The uncertainty in frequency
&
The prediction uncertainty

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Introduction

The CCTF has declared [*UTC*-*UTC*(*k*)] as published in monthly *BIPM Circular T* as the only Key Comparison in the Time and Frequency field.

The values of [*UTC*-*UTC*(*k*)] are reported at five-day intervals with their respective combined uncertainty values constant in a month.

From this, the corresponding deviation for frequency and its uncertainty are therefore available at 5 days.

The frequency uncertanty

The frequency uncertainty will be thus obtained by applying the law of uncertainty propagation on the mean frequency defined as:

$$\bar{y} = \frac{(UTC - UTC(k))_t - (UTC - UTC(k))_{t-\tau}}{\tau}$$

$$u_{\bar{y}}^2 = \frac{u^2_{(UTC-UTC(k))_t} + u^2_{(UTC-UTC(k))_{t-\tau}} - 2Cov\left((UTC - UTC(k))_t, (UTC - UTC(k))_{t-\tau}\right)}{\tau^2}$$

Considering the white phase noise and that the uncertainty of the difference [*UTC*-*UTC*(*k*)] remains constant over the whole period:

$$u_{\bar{y}}^2 = \frac{2(uA)^2}{\tau^2}$$

τ is minimum 5 days

Only the statistical component uA of the uncertainty of [*UTC*-*UTC*(*k*)] should be used considering that the calibration uncertainty (represented by the systematic uncertainty uB) does not affect the frequency measurements.

Discussion

The BIPM can gives results based on the information obtained from *Circular T*.

On the other hand the laboratories receive the result of [*UTC*-*UTC*(*k*)] and their uncertainty with a delay of about 15 days after the last date of data in a month, and in this period (45 days from the beginning of the month of data) they are not aware of the difference between the UTC and the local time scale UTC(k) and they neither know the uncertainty that should be declared.

In the case they cannot wait until the publication of *Circular T* to have the final values, they should add a “prediction component” to the global budget of frequency uncertainty. However, if the laboratories wait for the *Circular T* results they know the uncertainty information.

The prediction uncertainty

In the framework of the CCTF-K001.UTC the evaluation of the prediction uncertainty of UTC-UTC(k) has been declared. The time interval considered to declare the prediction uncertainty is 20 days.

The values declared for the prediction uncertainty from the different laboratories at 20 days show a big variation; from 20 ns to 200 ns.

A study started to indicate reasonable values for the prediction uncertainty of UTC-UTC(k) depending on the clock used and on the time transfer method.

Uncertainty components

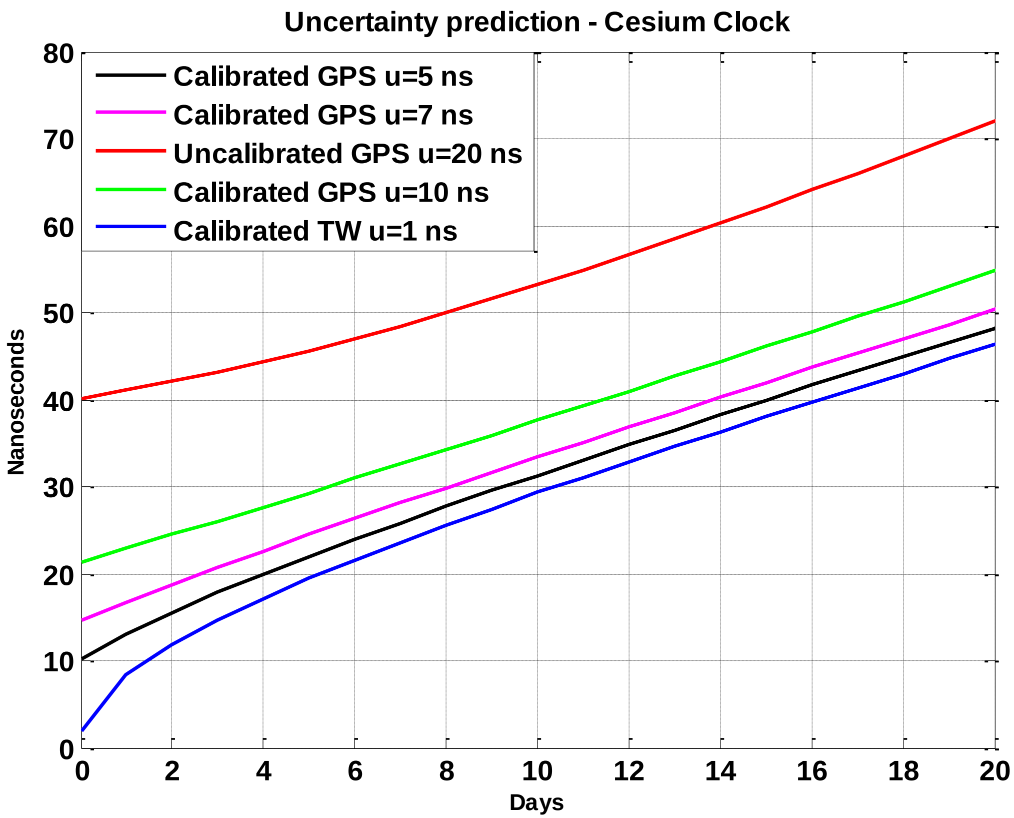
We consider a time scale generated by using free running Cesium clocks and H-masers:

The prediction uncertainty is depending on:

- The uncertainty on [UTC-UTC(*k*)] declared in the first section of Circular T
- The stability property of the atomic clock considered

The uncertainties reported in the first section of Circular T are linked to the link uncertainties reported in Section 6.

Prediction Uncertainty for Cs-clock



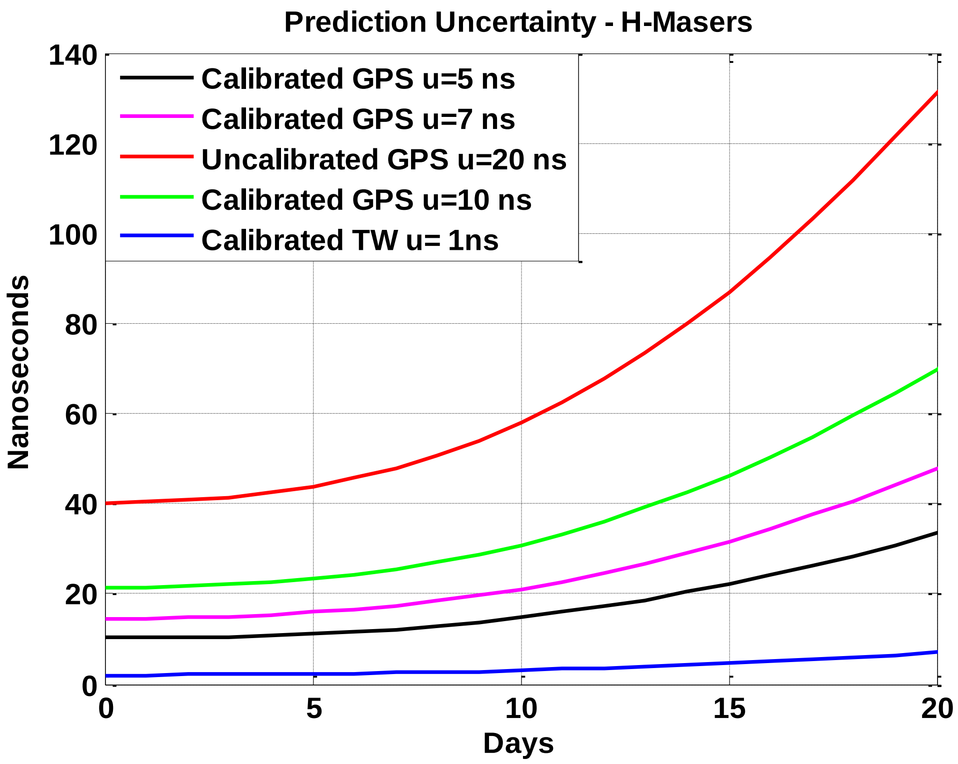
The proposed values

Measurement Uncertainty on UTC-UTC(k) / [ns]	Prediction uncertainty (2 sigma) at 20 days / [ns]	Prediction uncertainty (2 sigma) at 20 days / [ns]
1	12	81
5	19	83
7	24	84
10	32	87
20	57	98
	$\sigma_y(\tau) = 1 \times 10^{-14} / day$	$\sigma_y(\tau) = 7 \times 10^{-14} / day$

By analysing the values reported we observe the role of the clock generating the internal realization of UTC.

When UTC(*k*) is realized by a clock with good stability the prediction uncertainty is dependent on the time transfer performance; the contribution to the uncertainty of the time transfer is almost negligible when the clock is characterized by a larger instability.

Prediction uncertainty - H-masers



The proposed values

$\sigma_{WFN}(\tau) = 1\times10^{-15} / day$

$\sigma_{RWFN}(\tau) = 1\times10^{-16} / day$

Measurement Uncertainty on UTC-UTC(k) / [ns]	Prediction uncertainty (2 sigma) at 20 days / [ns]	Prediction uncertainty (2 sigma) at 20 days / [ns]
1	7	25
5	34	41
7	48	54
10	71	74
20	132	134

$\sigma_{WFN}(\tau) = 1\times10^{-14} / day$

$\sigma_{RWFN}(\tau) = 1\times10^{-15} / day$

When a good time transfer technique is used the atomic clocks has a significant impact on the uncertainty prediction but with an un-calibrated technique the results do not depend on the stability of the clock.