Requirements of a User on the IERS Conventions Models

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Some mathematical equations used in geodesy

static methods	dynamic methods	
geometric relation	integral equation	differential equation
observation equations of GPS, SLR, VLBI,. . .	Integral equations Energy integral Hammerstein integral e.g. for gravity field parameters	post-Newtonian equation for satellite orbits variational equations wrt dynamical parameters
VLBI GPS mostly	GRACE gravity field estimation	SLR DORIS

Parameters which influence the shape of an orbit = **dynamic parameters**

SLR : orbit ⟶ variational equations

orbit ↓ observation equations

variational equations ↓ (partial derivatives) observation equations

The geometric models of observation equations

Definition : parameter vector : $\boldsymbol{p} \in \mathbb{R}^n$ with a-priori values $\boldsymbol{p}^o \in \mathbb{R}^n$,

unknown corrections : $\Delta\boldsymbol{p} = \boldsymbol{p} - \boldsymbol{p}^0$

observation vector : $\boldsymbol{q} \in \mathbb{R}^m$

model function : $\boldsymbol{f}(\boldsymbol{p}) \in \mathbb{R}^m$

Linearization of $\boldsymbol{f}$ in a neighborhood of $\boldsymbol{p}^o$ $\longrightarrow$ **linearized observation equations**

$$\underbrace{\mathrm{grad}_{\boldsymbol{p}}\,\boldsymbol{f}(\boldsymbol{p}_0)}_{\text{design matrix}}\,\Delta\boldsymbol{p} \;=\; \sum_{i=1}^{n}\frac{\partial\boldsymbol{f}}{\partial p_i}(\boldsymbol{p}_0)\cdot\Delta p_i \;=\; \underbrace{\boldsymbol{q} \;-\; \boldsymbol{f}(\boldsymbol{p}_0)}_{\text{observed - computed}}$$

$\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{x}_{\mathrm{sat}}, \dot{\boldsymbol{x}}_{\mathrm{sat}}, \boldsymbol{x}_{\mathrm{sta}}, \dot{\boldsymbol{x}}_{\mathrm{sta}}, t, p)$ with

$\boldsymbol{x}_{\mathrm{sat}}(t),\ \dot{\boldsymbol{x}}_{\mathrm{sat}}(t)$: position & velocity coordinates of one or more satellites,

$\boldsymbol{x}_{\mathrm{sta}}(t),\ \dot{\boldsymbol{x}}_{\mathrm{sta}}(t)$: position & velocity coordinates of one or more stations.

$$\frac{\partial\boldsymbol{f}}{\partial p_i} = \frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x}_{\mathrm{sat}}}\frac{\partial\boldsymbol{x}_{\mathrm{sat}}}{\partial p_i} + \frac{\partial\boldsymbol{f}}{\partial\dot{\boldsymbol{x}}_{\mathrm{sat}}}\frac{\partial\dot{\boldsymbol{x}}_{\mathrm{sat}}}{\partial p_i} + \frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x}_{\mathrm{sta}}}\frac{\partial\boldsymbol{x}_{\mathrm{sta}}}{\partial p_i} + \frac{\partial\boldsymbol{f}}{\partial\dot{\boldsymbol{x}}_{\mathrm{sta}}}\frac{\partial\dot{\boldsymbol{x}}_{\mathrm{sta}}}{\partial p_i} + \frac{\partial\boldsymbol{f}}{\partial p_i}$$

Résumé :

For the geometric or static conventional models contained in the observation equations, we require its function value and its parameter and time derivatives.

The dynamic models of orbit correction methods

Variational equations for the dynamical parameters

$$\frac{d^2}{dt^2}\left(\frac{\partial \boldsymbol{x}_{\mathsf{sat}}}{\partial \boldsymbol{p}}\right) = \frac{\partial \boldsymbol{a}}{\partial \boldsymbol{x}_{\mathsf{sat}}}\left(\frac{\partial \boldsymbol{x}_{\mathsf{sat}}}{\partial \boldsymbol{p}}\right) + \frac{\partial \boldsymbol{a}}{\partial \dot{\boldsymbol{x}}_{\mathsf{sat}}}\frac{d}{dt}\left(\frac{\partial \boldsymbol{x}_{\mathsf{sat}}}{\partial \boldsymbol{p}}\right) + \frac{\partial \boldsymbol{a}}{\partial \boldsymbol{p}},$$

with external acceleration

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}_{\mathsf{sat}}, \dot{\boldsymbol{x}}_{\mathsf{sat}}, t, \boldsymbol{p}), \quad \boldsymbol{a} = \sum_i \text{model } \boldsymbol{a}_i$$

The partials of the total acceleration,

$$\partial \boldsymbol{a}/\partial \boldsymbol{x}_{\mathsf{sat}}, \quad \partial \boldsymbol{a}/\partial \dot{\boldsymbol{x}}_{\mathsf{sat}}, \quad \partial \boldsymbol{a}/\partial \boldsymbol{p}$$

are made up of the partials of the models $\boldsymbol{a}_i$.

Résumé :

For the dynamic conventional models contained in the variational equations, we require its function value and its derivatives with respect to parameters, satellite position, and satellite velocity.

Overview over required models

physical source	geometric models	dynamic models
gravity	—	earth gravitational accel. lunar gravitational accel. planetary gravitational accel.
earth tides	earth surface displacement	gravitational acceleration
ocean tides	sea surface displacement ocean loading	gravitational acceleration
solar radiation	—	radiation drag (eclipses)
earth radiation	—	albedo and infrared radiation
atmosphere	atmospheric loading atmospheric refraction	air drag (density model)
earth rotation	pole and lod tide coord. transformation	pole and lod tide coord. transformation
observation	instrumental models	satellite orientation

red : not yet contained in the IERS Conventions.

Résumé :

The IERS Conventions models are specific for VLBI, GPS and partly SLR.
The Integration of DORIS, Altimetry, and other observation methods is suggested.

Variety of model representations

Gravitational acceleration due to ocean tides

- finite elements all over the ocean
- spherical harmonics

I favour to use both approaches in my software.

Choice of coordinates :

input in cartesian coordinates	$\longrightarrow$	calculations in spherical coordinates	$\longrightarrow$	output in cartesian coordinates

The code in spherical coordinates just as taken from the IERS Conventions :

C Correction for the out-of-phase part of Love numbers (imaginary part of
C $h_2^{(0)}$ and $l_2^{(0)}$) induced by mantle inelasticity in the semi-diurnal band.

$$r_{\mathsf{sta}} = \|\boldsymbol{x}_{\mathsf{sta}}\| \,; \quad r_{\mathtt{M}} = \|\boldsymbol{x}_{\mathtt{M}}\| \,; \quad r_{\mathtt{S}} = \|\boldsymbol{x}_{\mathtt{S}}\|$$

$$\sin\varphi = z_{\mathsf{sta}}/r_{\mathsf{sta}} \,; \qquad \cos\varphi = \sqrt{x_{\mathsf{sta}}^2 + y_{\mathsf{sta}}^2}/r_{\mathsf{sta}}$$

$$\sin\lambda = y_{\mathsf{sta}}/\cos\varphi/r_{\mathsf{sta}} \,; \qquad \cos\lambda = x_{\mathsf{sta}}/\cos\varphi/r_{\mathsf{sta}}$$

$$\sin 2\lambda = 2\cos\lambda\sin\lambda \,; \qquad \cos 2\lambda = \cos^2\lambda - \sin^2\lambda$$

$$\delta r_{\mathtt{S}} = -3/4\,\delta h^{\mathsf{im}} \cos^2\varphi\; F_{\mathtt{S}}\left((x_{\mathtt{S}}^2 - y_{\mathtt{S}}^2)\sin 2\lambda - 2x_{\mathtt{S}}y_{\mathtt{S}}\cos 2\lambda\right)/r_{\mathtt{S}}^2$$

$$\delta r_{\mathtt{M}} = -3/4\,\delta h^{\mathsf{im}} \cos^2\varphi\; F_{\mathtt{M}}\left((x_{\mathtt{M}}^2 - y_{\mathtt{M}}^2)\sin 2\lambda - 2x_{\mathtt{M}}y_{\mathtt{M}}\cos 2\lambda\right)/r_{\mathtt{M}}^2$$

$$\delta n_{\mathtt{S}} = 3/2\,\delta l^{\mathsf{im}} \sin\varphi\cos\varphi\; F_{\mathtt{S}}\left((x_{\mathtt{S}}^2 - y_{\mathtt{S}}^2)\sin 2\lambda - 2x_{\mathtt{S}}y_{\mathtt{S}}\cos 2\lambda\right)/r_{\mathtt{S}}^2$$

$$\delta n_{\mathtt{M}} = 3/2\,\delta l^{\mathsf{im}} \sin\varphi\cos\varphi\; F_{\mathtt{M}}\left((x_{\mathtt{M}}^2 - y_{\mathtt{M}}^2)\sin 2\lambda - 2x_{\mathtt{M}}y_{\mathtt{M}}\cos 2\lambda\right)/r_{\mathtt{M}}^2$$

$$\delta e_{\mathtt{S}} = -3/2\,\delta l^{\mathsf{im}} \cos\varphi\; F_{\mathtt{S}}\left((x_{\mathtt{S}}^2 - y_{\mathtt{S}}^2)\cos 2\lambda + 2x_{\mathtt{S}}y_{\mathtt{S}}\sin 2\lambda\right)/r_{\mathtt{S}}^2$$

$$\delta e_{\mathtt{M}} = -3/2\,\delta l^{\mathsf{im}} \cos\varphi\; F_{\mathtt{M}}\left((x_{\mathtt{M}}^2 - y_{\mathtt{M}}^2)\cos 2\lambda + 2x_{\mathtt{M}}y_{\mathtt{M}}\sin 2\lambda\right)/r_{\mathtt{M}}^2$$

$$\delta r = \delta r_{\mathtt{S}} + \delta r_{\mathtt{M}}$$

$$\delta n = \delta n_{\mathtt{S}} + \delta n_{\mathtt{M}}$$

$$\delta e = \delta e_{\mathtt{S}} + \delta e_{\mathtt{M}}$$

$$\Delta\boldsymbol{x}_{\mathsf{sta}} = \begin{bmatrix} \cos\lambda\cos\varphi & -\sin\lambda & -\sin\varphi\cos\lambda \\ \sin\lambda\cos\varphi & \cos\lambda & -\sin\varphi\sin\lambda \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix} \begin{bmatrix} \delta r \\ \delta n \\ \delta e \end{bmatrix}$$

The same code for normalized cartesian coordinates $\bar{x} = x/r$, $\bar{y} = y/r$, $\bar{z} = z/r$:

$$
\begin{aligned}
C_{22} &= 3\left(F_{\mathtt{S}}(\bar{x}_{\mathtt{S}}^2 - \bar{y}_{\mathtt{S}}^2) + F_{\mathtt{M}}(\bar{x}_{\mathtt{M}}^2 - \bar{y}_{\mathtt{M}}^2)\right) \\
S_{22} &= 6\left(F_{\mathtt{S}}\bar{x}_{\mathtt{S}}\bar{y}_{\mathtt{S}} + F_{\mathtt{M}}\bar{x}_{\mathtt{M}}\bar{y}_{\mathtt{M}}\right) \\
\delta r &= 0.25\left(C_{22}(2\bar{x}_{\mathsf{sta}}\bar{y}_{\mathsf{sta}}) - S_{22}(\bar{x}_{\mathsf{sta}}^2 - \bar{y}_{\mathsf{sta}}^2)\right) \\
\Delta x_{\mathsf{sta}} &= (2\,\delta l^{\mathsf{im}} - \delta h^{\mathsf{im}})\,\delta r\;\bar{x}_{\mathsf{sta}} - \;0.5\,\delta l^{\mathsf{im}}(C_{22}\bar{y}_{\mathsf{sta}} - S_{22}\bar{x}_{\mathsf{sta}}) \\
\Delta y_{\mathsf{sta}} &= (2\,\delta l^{\mathsf{im}} - \delta h^{\mathsf{im}})\,\delta r\;\bar{y}_{\mathsf{sta}} - \;0.5\,\delta l^{\mathsf{im}}(C_{22}\bar{x}_{\mathsf{sta}} + S_{22}\bar{y}_{\mathsf{sta}}) \\
\Delta z_{\mathsf{sta}} &= (2\,\delta l^{\mathsf{im}} - \delta h^{\mathsf{im}})\,\delta r\;\bar{z}_{\mathsf{sta}}
\end{aligned}
$$

Tidal deformation in spherical formulation

$$\begin{aligned}
\triangle x_{\mathsf{r}} &= \frac{h_2}{2} \cdot \frac{r}{g}\left[\frac{\partial}{\partial r} V_{\mathsf{tg}}(r,\varphi,\lambda)\right] &&= \frac{h_2}{g} \cdot V_{\mathsf{tg}}(r,\varphi,\lambda) \\
\triangle x_{\mathsf{n}} &= l_2 \cdot \frac{r}{g}\left[\frac{1}{r}\frac{\partial}{\partial \varphi} V_{\mathsf{tg}}(r,\varphi,\lambda)\right] &&= \frac{l_2}{g} \cdot \frac{\partial}{\partial \varphi} V_{\mathsf{tg}}(r,\varphi,\lambda) \\
\triangle x_{\mathsf{e}} &= l_2 \cdot \frac{r}{g}\left[\frac{1}{r\cos\varphi}\frac{\partial}{\partial \lambda} V_{\mathsf{tg}}(r,\varphi,\lambda)\right] &&= \frac{l_2}{g} \cdot \frac{1}{\cos\varphi}\frac{\partial}{\partial \lambda} V_{\mathsf{tg}}(r,\varphi,\lambda)
\end{aligned}$$

where V_{tg} = tide generating potential.

Its cartesian equivalent reads

$$\triangle \boldsymbol{x} = \frac{1}{g}\left[l_2 \cdot r\,\mathrm{grad}_{\boldsymbol{x}} V_{\mathsf{tg}} + \left(\frac{h_2}{2} - l_2\right)\frac{2V_{\mathsf{tg}}}{r} \cdot \boldsymbol{x} \right]$$

Résumé :

Alternative representations of models are desired.

Modularization

physical source	geometric models	dynamic models
earth tides	earth surface displacement	gravitational acceleration
ocean tides	sea surface displacement ocean loading	gravitational acceleration
solar radiation	—	radiation drag
earth radiation	—	albedo and infrared radiation
atmosphere	atmospheric loading atmospheric refraction	air drag
earth rotation	pole and lod tide coord. transformation	pole and lod tide coord. transformation

red : models that use astronomical arguments, either in form of the Delaunay arguments (l, l', F, D, Ω) or the Doodson arguments. Both forms should be equivalent.

Results for different versions of that code do not coincide.

Résumé :

The modularization of conventional software should be considered.

Unification of terminology

"Equatorial radius of the earth", sometimes $= R_\oplus$, sometimes $= a_\oplus$.

$$\text{Probably we have to distinguish between} \begin{cases} \text{semi-major axis of an reference ellipsoid} & = a_\oplus, \\ \text{model-dependent equatorial radius} & = R_\oplus. \end{cases}$$

$$R_1(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix}, \qquad R_2(\varphi) = \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix}, \qquad R_3(\varphi) = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Similarity transformation of coordinates with rotation $R(\alpha) = R_1(\alpha_1)R_2(\alpha_2)R_3(\alpha_3)$:

$$H(\boldsymbol{x},\eta) = (1+\mu)R(\alpha)\cdot\boldsymbol{x} + \boldsymbol{d}, \qquad \eta = \begin{pmatrix} \mu \\ \alpha \\ \boldsymbol{d} \end{pmatrix} \in \mathbb{R}^7 .$$

A Taylor expansion of $\eta \longmapsto H(\boldsymbol{x},\eta)$ in $\eta^o = 0$ reads

$$H(\boldsymbol{x},\eta^o+\delta\eta) = H(\boldsymbol{x},0) + H_\eta(\boldsymbol{x},0)\cdot\delta\eta + \mathcal{O}(\|\delta\eta\|^2) \qquad \text{for } \delta\eta \to 0$$

where

$$H(\boldsymbol{x},0) = \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \qquad H_\eta(\boldsymbol{x},0) = \begin{bmatrix} x_1 & 0 & -x_3 & x_2 & 1 & 0 & 0 \\ x_2 & x_3 & 0 & -x_1 & 0 & 1 & 0 \\ x_3 & -x_2 & x_1 & 0 & 0 & 0 & 1 \end{bmatrix},$$

and $\delta\eta = \left[\, \delta\mu \; \delta\alpha \; \delta\boldsymbol{d} \,\right]^T$.

Signs of the rotational part being opposed to those of chapter 4.
Does chapter 4 use other elementary rotations than chapter 5 ?

Résumé :

Work has to be done to arrive at an uniform terminology throughout the conventions. Often it is not thoroughly clear, if different symbols mean the same object.

The effect of solid earth pole tide on geopotential is given as

$$
\begin{aligned}
\Delta C_{21} &= -1.333 \times 10^{-9}(m_1 - 0.115\, m_2) \\
\Delta S_{21} &= -1.333 \times 10^{-9}(m_2 + 0.115\, m_1)
\end{aligned}
$$

Formula with theoretical quantities only, values given in a table below.

The desired form of the example above could read for instance

$$
\begin{aligned}
\triangle C_{20} &= -\frac{1}{3}\frac{\overline{\omega}^2 a^3}{GM_\oplus}\Big(\mathrm{Re}\, k_{20}\, 2m_3\Big) + \mathcal{O}(2) \\
\triangle C_{21} &= -\frac{1}{3}\frac{\overline{\omega}^2 a^3}{GM_\oplus}\Big(\mathrm{Re}\, k_{21}\, m_1 + \mathrm{Im}\, k_{21}\, m_2\Big) + \mathcal{O}(2) \\
\triangle S_{21} &= -\frac{1}{3}\frac{\overline{\omega}^2 a^3}{GM_\oplus}\Big(\mathrm{Re}\, k_{21}\, m_2 - \mathrm{Im}\, k_{21}\, m_1\Big) + \mathcal{O}(2)
\end{aligned}
\qquad \text{with} \quad
\left\{
\begin{aligned}
m_1 &= \ (\, x_p - \bar{x}_p \,) \\
m_2 &= -(\, y_p - \bar{y}_p \,) \\
m_3 &= \frac{\triangle\omega}{\overline{\omega}}
\end{aligned}
\right.
$$

Résumé :

Give numerical values only separated from the formulas.

Recommendations

- The integration of DORIS, altimetry and other observation methods is suggested.
- A lot of conventional models require derivatives with respect to time, position, and parameters.
- Admission of alternative representations of models.
- The modularization of conventional software should be considered.
- Work has to be done to arrive at an uniform terminology throughout the conventions.
- Give numerical values only separated from the formulas.

Forum on the web

Has already been installed on the Conventions pages.

Thanks to you.