Microwave frequency standards and techniques

G. D. Rovera

BNM-SYRTE Observatoire de Paris

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The most commonly encountered method for characterization of the spectral purity of a signal is the phase noise spectral density $S_\varphi(\omega)$.

It is defined as the Fourier transform of the autocorrelation of the phsae noise process $\varphi(t)]$

The $S_\varphi(\omega)$ of a metrological quality oscillator has usually well defined slope, corresponding to a particular noise process.

A sinusoidal signal that is only affected by phase modulation noise can be written as:

$$v(t) = A_0 \sin\left[\omega_0 t + \varphi(t)\right] \qquad (1)$$

Only the total accumulated phase can be multiplied, producing a signal $v_n(t) = A_n \sin\left[n\omega_0 t + n\varphi(t)\right]$.

RF spectrum of a phase noise modulated carrier [Hal71, GL98]

type of noise	spectrum	linewidth grows as
White PM <<1 rad^2	dirac+pedestal	constant
White PM >>1 rad^2	only pedestal	n^2
White FM	lorenzian / gaussian	n^2 / n
Flicker FM	not defined	n
Random walk FM	not defined	n$^{2/3}$

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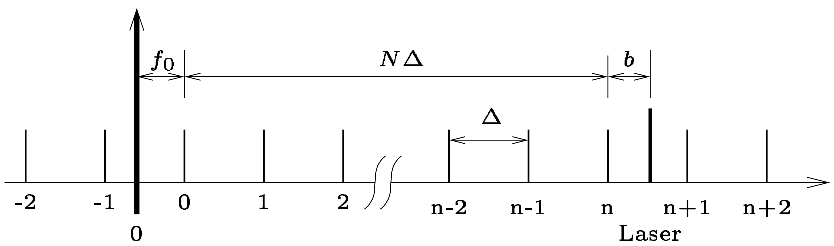
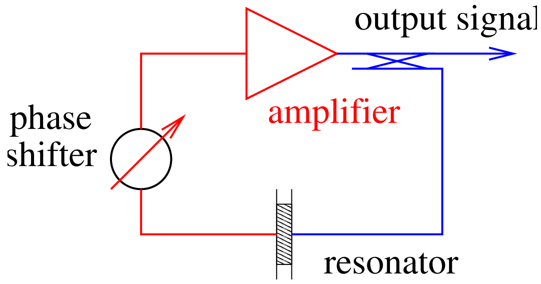
Example of frequency multiplication with a frequency comb



In the ideal case (no other noises) the noise of the measured signal $f_0 + b$ is affected by the noise of the synthesized signal $N\Delta$

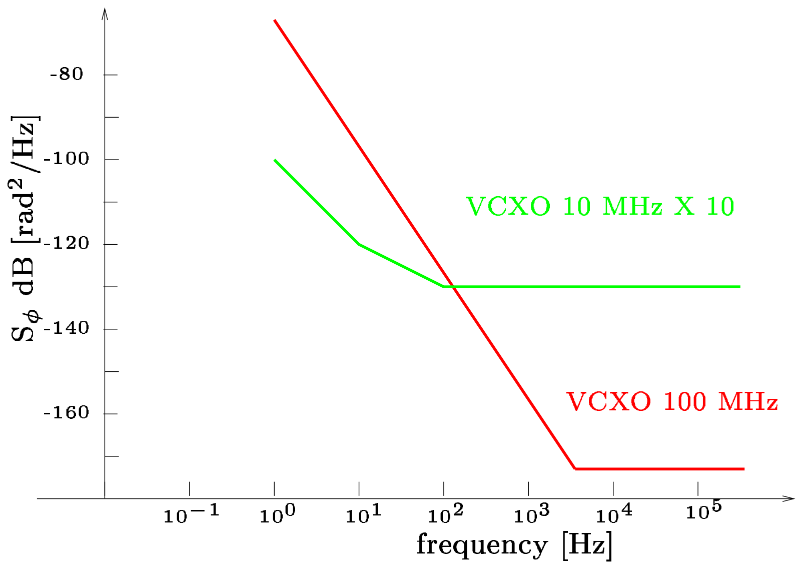
Leeson's model of feedback oscillator noise spectrum [Lee66]



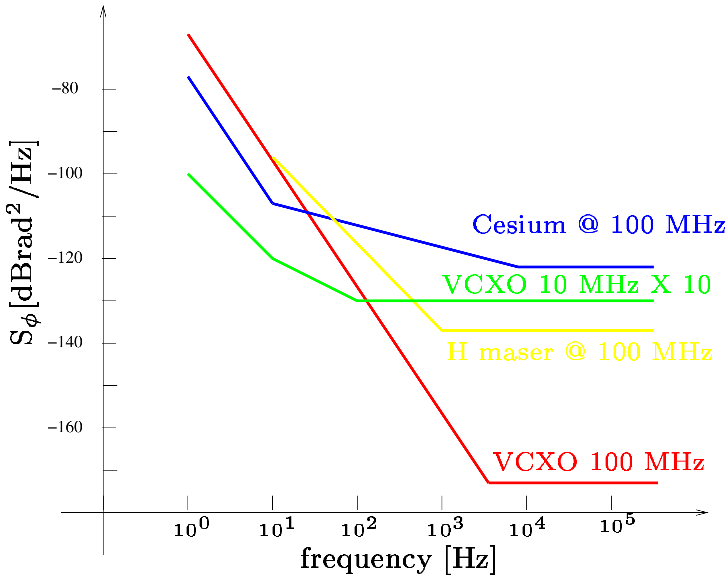
$$S_{\Delta\varphi} = bf^{-1} + \frac{Fk_BT}{P_S}; \qquad S_\varphi = S_{\Delta\varphi}\left[1+\left(\frac{f_0}{2Q_Lf}\right)^2\right]$$

$$S_\varphi = \left(\frac{f_0}{2Q_L}\right)^2 bf^{-3} + \left(\frac{f_0}{2Q_L}\right)^2 \frac{Fk_BT}{P_S} f^{-2} + bf^{-1} + \frac{Fk_BT}{P_S}$$

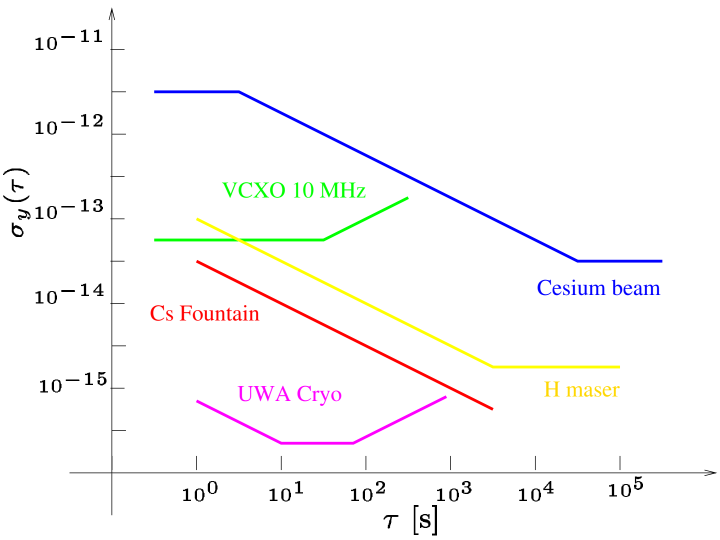
Typical phase noise spectrum of high performance quartz oscillators



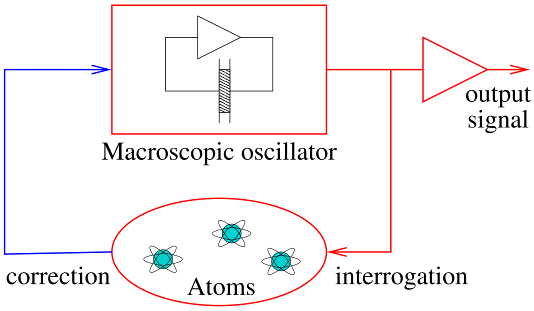
Typical phase noise spectrum of atomic frequency standards



Typical frequency (in)stability of microwave frequency standards

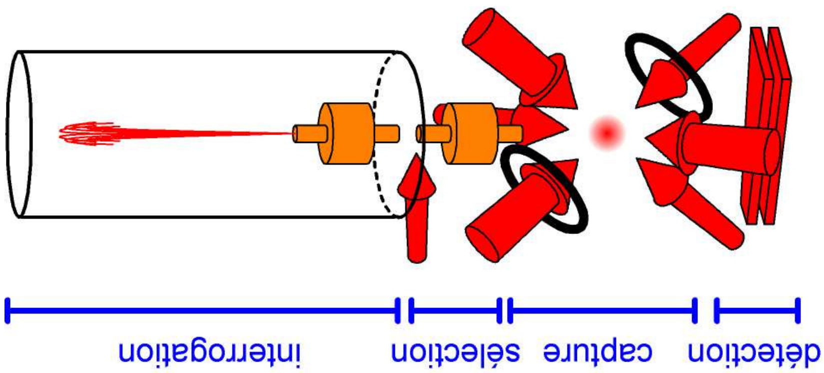


Passive atomic frequency standard

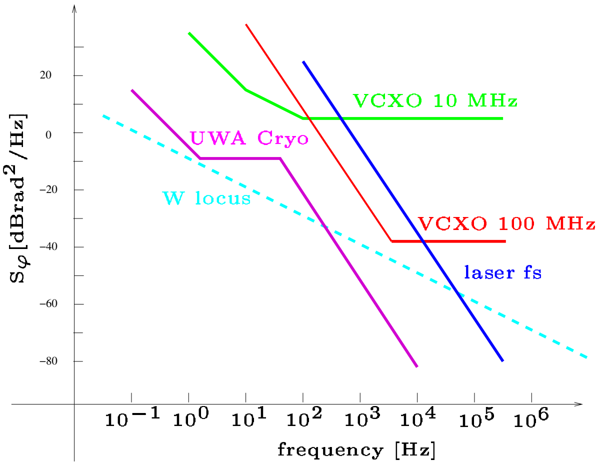


The short term stability is given by the macroscopic oscillator, the long term stability and the accuracy are due to the atoms.

Atomic fountain frequency standard

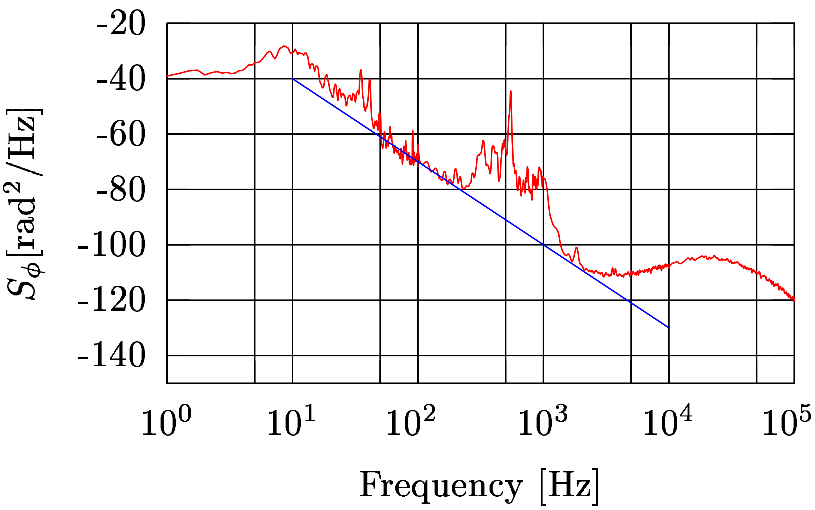


$S\varphi$ of some microwave oscillators multiplied to visible range.

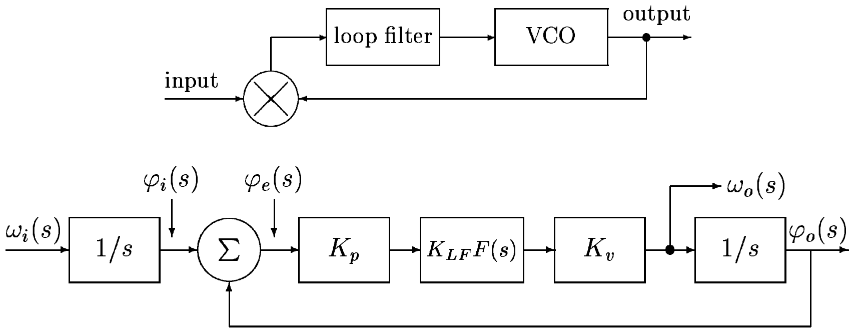


The dashed line can be used to infer empirically the fast linewidth of the multiplied signals.

Measured $S\varphi$ of the femtosecond repetition frequency



Phase noise filtering in the Fourier domain



$\varphi_i(s)$ $\varphi_o(s)$ $\varphi_e(s)$ = instantaneous phase of the input, output and error signal.

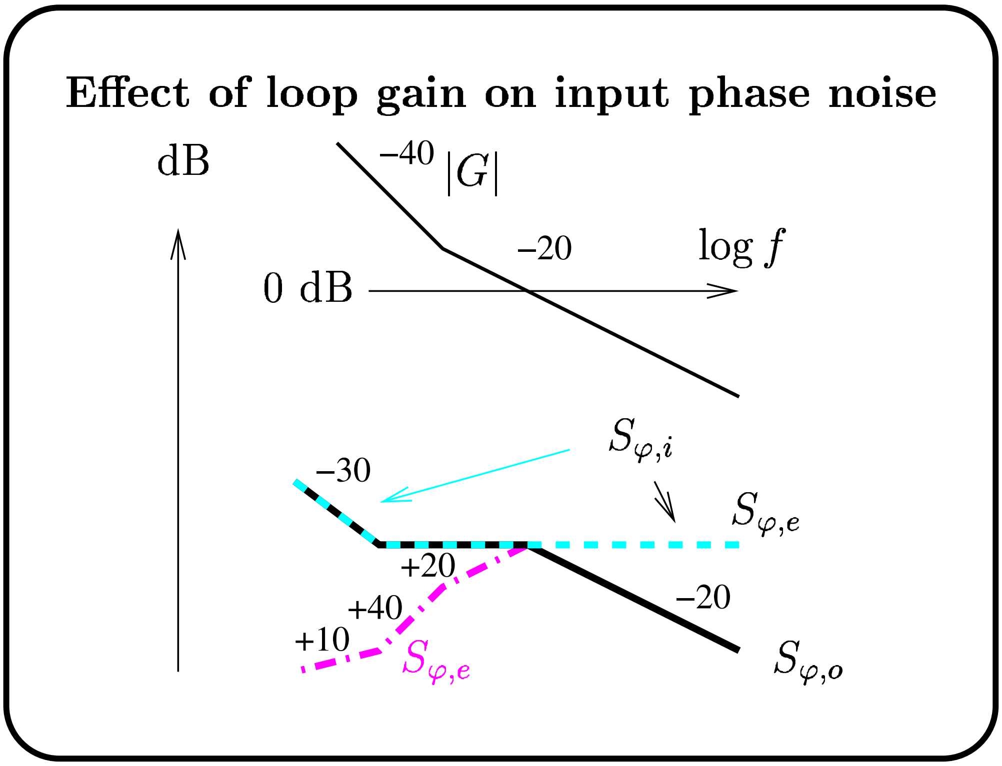
The block preceding $\varphi_i(s)$ establish the integral relationship $(1/s)$ between $\varphi_i(s)$ and the input frequency $\omega_i(s)$.

K_p = conversion factor of the phase detector K_{LF} = the gain of the loop filter with transfer function $F(s)$ K_v = sensitivity of the VCO.

Given $K = K_p K_{LF} Kv$ the open loop gain is: $G(s) = \frac{KF(s)}{s}$, and the closed-loop transfer function is:

$$H(s) = \frac{\varphi_i(s)}{\varphi_0(s)} = \frac{G(s)}{1 + G(s)} = \frac{1}{1 + s/KF(s)}$$

This demonstrate that, even in absence of a loop filter, there is always a true integration in the loop due to the conversion from frequency to phase.



BIPM, March 13, 2003 16-1

The first part of the talk is based on the article [RA00]. Many other information about frequency synthesis with old (chain) and new (fs-laser) techniques can be found in the same book. A reference to some recent result obtained in BNM-SYRTE with a fs-laser is also given [RDZ+02, CQK+03].

References

[CQK+03] Irène Courtillot, Audrey Quessada, Richard P. Kovacich, Anders Brusch, Dmitri Kolker, Jean-Jacques Zondy, Giovanni D. Rovera, and Lemonde Pierre. A clock transition for a future optical frequency standard with trapped atoms. *Phys. Rev. Lett.*, 2003. Submmitted on 6 March 2003.

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