Muonic hydrogen and the Proton Radius Puzzle

Nuclear structure from Laser spectroscopy of light muonic atoms

Randolf Pohl
for the CREMA collaboration
CREMA collaboration
Charge Radius Experiment with Muonic Atoms


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The proton rms charge radius measured with electrons: $0.8770 \pm 0.0045$ fm
muons: $0.8409 \pm 0.0004$ fm

7.9 $\sigma$
Outline

- **Muonic hydrogen**
  - 2 resonances measured in 2009
  - #1: Pohl et al., Nature (2010) $\rightarrow$ proton charge radius
  - #2: Antognini et al., Science (2013) $\rightarrow$ Zemach radius

- **Muonic deuterium**
  - 3 resonances measured in 2009. Analysis (nearly) done
  - Theory in $\mu d$
  - Results (still somewhat prel.)

- **Muonic helium-4 and helium-3**
  - 5 transitions measured in 2013/4:
    - Both transitions measured in in $(\mu^4\text{He})^+$ $\rightarrow$ Lamb shift and Fine structure
    - 3 transitions measured in in $(\mu^3\text{He})^+$ $\rightarrow$ Lamb shift, 2S-HFS, 2P-FS/HFS
The proton rms charge radius is not the most accurate quantity in the universe.

- **e-p scattering**:
  \[ r_p = 0.895(18) \text{ fm} \quad (\sigma_r = 2\%) \]

- **Hydrogen**:
  \[ r_p = 0.8760(78) \text{ fm} \quad (\sigma_r = 0.9\%) \]
The proton rms charge radius is not the most accurate quantity in the universe.

Electron scattering:

$$\langle r_p^2 \rangle = -6h^2 \frac{dG_E(Q^2)}{dQ^2} \bigg|_{Q^2=0} \Rightarrow \text{slope of } G_E \text{ at } Q^2 = 0$$

- electron scattering
- slope of $G_E$ at $Q^2 = 0$
- hydrogen spectr.
- Lamb shift (S-states)
The proton rms charge radius is not the most accurate quantity in the universe.

Hydrogen spectroscopy (Lamb shift):

\[ L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \text{ MHz} \]

\[ E_{nS} \simeq -\frac{R_{\infty}}{n^2} + \frac{L_{1S}}{n^3} \]

2 unknowns \( \Rightarrow \) 2 transitions

- Rydberg constant \( R_{\infty} \)
- Lamb shift \( L_{1S} \leftarrow r_p \)
Atomic physics

Wave functions of S and P states:

S states: max. at \( r=0 \)
- Electron sometimes inside the proton.
- S states are shifted.
- Shift is proportional to the size of the proton

P states: zero at \( r=0 \)
- Electron is not inside the proton.

Orbital pictures from Wikipedia
Atomic physics

S states: max. at r=0

Electron sometimes inside the proton.

S states are shifted.

Shift ist proportional to the size of the proton

P states: zero at r=0

Electron is not inside the proton.
Atomic and nuclear physics

S states: max. at r=0

Electron sometimes **inside** the proton.

S states are shifted.

Shift ist proportional to the **size of the proton**

P states: zero at r=0

Electron is **not** inside the proton.

Coulomb potential: \( V = \frac{1}{r} \)

radius [fm] 0 0.5 1 1.5 2 2.5

proton charge

Orbital pictures from Wikipedia
Atomic and nuclear physics

S states: max. at r=0

Electron sometimes inside the proton.

S states are shifted.

Shift ist proportional to the size of the proton

P states: zero at r=0

Electron is not inside the proton.

8S  ⎛ ⎞
4S  ⎜ ⎟
3S  ⎝ ⎠

2S ⎛ ⎞
2P ⎜ ⎟
3D ⎝ ⎠

radius [fm]

coulomb potential: V = 1/r

true potential

arb. units

proton charge

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Eltville, 2 Feb. 2015
Muonic hydrogen

Regular hydrogen:

electron $e^-$ + proton $p$

Muonic hydrogen:

muon $\mu^-$ + proton $p$

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Muonic hydrogen

Regular hydrogen:

\[ \text{electron } e^- + \text{ proton } p \]

Muonic hydrogen:

\[ \text{muon } \mu^- + \text{ proton } p \]

from Wikipedia
Muonic hydrogen

Regular hydrogen:

$e^- + \text{proton } p$

Muonic hydrogen:

$\mu^- + \text{proton } p$

Muon mass $m_\mu \approx 200 \times m_e$

Bohr radius $r_\mu \approx 1/200 \times r_e$

$\mu$ inside the proton: $200^3 \approx 10^7$

Muon much is more sensitive to $r_p$
Proton charge radius and muonic hydrogen

Lamb shift in $\mu p$ [meV]:

$$\Delta E = 206.0668(25) - 5.2275(10) r_p^2 \ [\text{meV}]$$

Proton size effect is 2% of the $\mu p$ Lamb shift

Measure to $10^{-5} \Rightarrow r_p$ to 0.05%

Experiment:

Theory summary:
$\mu^-$ stop in $H_2$ gas
⇒ $\mu p^*$ atoms formed ($n \sim 14$)

99%: cascade to $\mu p(1S)$, emitting prompt $K_\alpha$, $K_\beta$ ...

1%: long-lived $\mu p(2S)$ atoms
lifetime $\tau_{2S} \approx 1\mu s$ at 1 mbar $H_2$

fire laser ($\lambda \approx 6\mu m$, $\Delta E \approx 0.2$ eV)
⇒ induce $\mu p(2S) \rightarrow \mu p(2P)$
⇒ observe delayed $K_\alpha$ x-rays
⇒ normalize delayed $K_\alpha$ x-rays

**μp Lamb shift experiment: Principle**

Time spectrum of 2 keV x-rays (~13 hours of data @ 1 laser wavelength)
μp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays

"prompt" \( (t \sim 0) \)

- 99 %
- 1 %

\( n \sim 14 \)

1 S → 2 S → 2 P

2 keV
μp Lamb shift experiment: Principle

Time spectrum of 2 keV x-rays

“prompt” ($t \sim 0$)  “delayed” ($t \sim 1 \mu s$)

$1 S \rightarrow 2 S$  99% 

$2 S \rightarrow 2 P$  1% 

6 events per hour
µp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays

---

"prompt" (t ∼ 0)

```
1 S
2 S
2 P
```

99 %

1 %

"delayed" (t ∼ 1 µs)

```
Laser
2 P
2 S
```

 Normalize \( \frac{\text{delayed } K\alpha}{\text{prompt } K\alpha} \) \Rightarrow Resonance

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Swiss muons
Swiss muons
Swiss muons
Setup

- H2 target
- 5T solenoid
- Momentum filter
- Cyclotron trap
- Pion beam line
- Protons
- C-target
- Water vapor cell
- Raman cell
- Ti:Sa amplifier
- SHG
- Diode laser
- Disk-laser
- Ti:Sa cw laser
- FP cavity
- Ti:Sa oscillator
- SHG
Play movie: “Muon beam”
Muon beam line
The laser system

Main components:

- Thin-disk laser
  - fast response to detected $\mu^-$
- Frequency doubling
- TiSa laser:
  - frequency stabilized cw laser
  - injection seeded oscillator
  - multipass amplifier
- Raman cell
  - 3 Stokes: $708\,\text{nm} \rightarrow 6\,\mu\text{m}$
  - $\lambda$ calibration @ $6\,\mu\text{m}$
- Target cavity

The laser system

Yb:YAG thin–disk laser

- Oscillator 1030 nm
- Amplifier
- SHG
- 9 mJ
- 43 mJ
- 200 W
- 500 W

Thin-disk laser
- Large pulse energy: 85 (160) mJ
- Short trigger-to-pulse delay: \( \lesssim 400 \text{ ns} \)
- Random trigger
- Pulse-to-pulse delays down to 2 ms (rep. rate \( \gtrsim 500 \text{ Hz} \))

- Each single \( \mu^- \) triggers the laser system
- 2S lifetime \( \approx 1 \mu s \) \( \rightarrow \) short laser delay

cw TiSa laser

- Wave meter
- I\(_2\) / Cs
- cw TiSa 708 nm 400 mW
- 5 W

- Oscillator
- Amplifier
- SHG
- 1.5 mJ
- 7 mJ
- 23 mJ
- 515 nm
- 23 mJ

6 \( \mu m \) monitoring

- \( 20 \text{ m} \)
- \( 0.25 \text{ mJ} \)
- \( H_2O \)

Raman cell

- \( 6 \mu m \) cavity


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13
The laser system

Yb:YAG thin-disk laser

Oscillator 1030 nm
Amplifier
SHG

Oscillator 1030 nm
Amplifier
SHG

SHG

SHG

708 nm, 15 mJ

MOPA TiSa laser:
cw laser, frequency stabilized
- referenced to a stable FP cavity
- FP cavity calibrated with $I_2$, Rb, Cs lines

$$\nu_{FP} = N \cdot FSR$$

$$FSR = 1497.344(6) \text{ MHz}$$

$$\nu_{TiSa}^{cw}$$ absolutely known to 30 MHz

$$\Gamma_{2P-2S} = 18.6 \text{ GHz}$$

Seeded oscillator

$$\rightarrow \nu_{TiSa}^{pulsed} = \nu_{TiSa}^{cw}$$

(frequency chirp $\leq 200$ MHz)

Multipass amplifier (2f- configuration)

gain=10

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The laser system

Yb:YAG thin-disk laser

- Oscillator 1030 nm
- Amplifier
- SHG
- TiSa Amp.
- 200 W
- 9 mJ
- 43 mJ
- 515 nm
- 23 mJ
- 708 nm, 15 mJ
- 6 μm monitoring
- H₂O
- 0.25 mJ
- 20 m
- Ge-filter
- 6 μm cavity

Oscillator 1030 nm

Amplifier

SHG

SHG

Raman cell:

- 708 nm
- H₂
- 6.02 μm
- 6.02 μm
- 708 nm
- 4155 cm⁻¹
- ν=0
- ν=1
- ν=2
- ν=3
- 1.00 μm
- 1.72 μm
- 6.02 μm
- 1st Stokes
- 2nd Stokes
- 3rd Stokes

cw TiSa laser

- Wave meter
- Verdi
- 23 mJ
- 5 W
- 7 mJ
- TiSa Osc.
- 708 nm, 15 mJ
- 6 μm cavity

Verdi

FP

I₂ / Cs

SHG

SHG

Raman cell:

- ν=0
- ν=1
- ν=2
- ν=3
- 1.00 μm
- 1.72 μm
- 6.02 μm

P. Rabinowitz et. al., IEEE J. QE 22, 797 (1986)
The laser system

Yb:YAG thin−disk laser

Oscillator 1030 nm
Amplifier
SHG
200 W
500 W
43 mJ
9 mJ
9 mJ

Oscillator 1030 nm
Amplifier
SHG
200 W
500 W
43 mJ
9 mJ
9 mJ

cw TiSa laser

Wave meter
Verdi
[FP]
I₂ / Cs

43 mJ
5 W

Laser pulse

200 W
500 W

Oscillator
Amplifier
708 nm
SHG
7 mJ
400 mW

708 nm, 15 mJ

1.5 mJ
7 mJ

Design: insensitive to misalignment
Transverse illumination
Large volume

Dielectric coating with $R \geq 99.9\%$ (at 6 μm)

$\rightarrow$ Light makes 1000 reflections
$\rightarrow$ Light is confined for $\tau=50$ ns
$\rightarrow$ 0.15 mJ saturates the $2S − 2P$ transition

The laser system

Yb:YAG thin–disk laser

- Oscillator 1030 nm
- Amplifier
- SHG

- Oscillator 1030 nm
- Amplifier
- SHG

- cw TiSa laser
- Wave meter
- Verdi
- (FP)
- I₂ / Cs
- cw TiSa 708 nm
- 400 mW

- 200 W
- 500 W
- 9 mJ
- SHG
- 43 mJ
- SHG
- 23 mJ
- 708 nm, 15 mJ
- 515 nm
- 23 mJ

- SHG

- 1630 1640 1650 1660 1670 1680 1690 1700
- 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7

- 1200 1300 1400 1500 1600 1700 1800 1900 2000 2100
- 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7

Water absorption

- Water absorption

- Vacuum tube for 6 µm beam transport.
- Direct frequency calibration at 6 µm.
Target, cavity and detectors
The resonance: discrepancy, sys., stat.

Water-line/laser wavelength: 300 MHz uncertainty

Δν water-line to resonance: 200 kHz uncertainty

Discrepancy:

5.0σ ↔ 75 GHz ↔ δν/ν = 1.5 × 10^{-3}

The proton radius puzzle

The proton rms charge radius measured with electrons: $0.8770 \pm 0.0045$ fm
muons: $0.8409 \pm 0.0004$ fm


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Eltville, 2 Feb. 2015
The proton rms charge radius measured with electrons: $0.8770 \pm 0.0045$ fm

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The proton radius puzzle

The proton rms charge radius measured with

electrons: $0.8770 \pm 0.0045$ fm

muons: $0.8409 \pm 0.0004$ fm

$7.9 \sigma$

The proton radius puzzle

The proton rms charge radius measured with electrons: 0.8770 ± 0.0045 fm
muons: 0.8409 ± 0.0004 fm

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Muons in the news
Muons in the news
Muons in the news

J. Bernauer, RP

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What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}} (r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

- \( \mu p \) theory wrong?
- \( \mu p \) experiment wrong?
- \( H \) theory wrong?
- \( H \) experiments wrong? \( \rightarrow R_\infty \) wrong?
- AND e-p scattering exp. wrong?

Standard Model wrong?!?

What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}} (r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

\( \mu p \) experiment wrong?

- Frequency mistake by 75 GHz (\( \Leftrightarrow 0.15\% \))?

  That is \( > 100 \delta(\mu p) \) !

  \( \sigma_{\text{tot}} = 650 \text{ MHz} \), \( \Gamma = 19 \text{ GHz} \)

- 4 line widths !

- 2 resonances in \( \mu p \) give the same \( r_p \)

---

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What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}} (r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

- Frequency mistake by **75 GHz** (⇔ **0.15%**)?
- Wrong transition?
- FS, HFS huge.
- Next transition: \( \sim 1 \text{ THz away.} \)

---

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What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}} \left( r_p^{\text{CODATA}} \right) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

\( \mu p \) experiment wrong?

- Frequency mistake by 75 GHz (\( \Leftrightarrow 0.15\% \))?
- Wrong transition?
- Systematic error?

\begin{align*}
\text{Laser frequency (H}_2\text{O calibration)} & \quad 300 \text{ MHz} \\
\text{intrinsic H}_2\text{O uncertainty} & \quad 2 \text{ MHz} \\
\text{AC and DC stark shift} & \quad < 1 \text{ MHz} \\
\text{Zeeman shift (5 Tesla)} & \quad < 30 \text{ MHz} \\
\text{Doppler shift} & \quad < 1 \text{ MHz} \\
\text{Collisional shift} & \quad 2 \text{ MHz} \\
\end{align*}

\[ 300 \text{ MHz} \]

\( \mu p \) atom is small and not easily perturbed by external fields.
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}} (r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

- Frequency mistake by 75 GHz (\(\Leftrightarrow\) 0.15%)?
- Wrong transition?
- Systematic error?
- Molecular effects?


Experimentally:
- only 1 line observed (> 80% population)
- expected width
- \( pp\mu \) ion short-lived R. Pohl et al., PRL 97, 193402 (2006).
What may be wrong?

\[ \tilde{L}_{\mu P}^{\text{theo.}} (r_p^{\text{CODATA}}) - \tilde{L}_{\mu P}^{\exp.} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

- Frequency mistake by 75 GHz (\(\Leftrightarrow 0.15\%\))?
- Wrong transition?
- Systematic error?
- Molecular effects?
- Gas impurities?  
  
  M. Diepold, RP et al., PRA 88, 042520 (2013).
  
  Target gas contained 0.55(5) % air (leak).
  
  Back-of-the-envelope calculation:
  
  \[
  \lambda \approx 6 \cdot 10^3 \text{s}^{-1}
  \]
  
  \[
  \tau(2S) = 1 \mu\text{s}
  \]
  
  \(\Rightarrow\) Less than 1% of all \(\mu P(2S)\) atoms see any \(N_2\)
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}} \left( r_p^{\text{CODATA}} \right) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

- Frequency mistake by 75 GHz (⇔ 0.15%)?
- Wrong transition?
- Systematic error?
- Molecular effects?
- Gas impurities?

\( \mu p \) experiment probably not wrong by 100 \( \sigma \)
What may be wrong?

\[ \tilde{\mathcal{L}}_{\text{theo.}}(r_p^{\text{CODATA}}) - \tilde{\mathcal{L}}_{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

\[ \mu_p \text{ theory wrong?} \]

Discrepancy = 0.31 meV
Theory uncert. = 0.0025 meV
⇒ 120δ(theory) deviation

Some contributions to the \( \mu_p \) Lamb shift

\[ \Delta E = 206.0668(25) - 5.2275(10) r_p^2 \] [meV]

double-checked by many groups

5th largest term!

Theory summary:
A. Antognini, RP et al.
Annals of Physics 331, 127 (2013)
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}}(r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

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Discrepancy \( = 0.31 \text{ meV} \)
Theory uncert. \( = 0.0025 \text{ meV} \)
\[ \Rightarrow 120 \delta (\text{theory}) \text{ deviation} \]

double-checked by many groups

5th largest term!

Some contributions to the \( \mu p \) Lamb shift

\[ \Delta E = 206.0668(25) - 5.2275(10) r_p^2 \text{ [meV]} \]

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\[ \Delta E = 206.0668(25) - 5.2275(10) r_p^2 \] [meV]

\( \mu p \) theory probably not wrong by 100 \( \sigma \)
### Table 1

All known radius-independent contributions to the Lamb shift in $\mu p$ from different authors, and the one we selected. Values are in meV. The entry # in the first column refers to Table 1 in Ref. [13]. The “finite-size to relativistic recoil correction” (entry #18 in [13]), which depends on the proton structure, has been shifted to Table 2, together with the small terms #26 and #27, and the proton polarizability term #25. SE: self-energy, VP: vacuum polarization, LBL: light-by-light scattering, Rel: relativistic, NR: non-relativistic, RC: recoil correction.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>NR one-loop electron VP (eVP)</td>
<td>205.0074</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td>Rel. corr. (Breit–Pauli)</td>
<td>0.0169</td>
<td></td>
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<td>3</td>
<td>Rel. one-loop eVP</td>
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<tr>
<td>19</td>
<td>Rel. RC to eVP, $\alpha(Z\alpha)^4$ (incl. in #2)</td>
<td>−0.0041</td>
<td>−0.0041</td>
<td>205.0282</td>
<td>205.0282</td>
<td>−0.00208c</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Two-loop eVP (Källén–Sabry)</td>
<td>1.5079</td>
<td>1.5081</td>
<td>1.5081</td>
<td>1.50810</td>
<td>1.50810</td>
<td>[80]</td>
</tr>
<tr>
<td>5</td>
<td>One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^5$</td>
<td>0.1509</td>
<td>0.1509</td>
<td>0.1507</td>
<td>0.15102</td>
<td>0.15102</td>
<td>[80]</td>
</tr>
<tr>
<td>7</td>
<td>eVP corr. to Källén–Sabry</td>
<td>0.0023</td>
<td>0.00223</td>
<td>0.00223</td>
<td>0.00215</td>
<td>0.00215</td>
<td>[80]</td>
</tr>
<tr>
<td>6</td>
<td>NR three-loop eVP</td>
<td>0.0053</td>
<td>0.00529</td>
<td>0.00529</td>
<td>0.00529</td>
<td>0.00529</td>
<td>[87]</td>
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<tr>
<td>9</td>
<td>Wichmann–Kroll, “1:3” LBL</td>
<td>−0.00103</td>
<td>−0.00102</td>
<td>−0.00102</td>
<td>−0.00102</td>
<td>−0.00102</td>
<td>[84]</td>
</tr>
<tr>
<td>10</td>
<td>Virtual Delbrück, “2:2” LBL</td>
<td>0.00135</td>
<td>0.00115</td>
<td></td>
<td>0.00115</td>
<td>0.00115</td>
<td>[84]</td>
</tr>
<tr>
<td>New</td>
<td>“3:1” LBL</td>
<td>−0.00102</td>
<td></td>
<td>−0.00102</td>
<td>−0.00102</td>
<td>−0.00102</td>
<td>[89]</td>
</tr>
<tr>
<td>20</td>
<td>$\mu$SE and $\mu$VP</td>
<td>−0.6677</td>
<td>−0.66770</td>
<td>−0.66788</td>
<td>−0.66761</td>
<td>−0.66761</td>
<td>[80]</td>
</tr>
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<td>11</td>
<td>Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$</td>
<td>−0.005(1)</td>
<td>−0.00500</td>
<td>−0.004924d</td>
<td>−0.00254d</td>
<td>−0.00254</td>
<td></td>
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<tr>
<td>12</td>
<td>eVP loop in self-energy $\alpha^2(Z\alpha)^4$</td>
<td>−0.001</td>
<td>−0.00150</td>
<td>−0.00171f</td>
<td>−0.00171</td>
<td>−0.00171</td>
<td>[86]</td>
</tr>
<tr>
<td>21</td>
<td>Higher order corr. to $\mu$SE and $\mu$VP</td>
<td>−0.00169</td>
<td>−0.00171g</td>
<td></td>
<td>−0.00007</td>
<td>−0.00007</td>
<td>[74]</td>
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<td>13</td>
<td>Mixed eVP + $\mu$VP</td>
<td>0.00007</td>
<td>0.00007</td>
<td>0.00005</td>
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<td>eVP and $\mu$VP in two Coulomb lines</td>
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<td>0.00005</td>
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<tr>
<td>14</td>
<td>Hadronic VP $\alpha(Z\alpha)^4m_r$</td>
<td>0.0113(3)</td>
<td>0.01077(38)</td>
<td>0.011(1)</td>
<td>0.01121(44)</td>
<td>0.01121(44)</td>
<td>[93–95]</td>
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<tr>
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<td>Hadronic VP $\alpha(Z\alpha)^3m_r$</td>
<td>0.000047</td>
<td>0.000047</td>
<td>0.000047</td>
<td>0.000047</td>
<td>0.000047</td>
<td>[94,95]</td>
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<td>Rad corr. to hadronic VP</td>
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<td>−0.000015</td>
<td>−0.000015</td>
<td>−0.000015</td>
<td>[94,95]</td>
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<td>Recoil corr.</td>
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<td>0.05750</td>
<td>0.0575</td>
<td>0.05747</td>
<td>0.05747</td>
<td>[80]</td>
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<td>22</td>
<td>Rel. RC $(Z\alpha)^5$</td>
<td>−0.045</td>
<td>−0.04497</td>
<td>−0.04497</td>
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<td>Rel. RC $(Z\alpha)^6$</td>
<td>0.0003</td>
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<td>0.0002475</td>
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(continued on next page)
### Table 1 (continued)

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<th>#</th>
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<th>Nature</th>
<th>Borie-v6</th>
<th>Indelicato</th>
<th>Our choice</th>
<th>Ref.</th>
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<td>Rad. (only eVP) RC α(Zα)⁵</td>
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<td>[85] Eq. (64a)</td>
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<td>Rad. RC α(Zα)ⁿ (proton SE)</td>
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<td>−0.00960</td>
<td>−0.0100</td>
<td>−0.01080(100)</td>
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<td></td>
<td>Sum</td>
<td>206.0312</td>
<td>206.02915</td>
<td>206.02862</td>
<td>206.03399(109)</td>
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</table>

| a  | This value has been recalculated to be 0.018759 meV [77]. |
| b  | This correction is not necessary here because in #2 the Breit–Pauli contribution has been calculated using a Coulomb potential modified by eVP. |
| c  | Difference between Eqs. (6) and (4) in [78]: $E_{\text{rel}}^{(\text{2P}_1/2-2S_1/2)} - E_{\text{rel}}^{(\text{2P}_1/2-2S_1/2)} = 0.018759 - 0.020843 = -0.002084 \text{ meV}$ (see also Table IV). Using these corrected values, the various approaches are consistent. Pachucki becomes 205.0074 + 0.018759 = 205.0262 meV and Borie 205.0282 − 0.002084 = 205.0261 meV. |
| d  | In Appendix C, incomplete. |
| e  | Eq. (27) in [85] includes contributions beyond the logarithmic term with modification of the Bethe logarithm to the Uehling potential. The factor 10/9 should be replaced by 5/6. |
| f  | This term is part of #22, see Fig. 22 in [86]. |
| g  | Borie includes wave-function corrections calculated in [87]. The actual difference between Ref. [13] and Borie-v6 [79] is given by the inclusion of the Källén–Sabry correction with muon loop. |
| h  | This was calculated in the framework of NRQED. It is related to the definition of the proton radius. |

---

80 P. Indelicato, arXiv:1210.5828v2 [PRA 87, 022501 (2013)]
89 S.G. Karshenboim, E.Y. Korzinin, V.G. Ivanov, V.A. Shelyuto, JETP Lett. 92, 8 (2010)
92 A. Petermann, Phys. Rev. 105, 1931 (1957)
# Lamb shift in $\mu p$ 2: $r_p$-dependent

Table 2
Proton-structure-dependent contributions to the Lamb shift in $\mu p$ from different authors and the one we selected. Values are in meV, $\langle r^2 \rangle$ in fm$^2$. The entry # in the first column refers to Table 1 in Ref. [13] supplementary information [9]. Entry # 18 is under debate. TPE: two-photon exchange, VP: vacuum polarization, SE: self-energy, Rel: relativistic.

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<td>-5.1975 $\langle r^2 \rangle$</td>
<td>-5.1975 $\langle r^2 \rangle$</td>
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<td>-0.0018 $\langle r^2 \rangle$</td>
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<td>-5.1994 $\langle r^2 \rangle$</td>
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<td>-5.1994 $\langle r^2 \rangle$</td>
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<td>-5.2000 $\langle r^2 \rangle$</td>
<td>-5.2001 $\langle r^2 \rangle$</td>
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<td>-5.2000 $\langle r^2 \rangle$</td>
<td>-5.2001 $\langle r^2 \rangle$</td>
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<td>Finite size corr. to one-loop eVP</td>
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<td>-0.0110 $\langle r^2 \rangle$</td>
<td>-0.010 $\langle r^2 \rangle$</td>
<td>-0.0282 $\langle r^2 \rangle$</td>
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<td>Finite size corr. to one-loop eVP-it.</td>
<td>-0.0165 $\langle r^2 \rangle$</td>
<td>-0.0170 $\langle r^2 \rangle$</td>
<td>-0.017 $\langle r^2 \rangle$</td>
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<td>Finite-size corr. to Källén–Sabry</td>
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<td>Finite size corr. to $\mu$ self-energy</td>
<td>(0.00699)$^c$</td>
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<td>New</td>
<td>$\Delta E_{\text{TPE}}$ [46]</td>
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<td>Elastic (third Zemach)$^e$</td>
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<td>Measured $R_{(2)}^3$</td>
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<td>Yukawa</td>
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<td>25</td>
<td>Inelastic (polarizability)</td>
<td>0.0129(5) meV [101]</td>
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<td>0.012(2) meV</td>
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<tr>
<td>New</td>
<td>Rad. corr. to TPE</td>
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<td>26</td>
<td>eVP corr. to polarizability</td>
<td>-0.00062 $\langle r^2 \rangle$</td>
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### Table 2 (continued)

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<td>27</td>
<td>SE corr. to polarizability</td>
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<td>−0.00001 meV [95]</td>
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<tr>
<td>18</td>
<td>Finite-size to rel. recoil corr.</td>
<td>(0.013 meV)(^g) (^h)</td>
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<td>Higher order finite-size corr.</td>
<td>−0.000123 meV</td>
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<td>0.00001(10) meV</td>
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<td>0.00001(10) meV</td>
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<td>2P(_{1/2}) finite-size corr.</td>
<td>−0.0000519((r^2))(^i)</td>
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<td></td>
<td>(incl. above)</td>
<td>(incl. above)</td>
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\(^a\) Corresponds to Eq. (6) in [11] which accounts only for the main terms in \(F_{\text{REL}}\) and \(F_{\text{NREL}}\).

\(^b\) This contribution has been accounted already in both the \(-0.0110\) meV/fm\(^2\) and \(-0.0165\) meV/fm\(^2\) coefficients.

\(^c\) Given only in Appendix C. Bethe logarithm is not included.

\(^d\) This uncertainty accounts for the difference between all-order in \(Z\alpha\) and perturbative approaches [82].

\(^e\) Corresponds to Eq. (20).

\(^f\) This value is slightly different from Eq. (22) because here an all-order in finite-size and an all-order in eVP approaches were used.

\(^g\) See Appendix F of [96]. This term is under debate.

\(^h\) Included in \(\Delta E_{\text{TPE}}\). This correction of 0.018 − 0.021 = −0.003 meV is given by Eq. (64) in [10] and Eq. (25) in [11]. This correction is also discussed in [76] where the 6/7 factor results from 0.018/0.021.

\(^i\) Eq. (6a) in [79].

---

82  P. Indelicato, P.J. Mohr, 2012 (in preparation)
96  J.L. Friar, Ann. Phys. 122, 151 (1979)
# HFS in $\mu p$

Table 3

All known contributions to the 2S-HFS in $\mu p$ from different authors and the one we selected. Values are in meV, radii in fm. SE: self-energy, VP: vacuum polarization, Rel: relativistic, RC: recoil correction, PT: perturbation theory, p: proton, int: interaction, AMM: anomalous magnetic moment.

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<td>h1 Fermi energy, $(Z\alpha)^4$</td>
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<td>22.8054</td>
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<td>h2 Breit corr., $(Z\alpha)^6$</td>
<td>0.0026</td>
<td>0.00258</td>
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<td>h3 Dirac energy (+ Breit corr. in all-order)</td>
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<td></td>
<td>22.807995</td>
<td>22.807995</td>
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<td>h4 $\mu$ AMM corr., $\alpha(Z\alpha)^4$, $\alpha(Z\alpha)^4$</td>
<td>0.0266</td>
<td>0.02659</td>
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<td>h5 eVP in 2nd-order PT, $\alpha(Z\alpha)^5 (\epsilon_{\text{VP2}})$</td>
<td>0.0746</td>
<td>0.07443</td>
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<td>0.07437</td>
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<td>h6 All-order eVP corr.</td>
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<td>0.07437</td>
<td>0.07437</td>
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<td>h8 One-loop eVP in $1\gamma$ int., $\alpha(Z\alpha)^4 (\epsilon_{\text{VP1}})$</td>
<td>0.0482</td>
<td>0.04818</td>
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<td>0.04818</td>
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<td>h9 Two-loop eVP in $1\gamma$ int., $\alpha^2(Z\alpha)^4 (\epsilon_{\text{VP1}})$</td>
<td>0.0003</td>
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<td>h10 Further two-loop eVP corr.</td>
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<td>0.00037</td>
<td>0.00037</td>
<td>[113,114]</td>
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<td>h11 $\mu$VP (similar to $\epsilon_{\text{VP2}}$)</td>
<td>0.00091</td>
<td>(incl. in h13)</td>
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<td>0.00091</td>
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<td>h12 $\mu$VP (similar to $\epsilon_{\text{VP1}}$)</td>
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<td>0.00091</td>
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<td>h13 Vertex, $\alpha(Z\alpha)^5$</td>
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<td>$-0.00311$</td>
<td>$-0.00311$</td>
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<td>h14 Higher order corr. of (h13), (part with $\ln(\alpha)$)</td>
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<td>h15 $\mu$ SE with p structure, $\alpha(Z\alpha)^5$</td>
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<td>h16 Vertex corr. with proton structure, $\alpha(Z\alpha)^5$</td>
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<td>h17 &quot;Jellyfish&quot; corr. with p structure, $\alpha(Z\alpha)^5$</td>
<td>0.0005</td>
<td></td>
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<td>h18 Hadron VP, $\alpha^6$</td>
<td>0.0005(1)</td>
<td>0.00060(10)</td>
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<td>0.00060(10)</td>
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<td>h19 Weak interaction contribution</td>
<td>0.0003</td>
<td>0.00027</td>
<td></td>
<td>0.00027</td>
<td>[116]</td>
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<td>h20 Finite-size (Zemach) corr. to $\Delta E_{\text{Fermi}}$, $(Z\alpha)^5$</td>
<td>$-0.1518^b$</td>
<td>$-0.16037 r_Z$</td>
<td>$-0.16034 r_Z$</td>
<td>$-0.16034 r_Z$</td>
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<td>Higher order finite-size corr. to $\Delta E_{\text{Fermi}}$</td>
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<td>$-0.0022 r_E^2 + 0.0009$</td>
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<td>h22</td>
<td></td>
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<tr>
<td>Proton polarizability, $(Z\alpha)^5$, $\Delta E_{\text{pol}}^{\text{HFS}}$</td>
<td>0.0105(18)</td>
<td>0.0080(26)</td>
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<td>$0.00801(260)$</td>
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<td>h23</td>
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<td>(incl. in h20)</td>
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<td>[112]</td>
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<td>h24</td>
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<td>$-0.0018 r_Z - 0.00114 - 0.00114(20)$</td>
<td>Eq. (109) in [80]</td>
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<td>eVP corr. to finite-size (similar to $\epsilon_{\text{VP1}}$)</td>
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<td>Eq. (109) in [80]</td>
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<td>Sum</td>
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<td>22.8148(20)$^c$</td>
<td>22.9839(26)</td>
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<td>22.9858(26) − 0.1621(10) $r_Z - 0.0022(5) r_E^2$</td>
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<tr>
<td>Sum with $r_E = 0.841$ fm, $r_Z = 1.045$ fm [28]</td>
<td>22.8148 meV</td>
<td>22.8163 meV</td>
<td>22.8149 meV</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Includes a correction $(Z\alpha)^5$ due to $\mu$VP.
$^b$ Calculated using the Simon et al. form factor.
$^c$ The uncertainty is 0.0078 meV if the uncertainty of the Zemach term (h20) is included (see Table II of [72]).

80 P. Indelicato, arXiv:1210.5828v2 [PRA 87, 022501 (2013)]

A. Antognini, RP et al., Ann. Phys. 331, 127 (2013), Tab 3

Randolf Pohl
Eltville, 2 Feb. 2015
25
Theory in \( \mu p \): new papers

Papers that have appeared after we wrote A. Antognini, RP et al., Ann. Phys. 331, 127 (2013):

- Mohr, Griffith, Sapirstein, PRA 87, 052511 (2013), [1304.2076], “Bound-state field-theory approach to proton-structure effects in muonic hydrogen”

- Korzinin, Ivanov Karshenboim, PRD 88, 125019 (2013) [1311.5784]: “The \( \alpha^2(Z\alpha)^4m \) contributions to the Lamb shift and fine structure in light muonic atoms”

- Karshenboim, Ivanov, Korzinin, PRA 89, 022102 (2014) [1311.5789]: “Relativistic recoil effects for energy levels in a muonic atom within a Grotch-type approach. I. General approach”

- Ivanov, Korzinin, Karshenboim, PRA 90, 022103 (2014) [1311.5790]: “Relativistic recoil effects for energy levels in a muonic atom within a Grotch-type approach. II. An application to the one-loop electronic vacuum polarization”

- Alarcon, Lensky, Pascalutsa, EPJ C 74, 2852 (2014) [1312.1219]: “Chiral perturbation theory of muonic-hydrogen Lamb shift: polarizability contribution”

- Indelicato, Mohr, Sapirstein, PRA 89, 054017 (2014) [1402.0439]: “Coordinate-space approach to vacuum polarization”

- Peset, Pineda, arXiv 1403.3408 [hep-ph]: “Model independent determination of the muonic hydrogen Lamb shift and proton radius”


No big changes. Polarizability terms confirmed a couple of times.
We have measured two transitions in \( \mu p \)

\[ v_t = v\left(2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}\right) \]

\[ v_s = v\left(2S_{1/2}^{F=0} - 2P_{3/2}^{F=1}\right) \]

We have measured two transitions in $\mu p$

- Consider the two measurements \textit{separately}

Two independent determinations of $r_p$

$(\nu_t \rightarrow r_p, \nu_s \rightarrow r_p)$

Consistent results!

---

We have measured two transitions in $\mu p$

- Consider the two measurements separately

Two independent determinations of $r_p$

$$(\nu_t \rightarrow r_p, \nu_s \rightarrow r_p)$$  Consistent results!

- Combine the two measurements

Two measurements $\rightarrow$ determine two parameters

$$\nu_t, \nu_s \rightarrow \Delta E_L, \Delta E_{\text{HFS}} \rightarrow r_p, r_Z$$

$$\begin{align*}
\frac{3}{4} \nu_t + \frac{1}{4} \nu_s &= \Delta E_L(r_p) + 8.8123 \text{ meV} \\
\nu_s - \nu_t &= \Delta E_{\text{HFS}}(r_Z) - 3.2480 \text{ meV}
\end{align*}$$

Proton Zemach radius

2S hyperfine splitting in $\mu p$ is: $\Delta E_{\text{HFS}} = 22.9843(30) - 0.1621(10) r_Z$ [fm] meV

with $r_Z = \int d^3r \int d^3r' r \rho_E(r) \rho_M(r-r')$

We measured $\Delta E_{\text{HFS}} = 22.8089(51)$ meV

This gives a proton Zemach radius $r_Z = 1.082 (31)_{\text{exp}} (20)_{\text{th}} = 1.082 (37)$ fm

Rydberg constant


Hydrogen spectroscopy (Lamb shift):

\[ L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \text{ MHz} \]

\[ E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3} \]

2 unknowns \( \Rightarrow \) 2 transitions

- Rydberg constant \( R_\infty \)
- Lamb shift \( L_{1S} \leftarrow r_p \)


Rydberg constant

\[ R_\infty = 3.289 \ 841 \ 960 \ 249 \ 5 (10)^{rp} (25)^{QED} \times 10^{15} \text{ Hz/c} \]

[8 parts in $10^{13}$]


\[ R_\infty = 3.289\,841\,960\,249\,5^{(10)} r_p^{(25)} \times 10^{15}\,\text{Hz/c} \]

\[ [8\text{ parts in } 10^{13}] \]


Deuteron charge radius

H/D isotope shift: \( r_{d}^2 - r_{p}^2 = 3.82007(65) \text{ fm}^2 \)

C.G. Parthey, RP et al., PRL 104, 233001 (2010)

CODATA 2010 \( r_d = 2.1424(21) \text{ fm} \)

Deuteron charge radius [fm]
Deuteron charge radius

H/D isotope shift: \( r_d^2 - r_p^2 = 3.82007(65) \text{ fm}^2 \)

CODATA 2010 \( r_d = 2.1424(21) \text{ fm} \)

\( r_p = 0.84087(39) \text{ fm} \) from \( \mu \text{H} \) gives \( r_d = 2.12771(22) \text{ fm} \)

\[
\mu \text{H} + \text{iso H/D(1S-2S)}
\]

CODATA D + e-d

e-d scatt.

n-p scatt.

Deuteron charge radius [fm]
Deuteron charge radius

H/D isotope shift: \( r_d^2 - r_p^2 = 3.82007(65) \text{ fm}^2 \)

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Lamb shift in muonic DEUTERIUM
Muonic deuterium
muonic deuterium
muonic deuterium
2.5 resonances in muonic deuterium

- $\mu d \ [ \ 2S_{1/2}(F=3/2) \rightarrow 2P_{3/2}(F=5/2) \ ]$
  20 ppm (stat., online)

- $\mu d \ [ \ 2S_{1/2}(F=1/2) \rightarrow 2P_{3/2}(F=3/2) \ ]$
  45 ppm (stat., online)

- $\mu d \ [ \ 2S_{1/2}(F=1/2) \rightarrow 2P_{3/2}(F=1/2) \ ]$
  70 ppm (stat., online)
  only 5$\sigma$ significant
  identifies F=3/2 line
Deuteron charge radius

H/D isotope shift: $r_d^2 - r_p^2 = 3.82007(65) \text{ fm}^2$

CODATA 2010 $r_d = 2.1424(21) \text{ fm}$

$r_p = 0.84087(39) \text{ fm}$ from $\mu H$ gives

$\mu H + \text{ iso H/D}(1S-2S)$

CODATA D + e-d
e-d scatt.
n-p scatt.

Deuteron charge radius [fm]
Deuteron charge radius

H/D isotope shift: \( r_d^2 - r_p^2 = 3.82007(65) \text{ fm}^2 \)

C.G. Parthey, RP et al., PRL 104, 233001 (2010)

\[
r_d = 2.1424(21) \text{ fm} \\
r_p = 0.84087(39) \text{ fm from } \mu \text{H gives } r_d = 2.1277(2) \text{ fm} \\
\text{Lamb shift in muonic DEUTERIUM } \Rightarrow r_d = 2.1272(12) \text{ fm} \quad \text{PRELIMINARY!}
\]

### CODATA-2010

- \( \mu \text{D (preliminary)} \)
- \( \mu \text{H + iso H/D(1S-2S)} \)

### CODATA D + e-d

- e-d scatt.
- n-p scatt.

Deuteron charge radius [fm]
Deuteron charge radius

- $\mu_H$ and $\mu_D$ are consistent!
- Proton discrepancy $\mu_H$ vs. H: $4.5\sigma$
- Deuteron discrepancy $\mu_D$ vs. D: $4.4\sigma$
- Combined atomic physics discrepancy: $6.3\sigma$

$\mu_D$ (preliminary)

$\mu_H$ + iso H/D(1S-2S)

CODATA-2010

CODATA D + e-d

e-d scatt.

n-p scatt.
Proton-deuteron isotope shift

In other words: The muonic isotope shift agrees with the electronic one!

\[ r_d^2 - r_p^2: \]
- H/D isotope shift: \( 3.82007 \pm 0.00065 \text{ fm}^2 \)
- Muonic Lamb shift: \( 3.8221 \pm 0.0052 \text{ fm}^2 \) PRELIMINARY!
- Scattering: \( 3.764 \pm 0.045 \text{ fm}^2 \)

The muonic error is conservative (nucl. structure terms).

Randolf Pohl
Eltville, 2 Feb. 2015
Theory in $\mu$D


- Krutov, Martynenko, PRA 84, 052514 (2011): “Lamb shift in the muonic deuterium atom”
- several papers by Jentschura
- several papers by Karshenboim et al.
- Friar, PRC 88, 034003 (2013): “Nuclear polarization corrections to mu-d atoms in zero-range approximation”

...
Muonic helium ions.
Lamb shift in muonic helium

- CREMA collaboration: Charge Radius Experiment with Muonic Atoms
- PSI Experiment R10-01
- ERC Starting Grant for RP, 2011-2016
- Goal: Measure $\Delta E(2S-2P)$ in $\mu^4\text{He}$, $\mu^3\text{He}$
- $\Rightarrow$ alpha particle and helion charge radius to $3 \times 10^{-4}$ (0.0005 fm)
Lamb shift in muonic helium

- **CREMA** collaboration: Charge Radius Experiment with Muonic Atoms
- PSI Experiment R10-01
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<table>
<thead>
<tr>
<th>State</th>
<th>Energy (meV)</th>
<th>Wavelength (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2S_{1/2}$</td>
<td>206 meV</td>
<td>206 nm</td>
</tr>
<tr>
<td>$2P_{1/2}$</td>
<td>23 meV</td>
<td>898 nm</td>
</tr>
<tr>
<td>$2P_{3/2}$</td>
<td>8.4 meV</td>
<td>812 nm</td>
</tr>
<tr>
<td>$2P_{1/2}$</td>
<td>146 meV</td>
<td>146 nm</td>
</tr>
<tr>
<td>$2P_{3/2}$</td>
<td>145 meV</td>
<td>145 nm</td>
</tr>
<tr>
<td>$2S_{1/2}$</td>
<td>290 meV</td>
<td></td>
</tr>
<tr>
<td>$2P_{1/2}$</td>
<td>397 meV</td>
<td></td>
</tr>
</tbody>
</table>

Randolf Pohl
Eltville, 2 Feb. 2015
Lamb shift in muonic helium

- CREMA collaboration: Charge Radius Experiment with Muonic Atoms
- PSI Experiment R10-01
- ERC Starting Grant for RP, 2011-2016
- Goal: Measure $\Delta E(2S-2P)$ in $\mu^4\text{He}$, $\mu^3\text{He}$
  $\Rightarrow$ alpha particle and helion charge radius to $3 \times 10^{-4}$ (0.0005 fm)
- aims:
  - help to solve the proton size puzzle
  - absolute charge radii of helion, alpha
  - low-energy effective nuclear models: $^1\text{H}$, $^2\text{D}$, $^3\text{He}$, $^4\text{He}$
  - QED test with $\text{He}^+(1S-2S)$ [Udem @ MPQ, Eikema @ Amsterdam]

Lamb shift in muonic helium

Measured transitions:

$\mu^4\text{He}^+$

$2P - 2S_{1/2}$

- $2P_{3/2}$
- $2P_{1/2}$

- Sept. 23 – Dec. 23, 2013
Lamb shift in muonic helium

Measured transitions:

- $\mu^4\text{He}^+$
  - $2P - 2P_{3/2}$
  - $2P - 2P_{1/2}$
  - $2S_{1/2}$

- $\mu^3\text{He}^+$
  - $2P - 2P_{3/2}$
  - $2P - 2P_{1/2}$
  - $2S_{1/2}$

- Sept. 23 – Dec. 23, 2013
- May 15 – Aug. 6, 2014
1st resonance in muonic He-4

\[ \mu^4\text{He}(2S_{1/2} \rightarrow 2P_{3/2}) \] at \( \sim 813 \text{ nm} \) wavelength

Preliminary Randolf Pohl Eltville, 2 Feb. 2015
1st resonance in muonic He-4

\[ \mu^4\text{He}(2S_{1/2} \rightarrow 2P_{3/2}) \text{ at } \sim 813 \text{ nm wavelength} \]

2nd resonance in muonic He-4

\[ \mu^4\text{He}(2S_{1/2} \rightarrow 2P_{1/2}) \] at \( \sim 899 \text{ nm wavelength} \)
1st resonance in muonic He-3

$\mu^3\text{He}(2S^{F=1}_{1/2} \rightarrow 2P^{F=2}_{3/2})$ at $\sim 864$ nm wavelength
Muonic summary

Muonic hydrogen gives:
- Proton charge radius: $r_p = 0.84087 (39) \text{ fm}$
  - $7.9\sigma$ away from electronic average (H, e-p scatt.)
- Deuteron charge radius: $r_d = 2.12771 (22) \text{ fm}$ from $\mu H + H/D(1S-2S)$

Muonic deuterium:
- Deuteron charge radius: $r_d = 2.1272 (12) \text{ fm}$ (PRELIMINARY!)
  - consistent with muonic proton radius, but
  - $6.3\sigma$ away from CODATA value (H & D, e-p/d scatt.)

“Proton” Radius Puzzle is in fact “Z=1 Radius Puzzle”:
- combined $>\sim 8\sigma$ discrepancy

muonic helium-3 and -4 ions: No big discrepancy (PRELIMINARY)
Muonic summary

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- “Proton” Radius Puzzle is in fact “Z=1 Radius Puzzle”:
  - combined $>\sim 8\sigma$ discrepancy

- Muonic helium-3 and -4 ions: No big discrepancy (PRELIMINARY)

- Could ALL be solved if the **Rydberg constant** [ and hence the (electronic) proton radius ] was wrong.
What may be wrong?

\[ \tilde{L}^{\text{theo.}}_{\mu p} (r_p^{\text{CODATA}}) - \tilde{L}^{\exp.}_{\mu p} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

μp theory wrong? μp experiment wrong? 
H theory wrong? H experiments wrong? → \( R_\infty \) wrong?
AND e-p scattering exp. wrong?

Standard Model wrong?
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}} (r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

\( \mu p \) theory wrong? \( \mu p \) experiment wrong? 

H theory wrong? 
H experiments wrong? \( \rightarrow R_\infty \) wrong? 
AND e-p scattering exp. wrong? 

Standard Model wrong?
(Electronic) hydrogen.
Hydrogen spectroscopy

Lamb shift: \[ L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \text{ MHz} \]

\[ L_{nS} \approx \frac{L_{1S}}{n^3} \]

\[
\begin{array}{ccccccc}
8S & & & & & & \\
4S & & & & & & \\
3S & & & & & & 3D \\
2S & & & & & & 2P \\
1S & & & & & & \\
\end{array}
\]
Hydrogen spectroscopy

Lamb shift: \( L_{1S} (r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \) MHz

\[ L_{nS} \sim \frac{L_{1S}}{n^3} \]

\[
\begin{align*}
8S & \quad 4S & \quad 3S & \quad 3D & \quad 2S & \quad 2P & \quad 2\text{S-2P} \\
\end{align*}
\]

classical Lamb shift: 2S-2P

- Lamb, Retherford 1946
- Lundeen, Pipkin 1986
- Hagley, Pipkin 1994
- Hessels et al., 201x

- 9910 MHz = 40 µeV
- 1058 MHz = 4 µeV

Randolf Pohl
Eltville, 2 Feb. 2015
Hydrogen spectroscopy

Lamb shift: \( L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \) MHz

\[ L_{nS} \simeq \frac{L_{1S}}{n^3} \]

<table>
<thead>
<tr>
<th>8S</th>
<th>7S</th>
<th>6S</th>
<th>5S</th>
<th>4S</th>
<th>3S</th>
<th>2S-4P</th>
<th>2S-8D</th>
<th>3D</th>
</tr>
</thead>
</table>

\[ E_{nS} \simeq -\frac{R_{\infty}}{n^2} + \frac{L_{1S}}{n^3} \]

2 unknowns \( \Rightarrow \) 2 transitions

- Rydberg constant \( R_{\infty} \)
- Lamb shift \( L_{1S} \leftarrow r_p \)
Hydrogen spectroscopy

\[ \begin{align*}
2S_{1/2} - 2P_{1/2} & \\
2S_{1/2} - 2P_{3/2} & \\
1S-2S + 2S- & 4S_{1/2} \\
1S-2S + 2S- & 4D_{5/2} \\
1S-2S + 2S- & 4P_{3/2} \\
1S-2S + 2S- & 4P_{1/2} \\
1S-2S + 2S- & 6D_{3/2} \\
1S-2S + 2S- & 6D_{5/2} \\
1S-2S + 2S- & 8S_{1/2} \\
1S-2S + 2S- & 8D_{3/2} \\
1S-2S + 2S- & 8D_{5/2} \\
1S-2S + 2S- & 12D_{3/2} \\
1S-2S + 2S- & 12D_{5/2} \\
1S-2S + 1S - 3S_{1/2} & 
\end{align*} \]

proton charge radius (fm)
Hydrogen spectroscopy

\[
\begin{align*}
2S_{1/2} - 2P_{1/2} \\
2S_{1/2} - 2P_{1/2} \\
2S_{1/2} - 2P_{3/2} \\
1S - 2S + 2S &- 4S_{1/2} \\
1S - 2S + 2S &- 4D_{5/2} \\
1S - 2S + 2S &- 4P_{1/2} \\
1S - 2S + 2S &- 4P_{3/2} \\
1S - 2S + 2S &- 6S_{1/2} \\
1S - 2S + 2S &- 6D_{5/2} \\
1S - 2S + 2S &- 8S_{1/2} \\
1S - 2S + 2S &- 8D_{3/2} \\
1S - 2S + 2S &- 8D_{5/2} \\
1S - 2S + 2S &- 12D_{3/2} \\
1S - 2S + 2S &- 12D_{5/2} \\
1S - 2S + 1S &- 3S_{1/2}
\end{align*}
\]

\[\mu_p : 0.84087 \pm 0.00039 \text{ fm}\]

proton charge radius (fm)
Hydrogen spectroscopy

\[ 2S_{1/2} - 2P_{1/2} \]
\[ 2S_{1/2} - 2P_{3/2} \]
\[ 1S-2S + 2S- \quad 4S_{1/2} \]
\[ 1S-2S + 2S- \quad 4D_{5/2} \]
\[ 1S-2S + 2S- \quad 4P_{1/2} \]
\[ 1S-2S + 2S- \quad 4P_{3/2} \]
\[ 1S-2S + 2S- \quad 6S_{1/2} \]
\[ 1S-2S + 2S- \quad 6D_{5/2} \]
\[ 1S-2S + 2S- \quad 8S_{1/2} \]
\[ 1S-2S + 2S- \quad 8D_{3/2} \]
\[ 1S-2S + 2S- \quad 8D_{5/2} \]
\[ 1S-2S + 2S- \quad 12D_{3/2} \]
\[ 1S-2S + 2S- \quad 12D_{5/2} \]
\[ 1S-2S + \quad 1S - 3S_{1/2} \]

\[ H_{\text{avg}} = 0.8779 \pm 0.0094 \text{ fm} \]

\[ \mu_p : 0.84087 \pm 0.00039 \text{ fm} \]
Hydrogen spectroscopy

\[
\begin{align*}
2S_{1/2} - 2P_{1/2} \\
2S_{1/2} - 2P_{3/2} \\
1S-2S + 2S- &- 4S_{1/2} \\
1S-2S + 2S- &- 4D_{5/2} \\
1S-2S + 2S- &- 4P_{1/2} \\
1S-2S + 2S- &- 4P_{3/2} \\
1S-2S + 2S- &- 6S_{1/2} \\
1S-2S + 2S- &- 6D_{5/2} \\
1S-2S + 2S- &- 8S_{1/2} \\
1S-2S + 2S- &- 8D_{3/2} \\
1S-2S + 2S- &- 8D_{5/2} \\
1S-2S + 2S- &- 12D_{3/2} \\
1S-2S + 2S- &- 12D_{5/2} \\
1S-2S + 1S - &3S_{1/2}
\end{align*}
\]

\[ H_{\text{avg}} = 0.8779 \pm 0.0094 \text{ fm} \]
\[ \mu_p : 0.84087 \pm 0.00039 \text{ fm} \]
New measurements of the Rydberg constant are urgently needed to resolve the puzzle $\mu_p$ vs. $H$

- **Flowers @ NPL**: $2S - nS, D : n > 4$

- **Tan @ NIST**: Ne$^{9+}$
  [Tan et al., Phys. Scr. T144, 014009 (2011)]

- **Udem, RP et al. @ MPQ**: $2S - 4P$

- **Hessels @ York**: $2S - 2P$
  [Hessels et al., Bull. APS 57(5), Q1.138 (2012)]

- **Pachucki**: He

- **Udem @ MPQ, Eikema @ Amsterdam**: He$^+$
  [Herrmann et al., PRA 79, 052505 (2009)]

$\mu_p : 0.84087 \pm 0.00039$ fm
Rydberg constant from hydrogen

- Apparatus used for H/D(1S-2S)
- 486 nm at 90° + Retroreflector ⇒ Doppler-free 2S-4P excitation
- 1st oder Doppler vs. ac-Stark shift
- ∼ 2.5 kHz accuracy (vs. 15 kHz Yale, 1995)
- cryogenic H beam, optical excitation to 2S


C.G. Parthey, RP et al., PRL 104, 233001 (2010)
C.G. Parthey, RP et al., PRL 107, 203001 (2011)

Hydrogen spectroscopy

\[ \begin{align*}
2S_{1/2} - 2P_{1/2} \\
2S_{1/2} - 2P_{3/2} \\
1S - 2S + 2S_1 & 4S_{1/2} \\
1S - 2S + 2S_2 & 4D_{5/2} \\
1S - 2S + 2S_3 & 4P_{1/2} \\
1S - 2S + 2S_4 & 4P_{3/2} \\
1S - 2S + 2S_5 & 6S_{1/2} \\
1S - 2S + 2S_6 & 6D_{5/2} \\
1S - 2S + 2S_7 & 8S_{1/2} \\
1S - 2S + 2S_8 & 8D_{3/2} \\
1S - 2S + 2S_9 & 8D_{5/2} \\
1S - 2S + 2S_{10} & 12D_{3/2} \\
1S - 2S + 2S_{11} & 12D_{5/2} \\
1S - 2S + 1S & 3S_{1/2}
\end{align*} \]

Projected accuracy Garching 2S-4P

\[ H_{\text{avg}} = 0.8779 \pm 0.0094 \text{ fm} \]
\[ \mu_p : 0.84087 \pm 0.00039 \text{ fm} \]
Hydrogen spectroscopy

Posters on hydrogen 1S-2S and 2S-4P
Axel Beyer and Lothar Meisenbacher

Projected accuracy Garching 2S-4P

\[ H_{\text{avg}} = 0.8779 \pm 0.0094 \text{ fm} \]
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2S_{1/2} & - 2P_{1/2} \\
2S_{1/2} & - 2P_{3/2} \\
1S-2S + 2S- & 4S_{1/2} \\
1S-2S + 2S- & 4D_{5/2} \\
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1S-2S + 2S- & 6S_{1/2} \\
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1S-2S + 2S- & 12D_{3/2} \\
1S-2S + 2S- & 12D_{5/2} \\
1S-2S + 1S & 3S_{1/2} \\
\end{align*} \]

\[ H_{\text{avg}} = 0.8779 \pm 0.0094 \text{ fm} \]

\[ \mu_p : 0.84087 \pm 0.00039 \text{ fm} \]

Projected accuracy Garching 2S-4P

Posters on hydrogen 1S-2S and 2S-4P
Axel Beyer and Lothar Meisenbacher

Poster on muonic H/D/He
Marc Diepold, Julian Krauth, Bea Franke
Future muonic experiments

- Z=1:
  - Muonic hydrogen: HFS
  - Muonic deuterium: Lamb shift, HFS
  - Muonic tritium

- Z=2:
  - Muonic $^4$He: Fine structure
  - Muonic $^3$He: Lamb shift, fine and hyperfine structure

- Z=3, 4, 5:

Lamb shift: absolute charge radius
nuclear polarizability

2S-HFS: Zemach / magnetic radius
nuclear polarizability

Lamb shifts and fine-structure splittings for the muonic ions $\mu^-\cdot$Li, $\mu^-\cdot$Be, and $\mu^-\cdot$B:
A proposed experiment

G. W. F. Drake and Louis L. Byer*
Department of Physics, University of Windsor, Windsor, Ontario, Canada N9B 3P4
(Received 28 February 1985)

Detailed calculations are presented for the energy splittings of the states $2s_{1/2}$-$2p_{1/2}$ and $2s_{1/2}$-$2p_{3/2}$ for the muonic ions $\mu^-\cdot$Li, $\mu^-\cdot$Be, and $\mu^-\cdot$B obtained by numerical integration of the Dirac equation. It is shown that there is severe cancellation between the vacuum polarization and finite nuclear size contributions to the energy differences, leading to transition frequencies which lie in the visible region of the spectrum. As a consequence of the cancellation, a measurement of the transition frequency would provide a sensitive probe of nuclear size and structure. The system $\mu^-\cdot^7$Li appears to offer particularly good possibilities for performing such an experiment.
Future muonic experiments

- Z=1:
  - Muonic hydrogen: HFS
  - Muonic deuterium: Lamb shift, HFS
  - Muonic tritium

- Z=2:
  - Muonic $^4$He: Fine structure
  - Muonic $^3$He: Lamb shift, fine and hyperfine structure

- Z=3, 4, 5:

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<tr>
<th>Ion</th>
<th>R (fm)</th>
<th>$\lambda(2s_{1/2}-2p_{1/2})$</th>
<th>$\lambda(2s_{1/2}-2p_{3/2})$</th>
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<tr>
<td>$^4$He</td>
<td>1.674±0.012</td>
<td>8978.0±4±27</td>
<td>8118.0±3±22</td>
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<tr>
<td>$^6$Li</td>
<td>2.56 ±0.05</td>
<td>10097.0±33±1072</td>
<td>6275.0±13±414</td>
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<td>$^7$Li</td>
<td>2.39 ±0.03</td>
<td>7473.0±18±334</td>
<td>5147.0±9±159</td>
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<td>$^9$Be</td>
<td>2.520±0.012</td>
<td>-9520.0±116±703</td>
<td>11512.0±173±1048</td>
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<tr>
<td>$^{10}$B</td>
<td>2.45 ±0.12</td>
<td>-1393.0±3±354</td>
<td>-4033.0±27±2947</td>
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<tr>
<td>$^{11}$B</td>
<td>2.42 ±0.12</td>
<td>-1481.0±4±397</td>
<td>-4887.0±46±4286</td>
</tr>
</tbody>
</table>

Drake, Byer, PRA 32, 713 (1985)
Future muonic experiments

- **Z=1:**
  - Muonic hydrogen: HFS
  - Muonic deuterium: Lamb shift, HFS
  - Muonic tritium

- **Z=2:**
  - Muonic $^4$He: Fine structure
  - Muonic $^3$He: Lamb shift, fine and hyperfine structure

- **Z=3, 4, 5:**
  - (Electronic) isotope shifts have been measured very accurately.
    ⇒ (squared) charge radius differences are very well known.
  - Muonic Lamb shifts provide absolute charge radii.
  - Test of few-electron (QED) calculations.
  - *Ab initio* nuclear structure calculations.

- **Also:** 1S-2S in (electronic) tritium. “Missing link” at A=3
According to *Forbes* (Jul. 2012), the Higgs discovery cost

13.25 billion USD.
According to Forbes (Jul. 2012), the Higgs discovery cost 13.25 billion USD.

We shrunk the proton radius by 4%.
According to Forbes (Jul. 2012), the Higgs discovery cost 13.25 billion USD.

We shrunk the proton radius by 4%.

This decreased the p-p cross section by 8%.
The cost for LHC

According to Forbes (Jul. 2012), the Higgs discovery cost

13.25 billion USD.

We shrunk the proton radius by 4%.

This decreased the p-p cross section by 8%.

Cost increase for Higgs discovery: 1.06 billion USD.
The cost for LHC

My apologies.

:-)
Proton Size Investigators thank you for your attention
Backup slides.
### Table 1

All known radius-independent contributions to the Lamb shift in $\mu p$ from different authors, and the one we selected. Values are in meV. The entry # in the first column refers to Table 1 in Ref. [13]. The "finite-size to relativistic recoil correction" (entry #18 in [13]), which depends on the proton structure, has been shifted to Table 2, together with the small terms #26 and #27, and the proton polarizability term #25. SE: self-energy, VP: vacuum polarization, LBL: light-by-light scattering, Rel: relativistic, NR: non-relativistic, RC: recoil correction.

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<td>1</td>
<td>NR one-loop electron VP (eVP)</td>
<td>205.0074</td>
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<td>2</td>
<td>Rel. corr. (Breit–Pauli)</td>
<td>0.0169 $^a$</td>
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<td>3</td>
<td>Rel. one-loop eVP</td>
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<td>205.0282</td>
<td>205.0282</td>
<td>205.02821</td>
<td>205.02821</td>
<td>[80] Eq. (54)</td>
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<td>Rel. RC to eVP, $\alpha(Z\alpha)^4$ (incl. in #2) $^b$</td>
<td></td>
<td>−0.0041</td>
<td>−0.0041</td>
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<td>4</td>
<td>Two-loop eVP (Källén–Sabry)</td>
<td>1.5079</td>
<td>1.5081</td>
<td>1.5081</td>
<td>1.50810</td>
<td>1.50810</td>
<td>[80] Eq. (57)</td>
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<tr>
<td>5</td>
<td>One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^5$</td>
<td>0.1509</td>
<td>0.1509</td>
<td>0.1507</td>
<td>0.15102</td>
<td>0.15102</td>
<td>[80] Eq. (60)</td>
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<td>0.0023</td>
<td>0.00223</td>
<td>0.00223</td>
<td>0.00215</td>
<td>0.00215</td>
<td>[80] Eq. (62), [87]</td>
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<tr>
<td>6</td>
<td>NR three-loop eVP</td>
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<td>Wichmann–Kroll, “1:3” LBL</td>
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<td>−0.00103</td>
<td>−0.00102</td>
<td>−0.00102</td>
<td>−0.00102</td>
<td>[80] Eq. (64), [89]</td>
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<td>Virtual Delbrück, “2:2” LBL</td>
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<td>0.00135</td>
<td>0.00115</td>
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<td>0.00115</td>
<td>[74,89]</td>
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<td>New</td>
<td>“3:1” LBL</td>
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<td>−0.00102</td>
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<td>−0.00102</td>
<td>[89]</td>
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<td>$\mu$SE and $\mu$VP</td>
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<td>−0.6677</td>
<td>−0.66770</td>
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<td>−0.66761</td>
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<td>Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$</td>
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<td>−0.005(1)</td>
<td>−0.00500</td>
<td>−0.004924 $^d$</td>
<td>−0.00524 $^f$</td>
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<td>eVP loop in self-energy $\alpha^2(Z\alpha)^4$</td>
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<td>−0.001</td>
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<td>21</td>
<td>Higher order corr. to $\mu$SE and $\mu$VP</td>
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<td>−0.00169</td>
<td>−0.00171 $^g$</td>
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<td>13</td>
<td>Mixed eVP + $\mu$VP</td>
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<td>New</td>
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<td>0.00005</td>
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<td>14</td>
<td>Hadronic VP $\alpha(Z\alpha)^4 m_r$</td>
<td>0.0113(3)</td>
<td>0.01077(38)</td>
<td>0.011(1)</td>
<td>0.01121(44)</td>
<td>0.01121(44)</td>
<td>[93–95]</td>
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<td>Hadronic VP $\alpha(Z\alpha)^5 m_r$</td>
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<td>Rad. corr. to hadronic VP</td>
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<td>Recoil corr.</td>
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<td>0.05750</td>
<td>0.0575</td>
<td>0.05747</td>
<td>0.05747</td>
<td>[80] Eq. (88)</td>
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<td>Rel. RC $(Z\alpha)^5$</td>
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<td>−0.045</td>
<td>−0.04497</td>
<td>−0.04497</td>
<td>−0.04497</td>
<td>[80] Eq. (88), [74]</td>
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<td>23</td>
<td>Rel. RC $(Z\alpha)^6$</td>
<td></td>
<td>0.0003</td>
<td>0.00030</td>
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Lamb shift in μp 1: $r_p$ independent

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<td>0.000136</td>
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<td>Rad. RC $\alpha(Z\alpha)^n$ (proton SE)</td>
<td>$-0.0099$</td>
<td>$-0.00960$</td>
<td>$-0.0100$</td>
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<td>[43]^h [74]</td>
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<td>Sum</td>
<td>206.0312</td>
<td>206.02915</td>
<td>206.02862</td>
<td>206.03399(109)</td>
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a This value has been recalculated to be 0.018759 meV [77].

b This correction is not necessary here because in #2 the Breit–Pauli contribution has been calculated using a Coulomb potential modified by eVP.

c Difference between Eqs. (6) and (4) in [78]: $E_{\text{VP}}(2P_{1/2}-2S_{1/2}) - E_{\text{VP}}^{(0)}(2P_{1/2}-2S_{1/2}) = 0.018759 - 0.020843 = -0.002084$ meV (see also Table IV). Using these corrected values, the various approaches are consistent. Pachucki becomes 205.0074 + 0.018759 = 205.0262 meV and Borie 205.0282 − 0.0020843 = 205.0261 meV.

d In Appendix C, incomplete.

e Eq. (27) in [85] includes contributions beyond the logarithmic term with modification of the Bethe logarithm to the Uehling potential. The factor $10/9$ should be replaced by $5/6$.

f This term is part of #22, see Fig. 22 in [86].

g Borie includes wave-function corrections calculated in [87]. The actual difference between Ref. [13] and Borie-v6 [79] is given by the inclusion of the Källén–Sabry correction with muon loop.

h This was calculated in the framework of NRQED. It is related to the definition of the proton radius.

References:

80 P. Indelicato, arXiv:1210.5828v2 [PRA 87, 022501 (2013)]
89 S.G. Karshenboim, E.Y. Korzinin, V.G. Ivanov, V.A. Shelyuto, JETP Lett. 92, 8 (2010)
92 A. Petermann, Phys. Rev. 105, 1931 (1957)
**Table 2**

Proton-structure-dependent contributions to the Lamb shift in $\mu p$ from different authors and the one we selected. Values are in meV, $\langle r^2 \rangle$ in fm$^2$. The entry # in the first column refers to Table 1 in Ref. [13] supplementary information [9]. Entry # 18 is under debate. TPE: two-photon exchange, VP: vacuum polarization, SE: self-energy, Rel: relativistic.

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<td></td>
<td>Non-rel. finite-size</td>
<td>$-5.1973 \langle r^2 \rangle$</td>
<td>$-5.1975 \langle r^2 \rangle$</td>
<td>$-5.1975 \langle r^2 \rangle$</td>
<td>$-0.0009$ meV$^a$</td>
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<td>Rel. corr. to non-rel. finite size</td>
<td>$-0.0018 \langle r^2 \rangle$</td>
<td>$-0.0010 \langle r^2 \rangle$</td>
<td>$-0.010 \langle r^2 \rangle$</td>
<td>$-0.0282 \langle r^2 \rangle$</td>
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<td>Finite size corr. to one-loop eVP</td>
<td>$-0.0110 \langle r^2 \rangle$</td>
<td>$-0.0110 \langle r^2 \rangle$</td>
<td>$-0.010 \langle r^2 \rangle$</td>
<td>$-0.0282 \langle r^2 \rangle$</td>
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<td>Finite size corr. to Källén–Sabry b</td>
<td>$-0.0165 \langle r^2 \rangle$</td>
<td>$-0.0170 \langle r^2 \rangle$</td>
<td>$-0.017 \langle r^2 \rangle$</td>
<td>(incl. in $-0.0282$)</td>
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<td>(0.00699)$^c$</td>
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<td>$\Delta E_{\text{TPE}}$ [46]</td>
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<td>Elastic (third Zemach)$^e$</td>
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<td>Measured $R_2^\mu$</td>
<td>$0.0365(18) \langle r^2 \rangle^{3/2}$</td>
<td>$0.0363 \langle r^2 \rangle^{3/2}$</td>
<td>$0.0353 \langle r^2 \rangle^{3/2}$</td>
<td>$0.0353 \langle r^2 \rangle^{3/2}$</td>
<td>$0.0378 \langle r^2 \rangle^{3/2}$</td>
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<td>Inelastic (polarizability)</td>
<td>$0.0129(5)$ meV$[101]$</td>
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<td>$0.012(2)$ meV</td>
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<td>Rad. corr. to TPE</td>
<td>$-0.00062 \langle r^2 \rangle$</td>
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<td>eVP corr. to polarizability</td>
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### Table 2 (continued)

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<td>27</td>
<td>SE corr. to polarizability</td>
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<td>−0.00001 meV [95]</td>
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<td>Finite-size to rel. recoil corr.</td>
<td>(0.013 meV)(^g)</td>
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<td>Higher order finite-size corr.</td>
<td>−0.000123 meV</td>
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<td>0.00001(10)  meV</td>
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<td>0.00001(10) meV</td>
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<td>2P(_{1/2}) finite-size corr.</td>
<td>−0.0000519((r^2))(^i)</td>
<td>(incl. above)</td>
<td>(incl. above)</td>
<td>(incl. above)</td>
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</table>

\(^{a}\) Corresponds to Eq. (6) in [11] which accounts only for the main terms in \(F_{\text{REL}}\) and \(F_{\text{NREL}}\).

\(^{b}\) This contribution has been accounted already in both the −0.0110 meV/fm\(^2\) and −0.0165 meV/fm\(^2\) coefficients.

\(^{c}\) Given only in Appendix C. Bethe logarithm is not included.

\(^{d}\) This uncertainty accounts for the difference between all-order in \(Z\alpha\) and perturbative approaches [82].

\(^{e}\) Corresponds to Eq. (20).

\(^{f}\) This value is slightly different from Eq. (22) because here an all-order in finite-size and an all-order in eVP approaches were used.

\(^{g}\) See Appendix F of [96]. This term is under debate.

\(^{h}\) Included in \(\Delta E_{\text{TPF}}\). This correction of \(0.018−0.021=−0.003\) meV is given by Eq. (64) in [10] and Eq. (25) in [11]. This correction is also discussed in [76] where the 6/7 factor results from 0.018/0.021.

\(^{i}\) Eq. (6a) in [79].

82 P. Indelicato, P.J. Mohr, 2012 (in preparation)
96 J.L. Friar, Ann. Phys. 122, 151 (1979)
Table 3
All known contributions to the 2S-HFS in $\mu p$ from different authors and the one we selected. Values are in meV, radii in fm. SE: self-energy, VP: vacuum polarization, Rel: relativistic, RC: recoil correction, PT: perturbation theory, p: proton, int: interaction, AMM: anomalous magnetic moment.

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<td>Fermi energy, $(Z\alpha)^4$</td>
<td>22.8054</td>
<td>22.8054</td>
<td>22.807995</td>
<td>22.807995</td>
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<td>h2</td>
<td>Breit corr., $(Z\alpha)^6$</td>
<td>0.0026</td>
<td>0.00258</td>
<td>0.02659</td>
<td>0.02659</td>
</tr>
<tr>
<td>h3</td>
<td>Dirac energy (+ Breit corr. in all-order)</td>
<td>22.807995</td>
<td>22.807995</td>
<td>0.02659</td>
<td>Eq. (107) in [80]</td>
</tr>
<tr>
<td>h4</td>
<td>$\mu$ AMM corr., $\alpha(Z\alpha)^4$, $\alpha(Z\alpha)^4$</td>
<td>0.0266</td>
<td>0.02659</td>
<td>0.02659</td>
<td>Eq. (107) in [80]</td>
</tr>
<tr>
<td>h5</td>
<td>eVP in 2nd-order PT, $\alpha(Z\alpha)^2 (\epsilon_{VP2})$</td>
<td>0.0746</td>
<td>0.07443</td>
<td>0.07437</td>
<td>0.07437</td>
</tr>
<tr>
<td>h6</td>
<td>All-order eVP corr.</td>
<td>0.00056</td>
<td>0.00056</td>
<td>0.00056</td>
<td>0.00056</td>
</tr>
<tr>
<td>h7</td>
<td>Two-loop corr. to Fermi-energy ($\epsilon_{VP2}$)</td>
<td>0.00056</td>
<td>0.00056</td>
<td>0.00056</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h8</td>
<td>One-loop eVP in $1\gamma$ int., $\alpha(Z\alpha)^4 (\epsilon_{VP1})$</td>
<td>0.0482</td>
<td>0.04818</td>
<td>0.04818</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h9</td>
<td>Two-loop eVP in $1\gamma$ int., $\alpha^2(Z\alpha)^4 (\epsilon_{VP1})$</td>
<td>0.0003</td>
<td>0.00037</td>
<td>0.00037</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h10</td>
<td>Further two-loop eVP corr.</td>
<td>0.00037</td>
<td>0.00037</td>
<td>0.00037</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h11</td>
<td>$\mu$VP (similar to $\epsilon_{VP2}$)</td>
<td>0.00091</td>
<td>(incl. in h13)</td>
<td>0.00091</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h12</td>
<td>$\mu$VP (similar to $\epsilon_{VP1}$)</td>
<td>0.0004</td>
<td>(incl. in h13)</td>
<td>0.0004</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h13</td>
<td>Vertex, $\alpha(Z\alpha)^5$</td>
<td>$-0.00311$</td>
<td>$-0.00311$</td>
<td>$-0.00311$</td>
<td>$-0.00311$</td>
</tr>
<tr>
<td>h14</td>
<td>Higher order corr. of (h13), (part with $\ln(\alpha)$)</td>
<td>$-0.00017$</td>
<td>$-0.00017$</td>
<td>$-0.00017$</td>
<td>$-0.00017$</td>
</tr>
<tr>
<td>h15</td>
<td>$\mu$ SE with p structure, $\alpha(Z\alpha)^5$</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0010</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h16</td>
<td>Vertex corr. with proton structure, $\alpha(Z\alpha)^5$</td>
<td>$-0.0018$</td>
<td>$-0.0018$</td>
<td>$-0.0018$</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h17</td>
<td>“Jellyfish” corr. with p structure, $\alpha(Z\alpha)^5$</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h18</td>
<td>Hadron VP, $\alpha^6$</td>
<td>0.0005(1)</td>
<td>0.00060(10)</td>
<td>0.00060(10)</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h19</td>
<td>Weak interaction contribution</td>
<td>0.0003</td>
<td>0.00027</td>
<td>0.00027</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h20</td>
<td>Finite-size (Zemach) corr. to $\Delta E_{Fermi}$, $(Z\alpha)^5$</td>
<td>$-0.1518^a$</td>
<td>$-0.16037 r_Z$</td>
<td>$-0.16034 r_Z$</td>
<td>$-0.16034 r_Z$</td>
</tr>
</tbody>
</table>

(continued on next page)
Table 3 (continued)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>h21</td>
<td>Higher order finite-size corr. to $\Delta E_{\text{Fermi}}$</td>
<td>0.0105(18)</td>
<td>0.0080(26)</td>
<td>$-0.0022 r_E^2 + 0.0009$</td>
<td>Eq. (107) in [80]</td>
</tr>
<tr>
<td>h22</td>
<td>Proton polarizability, $(Z\alpha)^5$, $\Delta E_{\text{HFS}}^{\text{pol}}$</td>
<td>0.0022(20)</td>
<td>0.0010(260)</td>
<td>$0.00801(260)$</td>
<td>[117,118]</td>
</tr>
<tr>
<td>h23</td>
<td>Recoil corr. (incl. in h20)</td>
<td>0.02123</td>
<td>0.02123</td>
<td>0.02123</td>
<td>[112]</td>
</tr>
<tr>
<td>h24</td>
<td>eVP + proton structure corr., $\alpha^6$</td>
<td>$-0.0026$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h25</td>
<td>eVP corr. to finite-size (similar to $\epsilon_{\text{VP2}}$)</td>
<td>$-0.00114$</td>
<td></td>
<td>$-0.0018 r_Z - 0.0001$</td>
<td>Eq. (109) in [80]</td>
</tr>
<tr>
<td>h26</td>
<td>eVP corr. to finite-size (similar to $\epsilon_{\text{VP1}}$)</td>
<td>$-0.0014$</td>
<td></td>
<td>$-0.0018 r_Z - 0.0001$</td>
<td></td>
</tr>
<tr>
<td>h27</td>
<td>Proton structure corr., $\alpha(Z\alpha)^5$</td>
<td>$-0.0017$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h28</td>
<td>Rel. + radiative RC with p AMM, $\alpha^6$</td>
<td>$0.0018$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum</td>
<td>22.8148(20)$^c$</td>
<td>22.9839(26)</td>
<td>22.9858(26)</td>
<td>22.8149 meV</td>
<td></td>
</tr>
<tr>
<td>Sum with</td>
<td>$r_E = 0.841$ fm, $r_Z = 1.045$ fm [28]</td>
<td>$-0.1604 r_Z$</td>
<td>$0.1621(10) r_Z - 0.0022(5) r_E^2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a Includes a correction $\alpha(Z\alpha)^5$ due to $\mu\text{VP}$.
b Calculated using the Simon et al. form factor.
c The uncertainty is 0.0078 meV if the uncertainty of the Zemach term (h20) is included (see Table II of [72]).

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80 P. Indelicato, arXiv:1210.5828v2 [PRA 87, 022501 (2013)]
Theory in $\mu p$: new papers

Papers that have appeared after we wrote A. Antognini, RP et al., Ann. Phys. 331, 127 (2013):

- Mohr, Griffith, Sapirstein, PRA 87, 052511 (2013), [1304.2076], “Bound-state field-theory approach to proton-structure effects in muonic hydrogen”

- Korzinin, Ivanov Karshenboim, PRD 88, 125019 (2013) [1311.5784]: “The $\alpha^2(Z\alpha)^4 m$ contributions to the Lamb shift and fine structure in light muonic atoms”

- Karshenboim, Ivanov, Korzinin, PRA 89, 022102 (2014) [1311.5789]: “Relativistic recoil effects for energy levels in a muonic atom within a Grotch-type approach. I. General approach”

- Ivanov, Korzinin, Karshenboim, PRA 90, 022103 (2014) [1311.5790]: “Relativistic recoil effects for energy levels in a muonic atom within a Grotch-type approach. II. An application to the one-loop electronic vacuum polarization”

- Alarcon, Lensky, Pascalutsa, EPJ C 74, 2852 (2014) [1312.1219]: “Chiral perturbation theory of muonic-hydrogen Lamb shift: polarizability contribution”

- Indelicato, Mohr, Sapirstein, PRA 89, 054017 (2014) [1402.0439]: “Coordinate-space approach to vacuum polarization”

- Peset, Pineda, arXiv 1403.3408 [hep-ph]: “Model independent determination of the muonic hydrogen Lamb shift and proton radius”


No big changes. Polarizability terms confirmed a couple of times.
The proton rms charge radius measured with electrons: $0.8770 \pm 0.0045$ fm

muons: $0.8409 \pm 0.0004$ fm

What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}} \left( r_p^{\text{CODATA}} \right) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

- \( \mu p \) theory wrong?
- \( \mu p \) experiment wrong?
- H theory wrong?
- H experiments wrong? \( \rightarrow R_\infty \) wrong?
- AND e-p scattering exp. wrong?

Standard Model wrong?!?

What may be wrong?

\[
\tilde{L}_{\text{theo.}}(r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\exp.} = \begin{cases} 
75 \text{ GHz} \\
0.31 \text{ meV} \\
0.15 \% 
\end{cases}
\]

\(\mu p\) experiment wrong?

- Frequency mistake by 75 GHz (\(\Leftrightarrow 0.15\%\))?

  That is \(> 100 \delta(\mu p)\) !

  \(\sigma_{\text{tot}} = 650 \text{ MHz}, \quad [570 \text{ MHz}_{\text{stat}}, \quad 300 \text{ MHz}_{\text{syst}}]\)

  4 line widths !

  \(\Gamma = 19 \text{ GHz}\)

  2 resonances in \(\mu p\) give the same \(r_p\)
What may be wrong?

- Frequency mistake by 75 GHz (⇔ 0.15%)?
- Wrong transition?

FS, HFS huge.

Next transition: ∼ 1 THz away.
What may be wrong?

\[ \tilde{\mu}_p \text{ experiment wrong?} \]

Frequency mistake by 75 GHz (⇔ 0.15%)?
Wrong transition?
Systematic error?

<table>
<thead>
<tr>
<th>Source</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser frequency (H₂O calibration)</td>
<td>300 MHz</td>
</tr>
<tr>
<td>intrinsic H₂O uncertainty</td>
<td>2 MHz</td>
</tr>
<tr>
<td>AC and DC stark shift</td>
<td>&lt; 1 MHz</td>
</tr>
<tr>
<td>Zeeman shift (5 Tesla)</td>
<td>&lt; 30 MHz</td>
</tr>
<tr>
<td>Doppler shift</td>
<td>&lt; 1 MHz</td>
</tr>
<tr>
<td>Collisional shift</td>
<td>2 MHz</td>
</tr>
</tbody>
</table>

\[ \mu_p \text{ atom is small and not easily perturbed by external fields.} \]
What may be wrong?

\[ \tilde{\mu}_p \text{theo.} \left( r_p^{\text{CODATA}} \right) - \tilde{\mu}_p \text{exp.} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

- Frequency mistake by 75 GHz (⇔ 0.15%)?
- Wrong transition?
- Systematic error?
- Molecular effects?

\[ p \mu e \text{ molecular ion?} \]


Does not exist!

Experimentally:
- only 1 line observed (> 80% population)
- expected width
- \( pp\mu \) ion short-lived  

What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo}}(r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

\( \mu p \) experiment wrong?

- Frequency mistake by 75 GHz (⇔ 0.15%)?
- Wrong transition?
- Systematic error?
- Molecular effects?
- Gas impurities?  
  M. Diepold, RP et al., PRA 88, 042520 (2013).
  Target gas contained 0.55(5) % air (leak).

Back-of-the-envelope calculation:
- Collision rate  \( \lambda \approx 6 \cdot 10^3 s^{-1} \)
- 2S lifetime  \( \tau(2S) = 1 \mu s \)

⇒ Less than 1% of all \( \mu p(2S) \) atoms see any \( N_2 \)
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}} (r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

\( \mu p \) experiment wrong?

- Frequency mistake by 75 GHz (\( \Leftrightarrow 0.15\% \))?
- Wrong transition?
- Systematic error?
- Molecular effects?
- Gas impurities?

\( \mu p \) experiment probably not wrong by 100 \( \sigma \)
What may be wrong?

\[ \tilde{\mathcal{L}}_{\mu p}^{\text{theo.}}(r_p^{\text{CODATA}}) - \tilde{\mathcal{L}}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

\[ \mu p \text{ theory wrong?} \]

Discrepancy = 0.31 meV

Theory uncert. = 0.0025 meV

\[ \Rightarrow 120 \delta (\text{theory}) \text{ deviation} \]

double-checked by many groups

5\textsuperscript{th} largest term!

Some contributions to the \( \mu p \) Lamb shift

\[ \Delta E = 206.0668(25) - 5.2275(10) r_p^2 \ [\text{meV}] \]

Theory summary:

A. Antognini, RP \textit{et al.}

Annals of Physics 331, 127 (2013)
What may be wrong?

\[ \tilde{L}_{\text{theo.}}(r_p^{\text{CODATA}}) - \tilde{L}_{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

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**Theory summary:**
A. Antognini, RP et al.
Annals of Physics 331, 127 (2013)

\[ \Delta E = 206.0668(25) - 5.2275(10) r_p^2 \text{ [meV]} \]

**Some contributions to the \( \mu p \) Lamb shift**

- 1-loop eVP
- proton size
- 2-loop eVP
- \( \mu \)SE and \( \mu \)VP
- discrepancy
- 1-loop eVP in 2 Coul.
- recoil
- 2-photon exchange
- hadronic VP
- proton SE
- 3-loop eVP
- light-by-light
What may be wrong?

\[ \tilde{L}_{\mu p}^{\text{theo.}}(r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

\[ \Delta E = 206.0668(25) - 5.2275(10) r_p^2 \text{ [meV]} \]

\[ \mu_p \text{ theory probably not wrong by } 100 \sigma \]
Discussions: 3rd Zemach moment

- **PLB 693, 555**  De Rujula: “QED is not endangered by the proton’s size” (1008.3861)
  
  A large third Zemach moment \( \langle r_3^3 \rangle_{(2)} = \int d^3 r_1 d^3 r_2 \rho (r_1) \rho (r_2) |r_1 - r_2|^3 \)
  
  of the proton can explain all three measurements: \( \mu_p \), H, e-p
  
  \( \rho (r) \) is not a simple Dipole, but has “core” and “tail”

- **PRC 83, 012201**  Cloet, Miller: “Third Zemach moment of the proton” (1008.4345)
  
  Such a large third Zemach moment is impossible.

  \[
  \langle r_p^3 \rangle_{(2)} \text{ (De Rujula)} = 36.6 \pm 6.9 \text{ fm}^3
  
  \langle r_p^3 \rangle_{(2)} \text{ (Sick)} = 2.71 \pm 0.13 \text{ fm}^3
  \]

- **PLB 696, 343**  Distler et al: “The RMS radius of the proton and Zemach moments” (1011.1861)
  
  \[
  \langle r_p^3 \rangle_{(2)} \text{ (Mainz 2010)} = 2.85 \pm 0.08 \text{ fm}^3
  \]
Discussions: Proton polarizability

Proton polarizability aka. two-photon exchange

Seems to be the only contribution which might be able to solve the proton size puzzle by changing theory in $\mu_p$.

Keep in mind:
- Discrepancy: 0.31 meV
- Polarizability: 0.015(4) meV  20 times smaller!
Discussions: Proton polarizability

G.A. Miller et al., PRA 84, 020101(R) (2011)
“Toward a resolution of the proton size puzzle”
- New off-mass-shell effect $\sim \alpha \frac{m^4}{M^3}$ solves puzzle.

R.J. Hill, G. Paz, PRL, 107, 160402 (2011)
“Model independent analysis of proton structure for hydrogenic bound states”
- forward Compton amplitude’s $W_1(0, Q^2)$ is now well known
- “Crazy” functional behaviour can give any correction.
- No numbers given.

C.E. Carlson, M. Vanderhaeghen, PRA 84, 020102(R) (2011)
“Higher-order proton structure corrections to the Lamb shift in muonic hydrogen”
- All off-shell effects are automatically included in standard treatment.

C.E. Carlson, M. Vanderhaeghen, arXiv 1109.3779 (atom-ph)
“Constraining off-shell effects using low-energy Compton scattering”
- Off-shell effects are 100 times smaller than needed to explain the puzzle.
Discussions: Proton polarizability

  “Proton polarisability contribution to the Lamb shift in muonic hydrogen at fourth order in chiral perturbation theory”
  - Calculate $T_1(0,Q^2)$ in heavy-baryon chiral pert. theory.
  - Proton polarizability is not responsible for the radius puzzle.

- Gorchtein, Llanes-Estrada, Szczepaniak, PRA 87, 052501 (2013)
  “$\mu$-H Lamb shift: dispersing the nucleon-excitation uncertainty with a finite energy sum rule”
  - Sum rule + virtual photoabsorption data.
  - “We conclude that nucleon structure-dependent uncertainty by itself is unlikely to resolve the large discrepancy...”

- Karshenboim, McKeen, Pospelov, arXiv 1401.6156 [hep-ph]
  “Constraints on muon-specific dark forces”
  - “These estimates show that if indeed large muon-proton interactions are responsible for the $r_p$ discrepancy, one can no longer insist that theoretical calculations of the muon $g−2$ are under control. Thus, a resolution of the $r_p$ problem is urgently needed in light of the new significant investments made in the continuation of the experimental $g−2$ program.”
Discussions: Proton polarizability

- Proton off-shell effects can in principle shift the $\mu_p$ value.
- Evil subtraction function.
- Evidence is growing, that this effect can NOT solve the puzzle.
- Clarification needed $[(g - 2)_\mu!]$
What may be wrong?

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75 \text{ GHz} \\
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0.15 \% 
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Theory uncert. \(= 0.0025 \text{ meV}\)

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- H experiments wrong? \( \rightarrow R_\infty \) wrong?
- AND e-p scattering exp. wrong?
- Standard Model wrong?!?
Discussions: New Physics

Jaeckel, Roy, PRD 82, 125020 (2010)
“Spectroscopy as a test of Coulomb’s law - A probe of the hidden sector”
hidden photons, minicharged particles → deviations from Coulomb’s law.
µp transition can NOT be explained this. (contradicts Lamb shift in H)

“Lamb shift in muonic hydrogen – II. Analysis of the discrepancy of theory and experiment”
no millicharged particles, no unstable neutral vector boson.

Barger, Chiang, Keung, Marfatia, PRL 106, 153001 (2011)
“Proton size anomaly”
deck of Υ, J/ψ, π0, η, neutron scattering, muon g-2, μ24Mg, μ28Si
⇒ It’s NOT a new flavor-conserving spin-0, 1 or 2 particle

Tucker-Smith, Yavin, PRD 83, 101702 (2011)
“Muonic hydrogen and MeV forces”
MeV force carrier can explain discrepancies for rp and (g-2)μ
IF coupling to e, n is suppressed relative to coupling to μ, p
prediction for μHe+, μ+μ−
Discussions: New Physics

- Batell, McKeen, Pospelov, PRL 107, 011803 (2011)
  “New Parity-violating muonic forces and the proton charge radius”
  10...100 MeV heavy photon (“light Higgs”) can explain $r_p$ and $(g-2)_\mu$ prediction for $\mu\text{He}^+$, enhanced PNC in muonic systems

- Barger, Chiang, Keung, Marfatia, PRL 108, 081802 (2011)
  “Constraint on Parity-violating muonic forces”
  No missing mass events observed in leptonic Kaon decay.
  $\Rightarrow$ constraints on light Higgs.

- Pospelov (private comm.)
  Lack of missing mass events in leptonic Kaon decays no problem.
  Light Higgs is short-lived (decays inside the detector).

- C.E. Carlson, B.C. Rislow, PRD 86, 035013 (2012)
  “New physics and the proton radius problem”
  “New physics with fine-tuned couplings may be entertained as a possible explanation for the Lamb shift discrepancy.”
Discussions: New Physics

  “Proton puzzle and large extra dimensions” (arXiv 1303.4885)
  “Extra gravitational force between the proton and the muon at very short range provides an energy shift which accounts for the discrepancy…”

- Li, Chen, arXiv 1303.5146
  “Can large extra dimensions solve the proton radius puzzle?”
  “We find that such effect could be produced by four or more large extra dimensions which are allowed by the current constraints from low energy physics.”

  “Proton radius puzzle and quantum gravity at the Fermi scale” (1312.3469)
  “We show how the proton radius puzzle … may be solved by means of … an effective Yukawian gravitational potential related to charged weak interactions. […] Muonic hydrogen plays a crucial role to test possible scenarios for a gravitoweak unification, with weak interactions seen as manifestations of quantum gravity effects at the Fermi scale.”
What may be wrong?

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- AND e-p scattering exp. wrong?

Standard Model wrong?
(Electronic) hydrogen.
Hydrogen spectroscopy

Lamb shift: \[ L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \text{ MHz} \]

\[ L_{nS} \approx \frac{L_{1S}}{n^3} \]

8S
4S
3S
2S
1S

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3D

2P
Hydrogen spectroscopy

Lamb shift: \( L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \) MHz

\[ L_{nS} \approx \frac{L_{1S}}{n^3} \]

8S ———— 4S ———— 3S ———— 3D

2S ———— 2P

classical Lamb shift: 2S-2P

- Lamb, Retherford 1946
- Lundeen, Pipkin 1986
- Hagley, Pipkin 1994
- Hessels et al., 201x

\( 2S_{1/2} \): \( F=1 \) (1058 MHz = 4 \( \mu \)eV)

\( 2S_{1/2} \): \( F=0 \) (9910 MHz = 40 \( \mu \)eV)

\( 2P_{3/2} \): \( F=2 \)

\( 2P_{1/2} \): \( F=1 \) (32 MHz = 0.16 \( \mu \)eV)
Hydrogen spectroscopy

Lamb shift: \( L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \) MHz

\[
L_{nS} \simeq \frac{L_{1S}}{n^3}
\]

\[
E_{nS} \simeq -\frac{R_{\infty}}{n^2} + \frac{L_{1S}}{n^3}
\]

2 unknowns ⇒ 2 transitions

- Rydberg constant \( R_{\infty} \)
- Lamb shift \( L_{1S} \leftarrow r_p \)
Hydrogen spectroscopy

\[
\begin{align*}
2S_{1/2} - 2P_{1/2} \\
2S_{1/2} - 2P_{3/2} \\
1S-2S + 2S- & 4S_{1/2} \\
1S-2S + 2S- & 4D_{5/2} \\
1S-2S + 2S- & 4P_{1/2} \\
1S-2S + 2S- & 4P_{3/2} \\
1S-2S + 2S- & 6S_{1/2} \\
1S-2S + 2S- & 6D_{5/2} \\
1S-2S + 2S- & 8S_{1/2} \\
1S-2S + 2S- & 8D_{3/2} \\
1S-2S + 2S- & 8D_{5/2} \\
1S-2S + 2S- & 12D_{3/2} \\
1S-2S + 2S- & 12D_{5/2} \\
1S-2S + & 1S - 3S_{1/2} \\
\end{align*}
\]

proton charge radius (fm)
Hydrogen spectroscopy

\[ \begin{align*}
2S_{1/2} - 2P_{1/2} \\
2S_{1/2} - 2P_{3/2} \\
1S-2S + 2S- & 4S_{1/2} \\
1S-2S + 2S- & 4D_{5/2} \\
1S-2S + 2S- & 4P_{1/2} \\
1S-2S + 2S- & 4P_{3/2} \\
1S-2S + 2S- & 6S_{1/2} \\
1S-2S + 2S- & 6D_{5/2} \\
1S-2S + 2S- & 8S_{1/2} \\
1S-2S + 2S- & 8D_{3/2} \\
1S-2S + 2S- & 8D_{5/2} \\
1S-2S + 2S- & 12D_{3/2} \\
1S-2S + 2S- & 12D_{5/2} \\
1S-2S + 1S - 3S_{1/2} \\
\end{align*} \]

\[ \mu_p : 0.84087 \pm 0.00039 \text{ fm} \]
Hydrogen spectroscopy

- $2S_{1/2} - 2P_{1/2}$
- $2S_{1/2} - 2P_{3/2}$
- $1S - 2S + 2S - 4S_{1/2}$
- $1S - 2S + 2S - 4D_{5/2}$
- $1S - 2S + 2S - 4P_{1/2}$
- $1S - 2S + 2S - 4P_{3/2}$
- $1S - 2S + 2S - 6S_{1/2}$
- $1S - 2S + 2S - 6D_{5/2}$
- $1S - 2S + 2S - 8S_{1/2}$
- $1S - 2S + 2S - 8D_{3/2}$
- $1S - 2S + 2S - 8D_{5/2}$
- $1S - 2S + 2S - 12D_{3/2}$
- $1S - 2S + 2S - 12D_{5/2}$
- $1S - 2S + 1S - 3S_{1/2}$

$H_{\text{avg}} = 0.8779 \pm 0.0094 \text{ fm}$

$\mu p : 0.84087 \pm 0.00039 \text{ fm}$
Hydrogen spectroscopy

\[ \begin{align*}
2S_{1/2} & - 2P_{1/2} \\
2S_{1/2} & - 2P_{3/2} \\
1S-2S + 2S- & 4S_{1/2} \\
1S-2S + 2S- & 4D_{5/2} \\
1S-2S + 2S- & 4P_{1/2} \\
1S-2S + 2S- & 4P_{3/2} \\
1S-2S + 2S- & 6S_{1/2} \\
1S-2S + 2S- & 6D_{5/2} \\
1S-2S + 2S- & 8S_{1/2} \\
1S-2S + 2S- & 8D_{3/2} \\
1S-2S + 2S- & 8D_{5/2} \\
1S-2S + 2S- & 12D_{3/2} \\
1S-2S + 2S- & 12D_{5/2} \\
1S-2S + & 1S - 3S_{1/2}
\end{align*} \]

H_{avg} = 0.8779 ± 0.0094 fm

\[ \mu_p : 0.84087 ± 0.00039 \text{ fm} \]
Hydrogen spectroscopy

New measurements of the Rydberg constant are urgently needed to resolve the puzzle $\mu_p$ vs. H

- Flowers @ NPL: $2S - nS, D : n > 4$

- Tan @ NIST: Ne$^{9+}$
  [Tan et al., Phys. Scr. T144, 014009 (2011)]

- Udem, RP et al. @ MPQ: $2S - 4P$

- Hessels @ York: $2S - 2P$
  [Hessels et al., Bull. APS 57(5), Q1.138 (2012)]

- Pachucki: He

- Udem @ MPQ, Eikema @ Amsterdam: He$^+$
  [Herrmann et al., PRA 79, 052505 (2009)]

$\mu_p : 0.84087 \pm 0.00039$ fm
Rydberg constant from hydrogen

Apparatus used for H/D(1S-2S)

- 486 nm at 90° + Retroreflector ⇒ Doppler-free 2S-4P excitation
- 1st order Doppler vs. ac-Stark shift
- ~ 2.5 kHz accuracy (vs. 15 kHz Yale, 1995)
- cryogenic H beam, optical excitation to 2S

C.G. Parthey, RP et al., PRL 104, 233001 (2010)
C.G. Parthey, RP et al., PRL 107, 203001 (2011)

Hydrogen spectroscopy

\[ \begin{align*}
2S_{1/2} - 2P_{1/2} \\
2S_{1/2} - 2P_{3/2} \\
1S - 2S + 2S - 4S_{1/2} \\
1S - 2S + 2S - 4D_{5/2} \\
1S - 2S + 2S - 4P_{1/2} \\
1S - 2S + 2S - 4P_{3/2} \\
1S - 2S + 2S - 6S_{1/2} \\
1S - 2S + 2S - 6D_{5/2} \\
1S - 2S + 2S - 8S_{1/2} \\
1S - 2S + 2S - 8D_{3/2} \\
1S - 2S + 2S - 8D_{5/2} \\
1S - 2S + 2S - 12D_{3/2} \\
1S - 2S + 2S - 12D_{5/2} \\
1S - 2S + 1S - 3S_{1/2}
\end{align*} \]

Projected accuracy Garching 2S-4P

\[ H_{\text{avg}} = 0.8779 \pm 0.0094 \text{ fm} \]
\[ \mu_p : 0.84087 \pm 0.00039 \text{ fm} \]
Hydrogen spectroscopy

\[ 2S_{1/2} - 2P_{1/2}, \quad 2S_{1/2} - 2P_{3/2}, \quad 1S-2S + 2S-, \quad 4S_{1/2}, \quad 1S-2S + 2S-, \quad 4D_{5/2}, \quad 1S-2S + 2S-, \quad 4P_{1/2}, \quad 1S-2S + 2S-, \quad 4P_{3/2}, \quad 1S-2S + 2S-, \quad 6S_{1/2}, \quad 1S-2S + 2S-, \quad 6D_{5/2}, \quad 1S-2S + 2S-, \quad 8S_{1/2}, \quad 1S-2S + 2S-, \quad 8D_{3/2}, \quad 1S-2S + 2S-, \quad 8D_{5/2}, \quad 1S-2S + 2S-, 12D_{3/2}, \quad 1S-2S + 2S-, 12D_{5/2}, \quad 1S-2S + 1S - 3S_{1/2} \]

**Posters on hydrogen 1S-2S and 2S-4P**
Axel Beyer and Lothar Meisenbacher

Projected accuracy Garching 2S-4P

\[ H_{\text{avg}} = 0.8779 \pm 0.0094 \text{ fm} \]
\[ \mu_p = 0.84087 \pm 0.00039 \text{ fm} \]

proton charge radius (fm)
Hydrogen spectroscopy

Posters on hydrogen 1S-2S and 2S-4P
Axel Beyer and Lothar Meisenbacher

Poster on muonic H/D/He
Marc Diepold, Julian Krauth, Bea Franke

Projected accuracy Garching 2S-4P

$H_{avg} = 0.8779 \pm 0.0094 \text{ fm}$
$\mu_p = 0.84087 \pm 0.00039 \text{ fm}$
Old $\mu$He$^+$ resonances


$2S \rightarrow 2P_{3/2}$

Carboni et al, Phys. Lett. 73B, 229 (1978)

$2S \rightarrow 2P_{1/2}$
$\mu\text{He}^+(2S)$ lifetime

laser exp.: Dittus, PhD thesis ETH Zurich (1985)

Hauser et al., PRA 46, 2363 (1992)

1st resonance in muonic He-4