Acoustic Gas Thermometry
outline

Acoustic Gas Thermometry

historical development
- development of theory: Laplace was finally right
- wide ranging applications of acoustic gas thermometry ...
- ... including primary temperature standards

milestones along a successful road
- theory and practice of a sphere
- son et lumiere: advantages of the new SI

results
- determinations of the molar gas constant $R$ and the Boltzmann constant $k$
- determinations of $T - T_{90}$

ongoing work and future perspectives
- extending the temperature range of primary acoustic thermometers
- alternatives for dissemination: simplification of primary methods
- practical acoustic thermometers
To try exactly the time wherein sound is propagated

[...] let a man stand in a steeple, with a candle, veiled; and let another man stand a mile off: then let the person in the steeple strike a bell and at the same instant withdraw the veil; the other, at a distance, may measure the time between the light seen and the sound heard, for light is propagated instantaneously*

*Or what comes very near thereto, for in the space of seven or eight minutes it is thought by some to travel from the sun to the earth.

---

1627 - Francis Bacon

gedanken experiment

1635 - Pierre Gassendi

478 m/s

1656 - Viviani, Borelli

360 m/s

1708 - Derham, Flansted, Halley

348 m/s

Marin Mersenne 1636

448 m/s

transient method

steady-state method

at 20 °C, 1 atm, 50% relative humidity

speed of sound in air

344 m/s
Newton (isothermal)

\[ T = \text{const} \]

\[ \kappa_T = \frac{1}{p} \]

Laplace (adiabatic)

\[ S = \text{const} \]

\[ dQ = 0 \]

wavelength \( \equiv \lambda \gg l \equiv \text{mean free path} \)

1687 Newton

1st ed. *Philosophiae Naturalis Principia Mathematica*

pendulum analogy (elasticity/density)

\[ c^2 = \frac{p}{\rho} = \frac{1}{\kappa_T \rho} \]

(air) \( 295 \text{ m/s}^2 \)

1727 Euler

\[ c^2 = \frac{4}{\pi} \frac{p}{\rho} \]

(air) \( 375 \text{ m/s}^2 \)

1759 – Lagrange

generic acoustic perturbation

\[ u^2 = \left( \frac{p}{\rho} \right) \]

1816 Laplace

\[ c^2 = \gamma \frac{p}{\rho} = \frac{1}{\kappa_S \rho} \]

\[ \gamma = \frac{c_p}{c_v} \]

Experiment

1627 – Francis Bacon

proposal of virtual experiment

1636 – Mersenne

sources -> guns, musical instruments

time meas -> pulse beat

448 m/s

1656 – Viviani, Borrelli (Galilei’s disciples)

guns, pendulum

361 m/s

1708 – Derham, Flanstead, Halley

account for wind influence

348 m/s

1738 – Cassini de Thury

exchange source-receiver

measure of temperature

337 m/s

1868 – Regnault

Paris underground, in pipelines of different diameter, extrapolating results to free field, dry air at 0 °C

330.7 m/s

1953 – Hardy, Smith

dry air, 0 °C, 100 kPa

331.45 m/s
versatility of Laplace equation

\[ c^2 = \left( \frac{\partial p}{\partial \rho} \right)_S = \frac{1}{\kappa S \rho} \]

\[ \kappa_S = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_S \]

\[ \kappa_S = \frac{1}{\gamma \rho} \]

Longitudinal waves

bulk modulus

\[ c_L^2 = \left( \frac{\partial p}{\partial \rho} \right)_S = \frac{1}{\kappa_S \rho} = \frac{K}{\rho} \]

\[ c_L^2 = \frac{G}{\rho} \]

Shear waves

Shear modulus

\[ c_T^2 = \left( \frac{\partial p}{\partial \rho} \right)_S = \frac{G}{\rho} \]
Acoustic Thermometry

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(Received July 3, 1961; and in final form, January 8, 1962)

Fig. 2. Oscilloscope trace of voltage across transceiver. The transmitted burst occurs during the blank space and is followed by the reflected and "ringing" signals. From the decay of the ringing signal the cavity Q can be determined.

Fig. 5. Pictorial representation of experiment.

Simultaneous Ultrasonic and Line Reversal Temperature Determination in a Shock Tube

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Parametrics, Inc., Waltham, Massachusetts  
and  
G. W. Wars

Air Force Cambridge Research Laboratories, Bedford, Massachusetts

Fig. 7. Temperatures in reflected shock region in pure neon obtained by ultrasonics.
sonic anemometry (& thermometry)

Sonic anemometers are sensors that are able to estimate wind speed vector by measuring the influence of local wind speed on the transmission of ultrasound signals between pairs of emitters and receivers that configure acoustic paths. This estimation is normally assigned to the geometric center of acoustic path midpoints.

Sonic anemometers are used to evaluate the turbulent heat flux from the sonic kinematic heat flux (or “temperature flux”), i.e. the covariance of the vertical wind velocity $w$ and the sonic temperature $T$, the vertical wind velocity $w$ and the speed of sound $c$ being simultaneously measured by the anemometer, and the speed of sound being then corrected for the effect of the crosswind.

\[
\begin{align*}
T &= u^2 \frac{M}{\gamma R} \\
\end{align*}
\]

sonic kinematic heat flux

\[
\begin{align*}
T' &= w' T' \\
\end{align*}
\]

turbulent heat flux
Acoustic Thermometry of Ocean Climate (ATOC) precision 50 ms – accuracy 0.1 s = 0.05 K

3.3 kW at 57 Hz
206 dB

FIG. 1. Ray paths from source to receiver sites are refracted geodesics, i.e., great circles corrected for Earth flattening and horizontal sound speed gradients. The source array was suspended from R/V CORY CHOUEST 50 km southeast of Heard Island. Single dots indicate sites with single receivers. Dots connected by horizontal lines designate horizontal bottom-mounted arrays, vertical lines designate vertical arrays, and slanted lines designate arrays towed in the direction of the arrow. Signals were received at all sites except for the vertical array at Bermuda (which sank) and the Japanese station off Samoa.
**Abstract:** Measuring temperature changes of the deep oceans, important for determining the oceanic heat content and its impact on the Earth’s climate evolution, is typically done using free-drifting profiling oceanographic floats with limited global coverage. Acoustic thermometry provides an alternative and complementary remote sensing methodology for monitoring fine temperature variations of the deep ocean over long distances between a few underwater sources and receivers. We demonstrate a simpler, totally passive (i.e., without deploying any active sources) modality for acoustic thermometry of the deep oceans (for depths of ~500–1500 m), using only ambient noise recorded by two existing hydroacoustic stations of the International Monitoring System. We suggest that passive acoustic thermometry could improve global monitoring of deep-ocean temperature variations through implementation using a global network of hydrophone arrays.

(A) Locations of the two hydroacoustic stations (red dots) near Ascension and Wake Islands. (B) Zoomed-in schematic of the hydrophone array configurations for the Ascension and Wake Island sites, which both have a similar layout. Each hydroacoustic station consists of a northern and southern triangle array of three hydrophones (or triad), with each triangle side equal to approximately 2 km. The distance L between triad centers is equal to 126 km and 132 km for the Ascension and Wake Island stations, respectively. (C) Noise cross-correlation waveforms, averaged over three different years, obtained between all 9 pairwise combinations of the elements of the north and south triads for the Wake Island. (D) Same as C, but for Ascension Island. The beams shown by dashed lines in A are centered on the lines joining the centers of the south and north triads of each hydroacoustic station (yellow line) and which intersect the Polar regions where potential ice noise sources contributing to the coherent arrivals shown in C-D are located (18b).
ideal gas assumptions:
- the molecules of the gas are indistinguishable, small, hard spheres
- collisions are elastic and motion is frictionless (no energy loss)
- Newton’s laws apply
- average distance between molecules is much larger than molecular size
- the molecules constantly move in random directions with a distribution of speeds
- there are no attractive or repulsive forces between the molecules

ideal gas equation of state
\[ p = \rho RT / M \]

mean kinetic energy
\[ \frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} kT \]

\[ u_0^2(T) = \left( \frac{\partial p}{\partial \rho} \right)_s = \gamma_0 \frac{kT}{m} = \gamma_0 \frac{RT}{M} = \frac{1}{3} \gamma_0 \frac{v_{\text{rms}}^2}{\text{molecular mass}} \frac{1}{\text{molar mass}} \]

\[ \gamma_0 = 5/3 \]

monoatomic gas

\[ u_0^2 \approx \frac{v_{\text{rms}}^2}{2} \]

1873

ambien to 1000 °C

Alfred Marshall Mayer

Philosophical magazine

Wheatstone revolving mirror (1834)

Koenig flame manomter (1862)
Acoustical Thermometer

ultrasonic interferometer provides a new method for the precise measurement of low temperature.

Harmon H. Plumb and George Cataland

1965

$2 \, K + 20 \, K$

repeatability 3 mK

Fig. 4. A comparison, with values of the $T_a$ scale, of temperatures derived from (i) pressure-volume isotherm measurements, (ii) gas thermometer measurements, and (iii) acoustical-thermometer measurements (19). (Open circles, plain or bisected) Gas-thermometer values [Schmidt and Keesom (20)]; (solid circles) gas-thermometer values [Kistemaker (21)]; (crosses) isotherm values [Kistemaker (21)]; (open triangles) gas-thermometer values [Berman and Swenson (22)]; (open squares) He$^4$ isotherm values [Keller (22)]; (solid squares) He$^3$ isotherm values [Keller (22)]; (solid triangles) preliminary acoustical-thermometer values [Cataland et al. (24)]; (solid diamonds) acoustical-thermometer values obtained in the work described. $T_a$ represents actual values of temperature as determined by the experimenters.

Fig. 5. A comparison between the acoustically derived temperature scale [NBS Provisional Scale 2-20 (1965)] and the NBS (1955) temperature scale (25). The difference in the values of the two scales, as indicated by two germanium resistors, is plotted as a function of the Kelvin temperature.
Gas-filled spherical resonators: Theory and experiment

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Martin Greenspan(a)
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(Received 9 August 1985; accepted for publication 20 October 1985)

\[ u = \frac{2\pi a}{z_N} \left( f_N - \Delta f_{\text{thermal}} - \Delta f_{\text{viscous}} - \Delta f_{\text{shell}} - \Delta f_{\text{ducts}} \right) \]

\[ g_N = g_{\text{bulk}} + g_{\text{thermal}} + g_{\text{viscous}} + g_{\text{ducts}} \]

Measurement of the Universal Gas Constant R Using a Spherical Acoustic Resonator

M. R. Moldover, J. P. M. Trusler, (a) and T. J. Edwards (b)
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J. B. Mehl
Physics Department, University of Delaware, Newark, Delaware 19716

and

R. S. Davis
Length and Mass Division, National Bureau of Standards, Gaithersburg, Maryland 20899

(Received 6 November 1987)

\[ k = \frac{R}{N_A} = (1.3806513 \pm 0.0000025) \times 10^{-23} \text{ J mol}^{-1} \text{ K}^{-1} \quad (1.8 \text{ ppm}) \]
Measurement of the ratio of the speed of sound to the speed of light

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Michael R. Moldover
Thermophysics Division, National Bureau of Standards, Gaithersburg, Maryland 20899
(Received 2 June 1986)

\[ u(p,T) = f_{ac}(p,T) \frac{2\pi a(p,T)}{z_{ac}} \]
\[ c(p,T) = \langle f_{mw}(p,T) \rangle \frac{2\pi a(p,T)}{z_{mw}} \]
\[ \frac{u(p,T)}{c(p,T)} = \frac{f_{ac}(p,T)}{\langle f_{mw}(p,T) \rangle} \frac{z_{mw}}{z_{ac}} \]

\[
\begin{align*}
[\text{temperature}] &= \frac{[\text{energy}]}{[k_B]} = \frac{[\text{mass}][\text{velocity}]^2}{[k_B]} = \frac{[\text{mass}]}{[k_B]} c_0^2 \lim_{p \to 0} \frac{[\text{frequency}]^2}{[\text{frequency}]^2_{\text{light}}} \end{align*}
\]
Thermodynamic Temperatures of the Triple Points of Mercury and Gallium and in the Interval 217 K to 303 K

M.R. Moldover, S.J. Boyes, C.W. Meyer, and A. R. H. Goodwin

\[ \frac{T}{T_{TPW}} = \lim_{p \to 0} \left( \frac{u^2(p, T)}{u^2(p, T_{TPW})} \right) \]

Table 2. Standard uncertainties \( u \times 10^6 \) from various sources in the re-determination of \((T - T_0)/T\) is calculated twice: first, including Rows 4 and 5, but not Rows 6 and 7; and second, including

<table>
<thead>
<tr>
<th>Source</th>
<th>217 K</th>
<th>234 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microwave values for ([\sigma(T)/\sigma(T_w)]^2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Discrepancies among triplets</td>
<td>0.32</td>
<td>0.27</td>
</tr>
<tr>
<td>2. (\delta_{\sigma}(T)) calc. from resistivity ((0.04 \times p))</td>
<td>0.39</td>
<td>0.25</td>
</tr>
<tr>
<td>3. (\delta_{\sigma}(T)) calc. (-\delta_{\sigma}(T)) meas.</td>
<td>0.24</td>
<td>0.24</td>
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</tbody>
</table>

Acoustic (isotherm fits)

<table>
<thead>
<tr>
<th></th>
<th>217 K</th>
<th>234 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. Uncertainty of (A_0(T_w)/\alpha^2)</td>
<td>1.67</td>
<td>1.67</td>
</tr>
<tr>
<td>5. Uncertainty of (A_0(T)/\alpha^2)</td>
<td>2.31</td>
<td>1.85</td>
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Acoustic (surface fit)

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<th></th>
<th>217 K</th>
<th>234 K</th>
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</thead>
<tbody>
<tr>
<td>6. Uncertainty of (A_0(T_w)/\alpha^2)</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>7. Uncertainty of (A_0(T)/\alpha^2)</td>
<td>0.52</td>
<td>0.38</td>
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Thermometry

<table>
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<th>217 K</th>
<th>234 K</th>
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</thead>
<tbody>
<tr>
<td>8. SPRT &amp; bridge repeat. @ (T_w) ((10 \mu\Omega))</td>
<td>0.46</td>
<td>0.43</td>
</tr>
<tr>
<td>9. Difference between calibrations</td>
<td>0.22</td>
<td>0.15</td>
</tr>
<tr>
<td>10. Temperature gradient</td>
<td>0.46</td>
<td>0.43</td>
</tr>
<tr>
<td>11. Non-uniqueness of ITS-90</td>
<td>0.9</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Additional sources

<table>
<thead>
<tr>
<th></th>
<th>217 K</th>
<th>234 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. Thermal conductivity ((0.3 %))</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>13. Uncertainty of pressure zero</td>
<td>0.21</td>
<td>0.12</td>
</tr>
<tr>
<td>15. Surface fit: RSS</td>
<td>1.42</td>
<td>0.92</td>
</tr>
<tr>
<td>16. Isotherm fits: RSS (\times (T/mK))</td>
<td>0.68</td>
<td>0.61</td>
</tr>
<tr>
<td>17. Surface fit: RSS (\times (T/mK))</td>
<td>0.31</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Primary acoustic thermometry between $T = 90\, \text{K}$ and $T = 300\, \text{K}$

M. B. Ewing and J. P. M. Trusler

$90\, \text{K}$ to $300\, \text{K}$

FIGURE 1. The spherical resonator and its thermal environment: 1, spherical resonator; 2, isothermal shield; 3, vacuum jacket; 4, thermometer mount; 5, copper block; 6, copper block; 7, inlet tube; 8, to vacuum; 9, to vacuum gauge.
Progress in Primary Acoustic Thermometry at NIST: 273 K to 505 K

G. F. Strouse, D. R. Defibaugh, M. R. Moldover, and D. C. Ripple

TABLE 3. NIST Acoustic Thermometer uncertainty budget in millikelvins for the determination of $T - T_{90}$.

<table>
<thead>
<tr>
<th></th>
<th>Ga MP</th>
<th>In FP</th>
<th>Sn FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>303 K</td>
<td>0.6</td>
<td>1.5</td>
<td>3.0</td>
</tr>
<tr>
<td>430 K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>505 K</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$u_k = (T - T_{90}) / \text{mK}$

FIGURE 1. Schematic of the NIST acoustic thermometer inside the pressure vessel.

FIGURE 3. $T - T_{90}$ and corresponding uncertainty ($k = 1$) values for the NIST Acoustic Thermometer and results from the literature.
Acoustic measurements of the thermodynamic temperature between the triple point of mercury and 380 K

G Benedetto$^1$, R M Gavioso$^1$, R Spagnolo$^1$, P Marcarino$^2$ and A Merlone$^2$

$^1$ Acoustics Department, IEN, Istituto Elettrotecnico Nazionale Galileo Ferraris, Torino, Italy
$^2$ Thermometric Division, IMGC-CNR, Istituto di Metrologia G Colonnelli, Torino, Italy
Quasi-spherical cavity resonators for metrology based on the relative dielectric permittivity of gases

Eric F. May, a) Laurent Pitre, b) James B. Mehl, c) Michael R. Moldover, d) and James W. Schmidt
Process Measurement Division, National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8360
(Received 20 April 2004; accepted 23 June 2004; published 22 September 2004)

\[
\frac{\sigma f}{f} \approx 1 \cdot 10^{-8}
\]
precision

\[
\frac{\sigma f}{f} < 1 \cdot 10^{-6}
\]
accuracy
Acoustic thermometry: new results from 273 K to 77 K and progress towards 4 K

Laurent Pitre$^{1,2}$, Michael R Moldover$^1$ and Weston L Tew$^1$

Figure 1. Cylindrical and planar portions of a quasi-sphere.

Acoustic Thermometry Results from 271 to 552 K

D. C. Ripple • G. F. Strouse • M. R. Moldover

Fig. 1 Deviation of measured thermodynamic temperature $T$ from $T_{90}$ for recent acoustic determinations and the constant-volume gas thermometry (CVGT) of Edsinger and Schooley [4] and Schooley [5]. For the present work, standard uncertainties are only shown for the recommended Surface Fit.
Determination of the Boltzmann constant for the redefinition of the kelvin

F. A. Giuliano Albo · R. Cuccaro · L. Pitre · D. Truong

Characterization of the volume and shape of quasi-spherical resonators using coordinate measurement machines

M. de Podesta1, E. F. May2, J. B. Meli3, L. Pitre4, R. M. Gavilano1, G. Benedetto1, P. A. Giuliano Albo5, D. Truong1 and D. Hack5

(ka')² - \(\frac{\xi}{\xi^2}\)
composition (including isotopes) of test gases

$$\frac{3}{2} kT = \frac{1}{2} m v_{rms}^2$$

liquid He cold trap

2010

Contents list available at ScienceDirect
International Journal of Mass Spectrometry

Preparation of argon Primary Measurement Standards for the calibration of ion current ratios measured in argon

S. Valkiers a, *, D. Vandelbo c, M. Berglund c, M. de Podesta b

a Institute for Reference Materials and Measurements, EC-JRC, B-2200 Herentals, Belgium
b National Physical Laboratory, Hampton Road, Teddington TW11 8HJ, UK

2015

Improving acoustic determinations of the Boltzmann constant with mass spectrometer measurements of the molar mass of argon

Inseok Yang 1, Laurent Pitre 2, Michael R Moldover 3, Jintao Zhang 3, Xiaojuan Feng 4 and Jin Seog Kim 4

KRISS
Korea Research Institute of Standards and Science

June 2017

$$u_r(M_{Ar}) \leq 0.1 \text{ ppm}$$

$$0.03 \times 10^{-6} < \frac{3}{4} \text{He} < 1 \times 10^{-6}$$

$$\Delta M/M < 0.25 \times 10^{-6}$$

$$u_r(M_{Ar}) \approx 0.6 \text{ ppm}$$
**Figure 1:** Relative uncertainty in the value of Boltzmann’s constant. Also shown is the relative reproducibility of the practical temperature scales, in the vicinity of 100 °C.
Determinations of $k$ with AGT and other methods 2017 (yesterday).

- $u_r(k) = 0.59 \times 10^{-6}$
- $u_r(k) \approx 7.5 \times 10^{-6}$
- Johnson noise thermometry (JNT)
- AGT with cylindrical cavities $u_r(k) \approx 2 \times 10^{-6}$
- $u_r(k) = 0.70 \times 10^{-6}$
- $u_r(k) = 1.95 \times 10^{-6}$
2012 - 2015 EMRP Research Project
Implementing the new kelvin - InK

2012 - 2017 AGT data

(T - T_{90}) / mK
Temperature / K
2016 - 2019 EMRP Research Project
Implementing the new kelvin 2 - InK2

**T-T\textsubscript{90} by Acoustic Gas Thermometry (AGT) between 430 K and 933 K**

Inconel pressure vessel 1 MPa @ 1000 K
2016 -2019 EMRP Research Project
Implementing the new kelvin 2 - InK2

T-T\textsubscript{90} by Acoustic Gas Thermometry between 5 K and 200 K

Benefits
- 2-stage pulse-tube cryostat \(\rightarrow\) no LHe
- 2 thermal shields and 2 vacuum chambers
- minimum temperature < 4 K
- same calorimeter for \(T\) and \(T\textsubscript{90}\) realization
- houses a larger number of CSPRTs
- provides good stability especially in ranges 30 K – 77 K and 150 K – 234 K

Risks and drawbacks
- vibrations could affect measurements
- complex design, needed long realization time
extending the temperature range of primary thermometers

AGT at $T < 4$ K

The cold valve

The prototype sphere

The Cryogenic Current Comparator Amplification and its shield
termination of acoustic and microwave waveguides

\[ u_0^2(p, T) = u^2(p, T) - \left[ A_1(T) p + A_2(T) p^2 + \ldots \right] \]

measured \quad \text{calculable for He}

- material: \textbf{copper}; shape: \textbf{triaxial ellipsoid}; internal radius: 4 cm; internal volume: 260 cm\(^3\); thick wall to minimize shell coupling; the cavity is designed to be vacuum- and pressure tight
- excitation of \textbf{acoustic} and \textbf{microwave} resonances by \textbf{waveguides}
- embedded thermometer \textbf{wells} for \textbf{cSPRTs} and \textbf{long-stem SPRTs}
- working gas: \textbf{helium} (calculable properties) purity maintained by a getter
- temperature range of initial tests: \textbf{230 K to 430 K};
- aimed accuracy: \( \pm 5 \text{ ppm} \)

alternatives for dissemination: simplification of primary methods
Practical Acoustic Thermometry with Acoustic Waveguides

M. de Podesta · G. Sutton · R. Underwood · S. Legg · A. Steinitz

int J Thermophys
DOI 10.1007/s10765-010-0793-x

calibration zone with Peltiers

Microphone and Loudspeaker in stabilised zone