Determination of the fine structure constant with atom interferometry

Pierre Cladé
Fine structure constant:

$$\alpha = \frac{e^2}{4\pi \epsilon_0 \hbar c}$$  \hspace{1cm} (1)

- Magnetic moment of electron: \( g_e = 1 + C_1 \left( \frac{\alpha}{\pi} \right) + C_2 \left( \frac{\alpha}{\pi} \right)^2 + \ldots \)
- Measurement \( g_e \), determination of \( \alpha \), test of QED
Introduction

Rydberg constant:

\[ hR_\infty c = \frac{1}{2} m_e c^2 \alpha^2 \Rightarrow \alpha^2 = \frac{2R_\infty}{c} \times \left( \frac{h}{m_e} \right) \]  

(2)

Mass ratio are well measured: \( m_e \rightarrow m_{Rb} \)


Outlook:

- The “\( h/m \)” constant
- Principle of the experiment
- Discussion on systematic effects
Conversion factor for energies

- \( h\nu = mc^2 \) (Compton frequency)
- \( h\nu = \frac{1}{2} mc^2 \alpha^2 \) (Rydberg frequency)
- \( h\nu = \frac{1}{2} mv_r^2 \) (Recoil frequency)
- \( h\nu = mg\lambda \) (Bloch frequency)

Constant of quantum mechanics

\[
i \frac{\partial \psi}{\partial t} = -\frac{\hbar}{2m} \frac{\partial^2 \psi}{\partial x^2}
\]

(3)

“Diffusion coefficient” of the Schrödinger equation \((m^2.s^{-1})\)
Redefinition of the S.I.

Macroscopic \((M)\) to microscopic experiments \((m\) and \(h)\):

- Watt balance: measurement of \(h/M\)
- Silicon sphere: measurement of \(M/m\)

The link is done with a measurement of \(h/m\)

\[
\frac{h}{m} [m^2.s^{-1}] \times 10^{15} \approx 3.99031271 \times 10^8
\]

After the redefinition: link between AMU and SI.
Dispersion relation

Non relativistic equations:

\[ i \frac{\partial \psi}{\partial t} = -\frac{\hbar}{2m} \frac{\partial^2 \psi}{\partial x^2} \Rightarrow \omega = \frac{\hbar}{2m} k^2 \]  

(4)

- Second derivative
- \( k_A \) and \( k_B \): photon recoil
- Atom recoil
- Doppler effect
- Atom interferometer
Atom light interaction

\[ |b\rangle \]
\[ E = h \nu \]
\[ p = \pm \hbar k \]
\[ |a\rangle \]

\[ m \]
\[ \nu_1 \]

\[ \nu_r = \hbar k / m \]
Atom light interaction

\begin{align*}
|b\rangle & \quad \downarrow \quad 1 \\
E &= h\nu \\
p &= \pm \hbar k \\
|a\rangle & \quad \uparrow \quad 2 \\

\text{Spontaneous emission}
\end{align*}

$\nu = ??$
Atom light interaction

\[ |b\rangle \quad E = h\nu \quad p = \pm \hbar k \]

\[ |a\rangle \quad 1 \]

\[ m \quad \nu_1 \]

\[ \nu_r = \frac{\hbar k}{m} \]
Atom light interaction

Two photon transition to suppress spontaneous emission.

Same internal state

Two different internal states
Introduction to atom interferometry

Doppler effect

\[ \delta = (\vec{k}_1 - \vec{k}_2) \cdot \vec{v} \]

■ Selection of a subrecoil velocity distribution
Introduction to atom interferometry

Doppler effect

\[ \delta = (\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{v} \]

- Selection of a subrecoil velocity distribution
- Measurement of the final velocity distribution

Blow away beam

\[ \delta_{\text{sel}} \quad \delta_{\text{meas}} \]
Introduction to atom interferometry

Doppler effect

\[ \delta = (\vec{k}_1 - \vec{k}_2) \cdot \vec{v} \]

- Selection of a subrecoil velocity distribution
- Measurement of the final velocity distribution
Atom interferometry

Rabi

\[ \pi \]

Horizontal and vertical blow away beam

10 ms 10 ms

Ch. J. Bordé (1984 → ...)
Atom interferometry

Method of Separated Oscillatory Field (Ramsey)
Replace a $\pi$ pulse by two $\pi/2$ pulses.
Atom interferometry

Rabi

Method of Separated Oscillatory Field (Ramsey)
Replace a $\pi$ pulse by two $\pi/2$ pulses.

Ch. J. Bordé (1984→...)}
Measuring velocities

Horizontal and vertical blow away beam

\[ \delta_{\text{sel}} \]

\[ \delta_{\text{meas}} \]
Measuring velocities

Horizontal and vertical blow away beam

Raman

\[ \delta_{\text{sel}} - \delta_{\text{mes}} \]

100 Hz

\[ t \]

\[ \delta_{\text{sel}} \quad \delta_{\text{meas}} \]

Raman

10 ms 10 ms

100 Hz

\[ \delta_{\text{sel}} - \delta_{\text{mes}} \]
Coherent acceleration of atoms

Succession of stimulated Raman transitions
(same hyperfine level)

\[ \delta = \nu_1 - \nu_2 \propto t \]

Adiabatic passage: acceleration of the atoms

The atoms are placed in an accelerated standing wave: in its frame, they are submitted to an inertial force

→ Bloch oscillations in a periodic potential

(group of C. Salomon, LKB, Paris)
Apparatus

Magnetic shield

2D MOT

3D MOT

Bloch/Raman Beams

Micro-wave

Detection
6.834 GHz + δ + 2kgt

**Beat note**

- 80 MHz
- AOM
- Magnetic shield
- Bloch/Raman Beams
- Micro-wave
- Raman1
- Raman1
- 2D MOT
- 3D MOT
- AOM
- AOM
- AOM
- Detection
- Bloch/Raman

**Apparatus**

P. Cladé Determination of $\alpha$

Feb. , 2015
Measuring velocities

Horizontal and vertical blow away beam

Raman

10 ms

Raman

10 ms

$\delta_{\text{sel}} - \delta_{\text{mes}}$

100 Hz

$t$

$\delta_{\text{sel}}$

$\delta_{\text{mes}}$

$\delta_{\text{sel}} - \delta_{\text{mes}}$

100 Hz
Measuring velocities

Horizontal and vertical blow away beam

Raman 10 ms 10 ms Bloch Acceleration

\( \delta_{\text{sel}} \)  \( N = 500 \)

\( \delta_{\text{meas}} \)
Measuring velocities

Horizontal and vertical blow away beam

\[ N = -500 \quad \delta_{\text{sel}} \quad N = 500 \]

Acceleration \quad Deceleration

\[ \delta_{\text{meas}} \]
Measuring velocities

Horizontal and vertical blow away beam

\[ N = \pm 300 \quad \text{Elevator} \]
\[ N = -500 \quad \text{Acceleration} \]
\[ \delta_{\text{sel}} \]
\[ N = 500 \quad \text{Deceleration} \]
\[ \delta_{\text{meas}} \]
Measuring velocities

Horizontal and vertical blow away beam

\[ N = \pm 300 \]
Elevator

\[ N = -500 \]
Acceleration

\[ N = 500 \]
Deceleration

Raman

Bloch

\[ \delta_{\text{sel}} - \delta_{\text{mes}} \]
100 Hz

\[ 10 \text{ ms} \]

\[ \delta_{\text{sel}} - \delta_{\text{mes}} \]

BLB
170 measurements (14 hours)

Each measurement: $6 \times 10^{-9} \, (h/m)$ and $3 \times 10^{-9} \, (\alpha)$

Relative uncertainty on $h/m$: $4.4 \times 10^{-10}$ and $2.2 \times 10^{-10}$ on $\alpha$. 
Results (2010)

**Improvement of statistics**

- **Vibration isolation**

- **Reliability of the whole experiment (total time of 1000 h)**

Relative uncertainty on $h/m$ of $5 \times 10^{-9}$ in 3 mn.
### Error Budget ($\alpha$)

<table>
<thead>
<tr>
<th>Source</th>
<th>Correction</th>
<th>Uncertainty $10^{-10}$</th>
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Wavefront curvature and Gouy phase shift

What is the momentum of a photon? 

\[ p = \hbar \frac{\partial \phi}{\partial z} \text{ where } \phi \text{ is the phase of the laser beam.} \]

\[ p = \hbar k \text{ holds only for perfect plane-wave.} \]

For a Gaussian beam:

\[ \frac{\partial \phi}{\partial z} = k + \frac{1}{2k} \left( \frac{4}{w^2} - \frac{4r^2}{w^4} + \frac{r^2k^2}{R^2} \right) \]  \hspace{1cm} (5)

where \( w \) is the waist of the beam, \( R \) the wavefront curvature and \( r \) the distance from the propagation axes of the beam.

- We are now using a larger beam waist (smaller Gouy phase shift, better alignment)
SHG fibered laser system

- Laser at 1560 nm
- Amplifier : 30 W
- Freq doubling : PPLN crystal

Power at 1560.36 nm [%]

Crystal temperature : 79.95°C

Power at 780.18 nm [W]
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Magnetic field

- Mapping of the magnetic field
  - Spectroscopy (on \( m_F = 1 \)) at different positions

![Graph showing Zeeman effect vs position in cell and delay](image)

- Calculation (\( m_F = 0 \)) of the effect on each spectra and for \( h/m \)
  - Effect: 0.1 ppb
  - Measurement at different bias field
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### Current status

- Better statistics
- Improved: Gouy phase shift
- Improved: Beam alignment
- Improved: 2nd order Zeeman effect

However, we have a systematic effect correlated to the intensity of the laser used for Bloch oscillations that we don’t understand yet.
New project

High precision atom interferometry using large momentum transfer beamsplitter

- Evaporative cooling (technique used for BEC)
- Dipole trap
- Shot noise limited detection
- Vibration isolation
- Magnetic shield
- Optical frequency measurement
- Wavefront design

Design of a setup to reach the $2 \times 10^{-10}$ accuracy

Main systematic effects:

- Atom-atom interaction
- Wavefront curvature and Gouy phase shift
Ph.D Students
P. Cladé (2005)
M. Cadoret (2008)
R. Bouchendira (2012)
M. Andia
R. Jannin
C. Courvoisier

Postdoc
E. de Mirandes (2006-2007)

Permanents
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L. Julien
P. Cladé
S. Guellati-Khéïlifa
F. Nez
F. Biraben