

## Verifications for Multiple Solutions of Triaxial Earth Rotation

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**Abstract:** In this study, we provide several verifications on existence for multiple solutions of triaxial Earth rotation. In fact, the exact solution for Euler rotation equations of triaxial Earth case appears as elliptic functions of double periods in which the main term is Chandler wobble and the other is decadal polar motion of period 14.6 year. By quadratic polynomial approach, the triaxial condition of the principal moments of inertia satisfying inequality  $A < B < C$  simultaneously for the main inertia momentums guarantees that two stable solutions exist. A prime algorithm provides algebraic equations of two elliptic curves and a hyperbolic trajectory simultaneously so that shows the same consequence. As standard deduction for nonlinear differential equations, Jacobian matrix and eigen values indicate that there are two stable and an unstable solutions for rigid body rotation with true solution projection onto three principal planes. By observation data series of polar motion and LOD, filtering results illustrate explicit period of about 14 years. Phase portrait of the rotation model without solving equations is drawn with illustration of 2-torus graph in X—Y plane with spirals shows clearly existence of two periods and other two projections of phase portrait appear as explicit graphs of plane single pendulum with hyperbolic trajectories in it illustrating complex permanent consequence in multiple solutions of tentative trajectories.