4$He low-q data base excellent, data with small syst. errors
not only shape of large-r tail known
absolute value of density in tail also known

world data on p-$^4$He scattering + Forward Dispersion Relation analysis
yields residuum of closest singularity
this gives absolute normalization of tail density
tail steeply falling as $SE \sim 19.8$MeV
together with (e,e) produces more accurate value for rms-radius

![Graph showing $^4$He(e,e) analysis](image)

rms-radius = 1.681±0.004 fm, smallest relative error of all nuclei
relevant with regards to proton radius puzzle
Proton (charge) rms-radius
from electron scattering (world data w/o Bernauer)
radius = 0.887±0.008 \, fm
from muonic hydrogen (Pohl et al.)
radius = 0.8409±0.0004 \, fm
from electronic hydrogen 1S–nD
radius = 0.8779±0.0094 \, fm (Beyer et al.)

Unsolved problem, many speculations!

One idea: e- and \( \mu \) ”electromagnetic” interaction different
MUSE experiment at PSI
study of \( e^+ \) scattering at DESY, JLab, ...

\( e \leftrightarrow \mu \) for helium

relative error of \(^4\text{He} \) radius from (e,e) 4 times smaller than for proton

find agreement between (e,e) and \( \mu X \): 1.681±0.004 \( \leftrightarrow \) 1.679±0.001 \, fm!

(value from \( \mu X \) still preliminary; A. Antognini, CREMA collaboration)

good agreement only deepens puzzle
Lithium

$^6$Li and $^7$Li accessible to electron scattering
shifts of $A=8, 9, 11$ measured by laser spectroscopy (Nörtershäuser et al.)
matter radii for $A=6, 8, 9, 11$ from proton scattering (Dobrovolsky et al.)
in inverse kinematics
pronounced $A$-dependent shape changes (clustering)
interesting comparison to ab-initio calculations

Electron scattering

$^7$Li in past standard reference for rms-radius
not a good idea
data for $^6$Li more extensive ($86 \sigma$), more precise
$^7$Li experiments did not resolve 1. excited state
quadrupole contribution in $^7$ Li much more important, cannot be separated

Analysis of world data for $^6$Li (PRC84(11)024307)
use tail constraint as well
complication: p-tail or d-tail? (cluster structure of $^6$Li)
$SE_p=4.6$MeV, $SE_d=1.5$MeV
as GFMC calculation (Pieper et al.) gives correct BE: use GFMC

Result

charge rms-radius $= 2.589 \pm 0.039 \, fm$
comparatively large uncertainty due to poor low-q data
Theoretical understanding
GFMC calculation
V18+Urbana 3BF
Wiringa et al.
MEC included, for C0 small

C0 well understood
M1 problematic
Isotope shifts measured by laser spectroscopy with stored ions
Nörtershäuser et al.
$^{11}\text{Li} = \text{Borromean nucleus (}2n, ^{10}\text{Li unbound)}$
$2n$-separation energy only 369KeV
Extreme case of tail-importance: matter radii
Not emphasized: magnetic form factors + radii
  data in general not as good
  understanding more involved (MEC)
rms-radii even more difficult to measure
  at low q σ is dominated by $F_{ch}$
polarization transfer useful only for p
  best results from (old) 180° facilities
small contribution from $F_m$ enhances effect of systematic errors
  example: proton
information from HFS limited

Heavier p-shell nuclei
  complication: spin=$3/2$ ($^9$Be, $^{11}$B), =3 ($^{10}$B)
little accurate data available
could do accurate experiment on $^9$Be
  despite loss of knowhow
as accuracy for $^{12}$C excellent, could do Be/C ratio measurement
  produce precise reference radius for isotope shift data

for review: see I.Sick, Prog. Part. Nucl. Phys. 47 (01) 245