The $g$-Factor of the Proton and the Antiproton

Outlook

• Motivation

  • $g$-factor measurement principle in a Penning trap

    • Observation of single spin flips of a single proton

    • Measurement of proton $g$–factor

  • Status on Antiproton $g$-factor at BASE - CERN
Test of CPT invariance

- CPT invariance is the most fundamental symmetry in the Standard Model.
- Strategy: Compare properties of matter and antimatter conjugates with high precision.
Proton / Antiproton Magnetic Moments

Proton Magnetic Moment

\[ g_p = g_e \cdot \frac{m_p}{m_e} \cdot \frac{g_p}{g_p(H)} \cdot \frac{g_e(H)}{g_e} \cdot \frac{\mu_p(H)}{\mu_e(H)} \]

0.4 ppb

10 ppb

0.78 ppt

\( \approx \text{ppb} \)

Requires theoretical corrections at the level of 17.7 ppm.


Antiproton Magnetic Moment

- Until 2012: Exotic atom spectroscopy (ASACUSA) with at per-mille level

- 2012: First direct single particle measurement with 4.4 ppm in precision

J. DiSciascia et al., PRL 110, 130801 (2013)
Determination of the $g$-factor

**Determination of Larmor frequency in a given magnetic field**

\[ \omega_L = g \frac{e}{2m_p} B \]

**Monitoring magnetic field via simultaneous measurement of the free cyclotron frequency**

\[ \omega_c = \frac{e}{m_p} B \]

\[ g = 2 \frac{\omega_L}{\omega_c} = 2 \frac{\nu_L}{\nu_c} \]
The Penning trap

Superposition of homogeneous magnetic field and electrostatic quadrupole potential

\[ \vec{B} = B\hat{e}_z \]

\[ \Phi(z, \rho) = U_0 c^2 \left( z^2 - \frac{\rho^2}{2} \right) \]

**Invariance Theorem:**

\[ \nu_c^2 = \nu_+^2 + \nu_z^2 + \nu_-^2 \]

Measurement of eigenfrequencies
Ion in thermal equilibrium - axial frequency

- Detection of tiny image currents [fA]
- Highly sensitive detection circuits – High-Q

Particle acts as a perfect short

\[
\delta v_z \propto N_p
\]

Auto-resonant excitation for energy selective particle reduction

\[ Q = 12000 \]
\[ R_p = 120M\Omega \]

Signal \( \sim R_p \sim Q \)
Measurement of eigenfrequencies
Access to radial modes

Coupling of modes via rf-sideband coupling, e.g. \( \nu_{rf} = \nu_+ - \nu_z \)

Amplitude modulation of the axial motion

\[
\nu_+ = \nu_{rf} + \nu_l + \nu_r - \nu_z
\]

Same principle for remaining magnetron mode

\[
\nu_- = \nu_{rf} - \nu_l - \nu_r + \nu_z
\]

\[
\nu_c^2 = \nu_+^2 + \nu_z^2 + \nu_-^2
\]

\[
\frac{\Delta \nu_c}{\nu_c} \approx 10^{-9}
\]
Detection of the spin state
The continuous Stern-Gerlach effect

Introduce magnetic inhomogeneity, the magnetic bottle

\[ B_z = B_0 + B_2 \left( z^2 - \frac{\rho^2}{2} \right) \]

Spin flip results in shift of the axial frequency

\[ \nu_z \propto \frac{\mu_p}{m} B_2 \]

Coupling of spin moment to axial oscillation

\[ \Phi_z = \pm \mu_p B_z \]
Detection of spin state
Challenge

Applied with great success for electron $g$-factors – Bohr magneton

$$v_z \propto \pm \frac{1}{2\pi^2 v_{z,0}} \frac{\mu_z}{m} B_2$$

Dealing with nuclear magneton requires huge magnetic bottle of

$$B_2 = 30 \text{T/cm}^2$$

to obtain frequency jump due to spin transition of

$$\Delta v_z = 190 \text{ mHz} \rightarrow \Delta v_z/v_z = 2 \times 10^{-7}$$

BUT

$$v_z \propto + \frac{1}{2\pi v_{z,0}} \frac{B_2}{B_0} E_{\text{radial}}$$

Challenging!
Tiny energy fluctuations in radial modes cause huge axial frequency shifts

$$\Delta v_z / E_+ = 1 \text{ Hz/\mu eV}$$
Frequency fluctuation \( \Xi \)

\[ \Delta v_z = (v_z(t+T) - v_z(t)) \]

\[ \Xi^2 = \frac{1}{n} \sum (\Delta v_z - \bar{\Delta v}_z)^2 \]

\( \Xi = 150 \text{mHz} \) - not stable enough for observation individual spin transition

Axial frequency fluctuation \( \Xi \) increases due to frequency jump caused by spin transitions

\[ \Xi_{SF} = \sqrt{\Xi_{ref}^2 + P_{SF} \Delta v_{z,SF}^2} \]

Measure \( \Xi_{SF} \) and \( \Xi_{ref} \) → obtain SF-Probability!!!

Detecting spin transitions in a statistical measurement!
$g$-Factor measurement

relative precision of $10^{-4}$


reduction of line broadening using feedback cooling


- Larmor frequency measurement with a relative uncertainty of $1.8 \times 10^{-6}$
- With cyclotron frequency measurement

$g = 5.585696 (50)$

Limited by magnetic field inhomogeneity

similar method used by Harvard group – relative precision of 2.5 ppm

di Sciacca et al., PRL 108, 153001 (2012)
Double Penning trap technique

- High Precision measurement demands homogeneous magnetic field

Demands detection of every single spin transition!
Improvement of frequency stability

White noise

Higher signal-to-noise ratio

\[ SNR \propto R_p \propto Q \]

results in improved frequency measurement in but

\[ \delta v_z = \frac{1}{2\pi} \frac{q^2 R_p}{m D} \]

Can be overcome by increased effective Electrode distance \( D \)

Novel toroid detection coils allow for 4 times faster and precise frequency measurements

\[ Q = 12000 \]
\[ R_p = 120 \, M\Omega \]
\[ u_n = 0.9 \, nV/\sqrt{Hz} \]
Improvement of frequency stability

White noise

Higher signal-to-noise ratio

\[ SNR \propto R_p \propto Q \]

results in improved frequency measurement in but

Novel toroid detection coils allow for 4 times faster and precise frequency measurements
Quality of spin state detection
Bayes and threshold method

**Threshold method**: Accept spin flip if frequency jump above given threshold

**Bayes rule** – conditional probability of having a spin state

\[ P(S \mid f_2, f_1) \propto P(f_2 \mid S, f_1)P(S, f_1) \]

Update of state probability given complete frequency, noise and previous state information

**Fidelity**: fraction of correctly assigned spin states in a series of measurements

Bayes method superior to threshold method - Optimal fidelity of 88%
Observation of Single Spin Flips

Series of axial frequency measurements in AT
Apply resonant and off-resonant spin flip drives – background check

Algorithms initialized with maximum uncertainty (p=50%)
No extraordinary frequency jumps at off-resonant drives – cyclotron mode not affected

Double Penning trap technique

- Additionally spin-state has to be detected two times
  → Reduction of detection fidelity

- Cyclotron frequency measurement heats cyclotron mode to 30 meV
- Low energies required in analysis trap for high fidelity spin state detection

- Coupling to thermal bath in precision trap
- Preparation of subthermal $E_+$

3 hours for one spin flip trail in precision trap with fidelity of 75%
Demonstration of double Penning trap technique

Measurement:
- Detect spin state - magnetic bottle in analysis trap
- Excite spin transition in precision trap
- Detect spin state - magnetic bottle in analysis trap

After two weeks of data taking
Observation of spin flips excited in the homogeneous magnetic field

Finite spin flip probability for off-resonant drive due to finite spin state detection fidelity

The $g$-factor of the proton

Sweep spin flip excitation frequency to obtain $g$-factor resonance

- To avoid systematic effects spin flip excitation frequency randomly chosen
- Blindfold analysis of axial frequency in analysis trap for spin state detection using Bayesian analyses
- Line width: due to residual $B_2$ in precision trap and saturation

$$g = 5.585694704(14)$$ that is 2.6 ppb
The $g$-factor of the proton

Systematic errors

<table>
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- Octupolar contribution - energy dependent frequency shift (not for Larmor frequency)

\[
\frac{\Delta \omega_z}{\omega_z} = \frac{3}{2} \left( \frac{C_4}{C_2^2} + \frac{5}{4} \frac{C_6}{C_3} \left( \frac{E_z}{qV_0} \right) \right) \frac{E_z}{qV_0}
\]

with $C_4, C_6 \propto TR$

$C_4$ contribution can be optimized to better than $10^{-5}$ hence uncertainty of $\frac{\Delta \omega_z}{\omega_z} = 10^{-9}$ which contribute 0.2ppb to cyclotron frequency via double dip.

Direct cyclotron frequency shift

\[
\frac{\Delta \omega_p}{\omega_p} \propto \left( \frac{\omega_z}{\omega_p} \right)^2 E_z \sim 1 \text{ppt}
\]
The $g$-factor of the proton

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<td>-0.088 ppb</td>
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• Negligible systematic shifts due to
  • Relativistic effects - proton at cryogenic temperature
    \[
    \frac{\Delta \omega_p}{\omega_p} = -\frac{1}{mc^2} k_b T_p
    \]
    Compare electron:
    • Groundstate cooling
    • Cyclotron quantum jump spectroscopy
  • Additional Electrostatic Potential due to image charge - single charge
    \[
    \frac{\Delta \omega_c}{\omega_c} = \left( \frac{\omega_-}{\omega_c} - \frac{\omega_+}{\omega_c} \right) \frac{q^2}{4\pi \varepsilon_0 m r_0^3 \omega_c^2}
    \]
    Compare highly charged ions $^{12}$C$^{5+}$:
    • Dominant systematic shift
    • Increase trap radius $r_0$
The $g$-factor of the proton
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<td>Nonlinear Magnetic Field Drift</td>
<td>0</td>
<td>2 ppb</td>
</tr>
<tr>
<td>Cyclotron Cooling</td>
<td>-0.51 ppb</td>
<td>0.08 ppb</td>
</tr>
<tr>
<td>Voltage Stability</td>
<td>-0.07 ppb</td>
<td>0.35 ppb</td>
</tr>
<tr>
<td><strong>Total Systematic Shift</strong></td>
<td><strong>-0.64 ppb</strong></td>
<td><strong>2 ppb</strong></td>
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- Voltage Stability – accounts for possible drift of applied potentials due to DC-Filters after transport from AT to PT
- In addition at level of $<<$ ppt:
  - Detector damping – damped harmonic oscillator
  - Bloch Siegert Shift – linear polarization of SF-driving field
  - retarded part of image charge found negligible
The $g$-factor of the proton

- First direct and high precision measurement of the proton magnetic moment.
- Improves 42 year old MASER value by factor of 3.3 \( (P. F. \text{ Winkler et al., Phys. Rev. A 5, 83 (1972)}) \)
- Value in agreement with accepted CODATA value, but 2.5 times more precise

\[
g = 5.585 \, 694 \, 700(14)_{\text{stat}}(11)_{\text{sys}}
\]

Letter

Direct high-precision measurement of the magnetic moment of the proton

A. Moosér\textsuperscript{1,2}, S. Ulmer\textsuperscript{3}, K. Blaum\textsuperscript{4}, K. Franke\textsuperscript{2,3}, H. Kracke\textsuperscript{3,2}, C. Leiteritz\textsuperscript{1}, W. Quint\textsuperscript{2,6}, C. C. Rodegerdt\textsuperscript{3,4}, C. Smorra\textsuperscript{2} & J. Walz\textsuperscript{1,2}

Antiproton

- GERN
New Advanced Penning Trap System

Catching trap and reservoir for antiprotons.

Cooling trap: Weak magnetic bottle with cyclotron energy resolution of 5Hz/K. Small trap with improved cooling time constant.

Procedure:
Alternate fast coupling and measuring cycles. Preparation of particle with single spin flip resolution within a few minutes.

Improved detectors:
\[ \sigma \propto \sqrt{\frac{1}{4\pi} \cdot \frac{\Delta v}{t \sqrt{SNR}}} \]
\[ \frac{S}{N} = \frac{\sqrt{4\pi k T R_p}}{e_n} \]
two times faster detection
BASE-CERN Apparatus

- **Experiment was approved in June 2013.**

  Constructed new apparatus

  Developed advanced trapping system

  Constructed antiproton transfer line

  Implemented system into AD facility
Installation

...about 1 and a half year ago there was nothing...
Methods

- Established meanwhile all standard techniques used in the antiproton community:
  - Catching
    - Deceleration of 5.3 MeV antiprotons using degrader foils.
    - Fast HV catching pulses to confine the slow antiprotons up to 5 keV.
  - Electron cooling
    - Electron and resistive cooling to 4 K thermal equilibrium energy
      \(~ 320 \, \mu\text{eV}\)
  - Electron kick-out
  - Trap cleaning
  - Single particle preparation

The Reservoir Trap

Basic idea: serve as antiproton reservoir – survive accelerator shutdown

Initial state: antiproton cloud in trap 1

step 1: separation of particle cloud
step 2: adiabatic transport to second trap

Final state: cloud ion reservoir, single particle in experiment cycle
Realization

- Found that adequate potential ramps are most efficient to perform this scheme.
- Potential-tweezer scheme:
  - All experiments were performed with the same cloud of particles -> also merging of particle clouds works.
  - No particle loss during separation/merging experiments
  - One separation cycle takes only 12s
Noise

- Careful electronics layout: No issues with electrical interference
- Magnetic noise is a pain:

Cyclotron frequency fluctuations of 500ppt – proves magnetic inhomogeneity problems seen in Mainz solved
- With respect to magnetic field stability 500ppt measurement feasible
Conclusion

• Detection of single proton spin transitions
  • Demonstration of double Penning-trap technique
    • Most precise and direct high-precision measurement of proton $g$-factor
• BASE experiment successfully installed
  • Captured and prepared first single antiproton

BASE Collaboration: Stefan Ulmer, Christian Smorra, Takashi Higuchi, Andreas Mooser, Kurt Franke, Peter Koss, Nathan Leefer, Clemens Leiteritz, Hiroki Nagahama, Georg Schneider, Simon Van Gorp, Klaus Blaum, Yasuyuki Matsuda, Christian Ospelkaus, Wolfgang Quint, Jochen Walz, Yasunori Yamazaki
Thank you for your attention